

# POLITICS AS A (VERY) COMPLEX SYSTEM: A NEW METHODOLOGICAL APPROACH TO STUDYING FRAGMENTATION WITHIN A COUNCIL

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# Politics as a (very) complex system: A new methodological approach to studying fragmentation within a council

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## Abstract

The contemporary political landscape presents a complex and highly dynamic system characterized by fragile equilibria. We argue that while traditional cooperative game theory tools remain valuable, complex political processes need a more comprehensive analysis. By exploiting the natural isomorphism between simple cooperative games and hypergraph structures, we propose novel analytical frameworks for modeling and interpreting complex political scenarios. We apply hypergraph proper coloring, chromatic number analysis and a new measure of fragmentation to examine voting patterns within a council. We apply our analytical framework to the case of the United Nations Security Council. We formalize a persistent ideological division between Western and Non-Western member states, but we also reveal a certain fragmentation across years, in particular between Western states. This methodological approach offers promising insights for anticipating and interpreting future developments in complex political systems.

**Keywords:** Voting system, UNSC, Hypergraphs, Chromatic number, Fragmentation

**JEL:** C71, D72, D85

# 1 Introduction

Recent political developments, including Russia’s 2022 invasion of Ukraine, coupled with the new outbreak Israeli-Palestinian conflict that escalated in 2023, or the global shift towards the far right in recent political elections, have led to a stark change in global power dynamics. More recently, the election of Donald Trump as the 47th President of the United States, along with his domestic and international political stances and the resulting reactions from various countries, is reshaping the global balance in unprecedented ways. All these events highlight a far more dynamic and complex political scenario and power game than what traditional analytical techniques reveal. Never before in the last decades has the political landscape appeared as a complex and highly dynamic system with often fragile balances.

Cooperative game theory provides a robust framework for analyzing political scenarios, offering insights into how groups form, negotiate, and allocate resources. In political science, alliances—whether between nations, parties, or interest groups—often emerge from strategic cooperation where participants seek to maximize their individual or collective benefits. Cooperative game models help explain the stability of such alliances, the distribution of power among members, and the mechanisms that encourage or deter cooperation. Concepts such as coalition formation (Bloch, 1996; Ray, 2007), bargaining solutions (Nash, 1950; Chae and Heidhues, 2004), and power indices (Shapley and Shubik, 1954; Banzhaf, 1965) allow scholars to quantify and predict political dynamics in the voting process, from legislative negotiations to international treaties.

We argue that, in addition to traditional cooperative game theory tools, which analyze situations dictated by more static and structured voting and power share rules, politics necessitates a dynamic complexity framework for a deeper understanding. While this idea is not new—previous literature has explored cooperation, competition, and conflict in politics as complex systems (Cairney, 2012), or more in general has proposed complex theory applied to social science (Byrne, 2002; Byrne and Callaghan, 2022)—we suggest a more methodological approach. By exploiting the natural isomorphism between simple cooperative games and hypergraphs<sup>1</sup>—a generalization of graphs in which connection may be between more than two nodes—it is possible to bridge these two domains and develop novel analytical frameworks for the modeling and the comprehension of complex political scenarios. Our

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<sup>1</sup>More generally, the isomorphism applies to cooperative games and weighted hypergraphs as a whole. In this paper, however, we focus on the simpler subclass of simple cooperative games, which are most often applied to voting models.

analysis represents an initial step toward a broader, yet-to-be-formalized research agenda aimed at bridging the gap between these two literatures.

This paper specifically focuses on the concepts of proper coloring and of chromatic number, along with the newly defined concept of fragmentation of a hypergraph. Here, the chromatic number serves as an indicator of system complexity, with lower values indicating simpler, more cohesive structures and higher values suggesting more complex, potentially less stable configurations. In our application, the chromatic number—along with an investigation of the structure of the corresponding proper colorings—serves as a measure of fragmentation of a voting system.

We apply these measures to the United Nations Security Council (UNSC), where voting sessions provide a clear reflection of the evolving global political balance. Remarkably, 35 years after the historic fall of the Berlin Wall, our proper coloring and fragmentation analysis reveals that the fundamental ideological division between Western and Non-Western member states continues to shape international governance, despite the substantial geopolitical transformations that have occurred during this period. However, we highlight a greater level of cohesion among Non-Western countries and a persistent lack of unity within the Western bloc.

This methodological approach offers promising insights for anticipating and interpreting future developments in complex political systems. In cases where traditional cooperative game theory fails to fully account for the specific dynamics underlying recent political developments, we believe that complexity theory and its many tools can offer complementary and valuable insights.

## 2 Related work

Parallel to (cooperative) game theory, the study of graphs has emerged as a crucial tool for analyzing relationships and interactions. Graph studies<sup>2</sup> encompass community detection (Fortunato, 2010), diffusion (Kempe et al., 2003) and learning processes (Golub and Jackson, 2010), link formation (Bala and Goyal, 2000), and centrality measures (Bloch et al., 2023). These concepts, which originate in graph theory, have become central to fields such as computer science, economics, telecommunications, and social sciences.

The intersection of cooperative game theory and graph studies has led to significant advancements. Graph structures have been integrated into

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<sup>2</sup>For consistency with the terminology of this paper, we prefer the term “graph.” However, in the literature in many fields, the term “network studies” is more commonly used.

cooperative games to analyze cooperation constraints imposed by relational limitations among players (Myerson, 1977). Research has also explored how social networks shape individual and collective learning (Goyal, 2011). Furthermore, cooperative game concepts have been adapted to graph studies, leading to alternative centrality measures (Mazalov et al., 2016; Mazalov and Khitraya, 2021) and community detection methods (Avrachenkov et al., 2018).

Hypergraphs (Bretto, 2013) extend traditional graph models by allowing links to connect multiple nodes, providing a powerful framework for understanding complex interactions. For example, social interactions modeled as hypergraphs have proven effective in describing international trade agreements (Chessa et al., 2023) or influence in social networks (Zhu et al., 2018).

Recent research has developed a utility-based foundation for centrality measures using cooperative game theory (van Den Brink and Rusinowska, 2024). Exploiting the natural isomorphism between cooperative games and weighted hypergraphs, the authors also provide a foundation for the Shapley value—the most well-known solution concept in cooperative games—as a centrality measure for weighted hypergraphs. Their paper represents one of the rare attempts to transfer classic concepts from cooperative game theory to the study of (weighted) hypergraphs, thanks to the strong connection between the two frameworks. By contrast, our contribution moves in the opposite direction: it transfers concepts from hypergraph theory to the study and understanding of social interactions modeled as (simple) cooperative games—specifically, voting in a council.

Finally, scholars have occasionally explored the isomorphism between simple cooperative games and hypergraphs as an analytical tool for examining game structures. For instance, Axenovich and Roy (2010) employ the same hypergraph-related concepts we utilize—such as proper coloring and chromatic number—to analyze the structure of minimal winning coalition families in maximal and proper simple voting games. However, their approach merely adopts these mathematical tools to formalize proofs rather than developing substantive interpretations of these concepts. Our work extends beyond mathematical formalism to provide meaningful interpretations and applications of these theoretical constructs in voting system analysis.

## 3 Theoretical framework

### 3.1 Simple games

Cooperative game theory models situations where agents collaborate toward a common goal, addressing two fundamental questions: (1) Which coalitions will form? and (2) How will the generated benefits be distributed among coalition members? (Maschler, 1992).

Voting systems are usually represented in cooperative game theory as simple games, a special class of cooperative games with transferable utility. A *simple game* is a couple  $(N, v)$ , where  $N = \{1, \dots, n\}$  is the *set of players* and the *characteristic function* is binary-valued,  $v : 2^N \rightarrow \{0, 1\}$ . A coalition  $S \subseteq N$  is called *winning* if  $v(S) = 1$  and *losing* if  $v(S) = 0$ . A simple game is normalized such that the empty set is always losing ( $v(\emptyset) = 0$ ) and the grand coalition is always winning ( $v(N) = 1$ ). A simple game satisfies the property of *monotonicity*: if  $S$  is winning ( $v(S) = 1$ ), then any  $T \subseteq N$  such that  $S \subseteq T$  is also winning ( $v(T) = 1$ ). We denote by  $\mathcal{W}^m = \{S \subseteq N \mid v(S) = 1 \text{ and } v(T) = 0 \forall T \subset S\}$  the set of *minimal winning coalitions*. A simple game is uniquely characterized by its set of minimal winning coalitions, and it is often denoted as  $(N, \mathcal{W}^m)$ .

Given a simple game  $(N, \mathcal{W}^m)$ , a player  $i \in N$  is a *veto player* if  $i \in S$  for each  $S \in \mathcal{W}^m$ . A veto player belongs to every (minimal) winning coalition.

### 3.2 Hypergraphs

The concept of hypergraph is a natural generalization of the concept of graph. A *hypergraph* is a pair  $(N, H)$ , where  $N = \{1, \dots, n\}$  is a finite *set of nodes*, and  $H \subseteq \{T \subseteq 2^N\}$  is a *set of hyperlinks* between any set of nodes in  $N$  (Berge and Minieka, 1973).

Observe how a hyperlink structure is naturally seen as a collection of sets on  $N$ . Thus, there is a trivial correspondence between a simple game and an hypergraph, where the nodes are the players<sup>3</sup> and the hyperlinks are the minimal winning coalitions ( $\mathcal{W}^m \equiv H$ ). In this case, we refer to this hypergraph such as to the *formal rules hypergraph* of the voting system. An hypergraph can instead represents voting patterns. In this case, an hyperlink groups together those players that have casted the same vote at least once in a given period of time. We refer to this hypergraph as to the *voting behavior hypergraph*.

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<sup>3</sup>This explains why, for the sake of simplicity and with a slight abuse of notation, we denoted both the set of players and the set of nodes using the same notation.

A *proper coloring* of a hypergraph  $(N, H)$  is an assignment of colors to the vertices such that no hyperlink contains vertices all of the same color (Erdős and Hajnal, 1966). Formally, a proper coloring is a function  $c : N \rightarrow C$ , where  $C$  is the set of colors, such that  $\forall h \in H, \exists i, j \in h, i \neq j$ , such that  $c(i) \neq c(j)$ . Let  $\mathcal{C}$  be the set of proper colorings of hypergraph  $(N, H)$ . Without loss of generality, we denote the colors as the first  $K$  natural numbers, i.e.,  $C = \{1, \dots, K\}$ . The *chromatic number* of hypergraph  $(N, H)$ , denoted  $\chi(N, H)$ , is the minimum number of colors used by a proper coloring of  $(N, H)$ , thus

$$\chi(N, H) = \min\{k \in K \mid \exists c \in \mathcal{C}, c : N \rightarrow \{1, \dots, k\} \text{ s.t. } \forall h \in H, \exists i, j \in h, c(i) \neq c(j)\}.$$

Observe that given the chromatic number  $\chi(N, H) = k$ , there may exist many different proper colorings employing the given number of colors, that we call *proper  $k$ -colorings*. We denote as  $\mathcal{C}_k$  the set of proper  $k$ -colorings. To introduce the following analysis, we present a new definition.

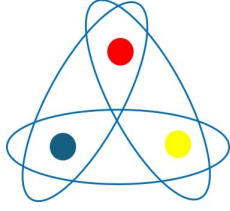
**Definition 1.** For a hypergraph  $(N, H)$  with chromatic number  $\chi(N, H) = 2$ , we define the fragmentation  $f(N, H)$  as follows. Among all proper 2-colorings, we consider, for each coloring, the minimum between the number of nodes assigned to each color. We then take the minimum of these values over all proper 2-colorings. Formally, ,

$$f(N, H) = \min_{c \in \mathcal{C}_2} \min\{|N_1^c|, |N_2^c|\}$$

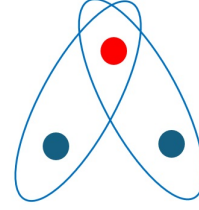
where given  $c \in \mathcal{C}_2$ ,  $\{N_1^c, N_2^c\}$  is the partition of  $N$  such that  $\forall i \in N_1^c, c(i) = 1$  and  $\forall j \in N_2^c, c(j) = 2$ .

### 3.3 Example

Consider the simple game  $(N, W^m)$ , with set of players  $N = \{1, 2, 3\}$ , and set of minimal winning coalitions  $W^m = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ , representing a 3-player voting situation under simple majority rule. The corresponding voting rule hypergraph has a chromatic number equal to 3. A proper coloring—i.e., an assignment of colors to the vertices such that no hyperlink contains vertices all of the same color—is only possible if each of the three nodes is assigned a different color, for instance red, blue, and yellow (see Figure 1-(a)). Now consider the game  $(N, \widetilde{W}^m)$  with  $N = \{1, 2, 3\}$  and  $\widetilde{W}^m = \{\{1, 2\}, \{1, 3\}\}$ , which represents a similar voting situation but where player 1 acts as veto player. The corresponding voting rule hypergraph has chromatic number equal to 2 and fragmentation equal to 1. A proper coloring is given by assigning the red color to the veto player 1, and the blue color to the other two nodes (see Figure 1-(b)).



(a) Simple majority rule.



(b) Simple majority rule with veto player.

Figure 1: Proper coloring of a hypergraph representing a 3-player voting situation under simple majority rule (a) and a 3-player voting situation under simple majority rule with a veto player.

## 4 Chromatic Number and Fragmentation as Structural Indicators of the UNSC

We apply the concepts of proper coloring, chromatic number and fragmentation to investigate the voting structure and the voting behavior of the United Nations Security Council (UNSC). The UNSC is the foremost international body responsible for the maintenance of international peace and security. It serves as one of the most well known examples in the voting literature (Felsenthal and Machover, 2014; Dreher et al., 2014).

While the voting system of the UNSC is straightforward to describe, its political dynamics in the voting process are particularly complex. The UNSC consists of 15 members: five permanent members (China, France, the Russian Federation, the United Kingdom, and the United States), and ten non-permanent members who are elected for two-year terms by the General Assembly, with their term ending after two years. At current date in March 2025, Algeria, Guyana, Republic of Korea, Sierra Leone, Slovenia (with term ending in 2025), and Denmark, Greece, Pakistan, Panama and Somalia (with term ending in 2026).<sup>4</sup>

According to Article 27 of the UN Charter, each member of the UNSC has one vote. Decisions are made by the affirmative vote of at least nine members. When the United Nations Charter was drafted, the founders envisioned that the five permanent members, due to their pivotal roles in the

<sup>4</sup>See <https://main.un.org/securitycouncil/en/content/current-members> for an updated list of UNSC members.



establishment of the United Nations, would continue to play a significant role in maintaining international peace and security. As a result, these five countries were granted special status not only as permanent members of the UNSC but also with a unique voting power known as the “right to veto” on non-procedural matters. Under this agreement, if any one of the five permanent members casts a negative vote, the resolution or decision cannot be approved. Therefore, decisions of the UNSC on all non-procedural matters require the affirmative votes of nine members, including the concurring votes of the permanent members.

The hypergraph representation is particularly valuable for the UNSC, as it can model both the formal rules (accounting for the veto power in non-procedural matters) and voting behavior patterns.

## 4.1 Formal rules hypergraphs

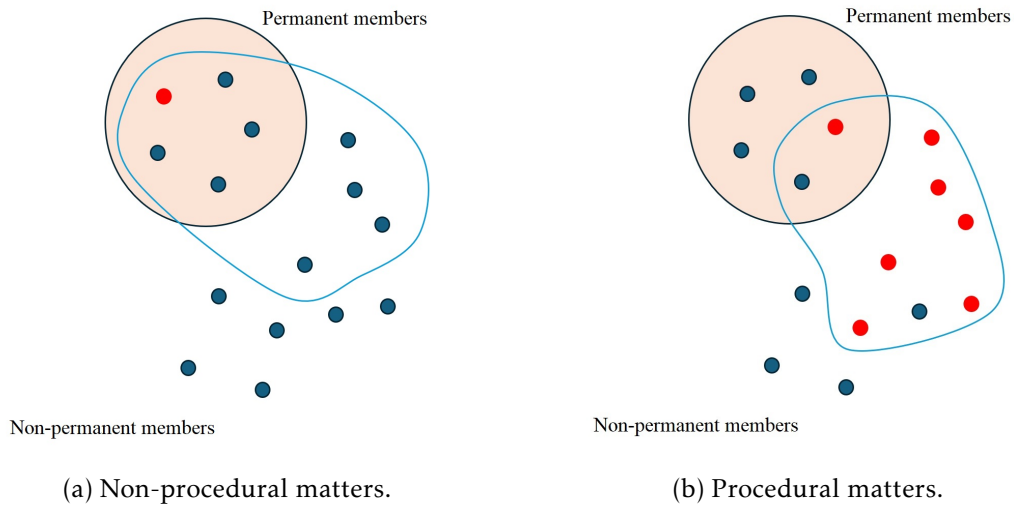


Figure 2: UNSC, example of hyperlink of the formal rules hypergraph for non-procedural matters (a), and for procedural matters (b), and examples of proper coloring.

In Figure 2-(a), we illustrate the structure of a hyperlink in the UNSC formal rules hypergraph for votes on non-procedural matters. A hyperlink contains the 5 permanent members and 4 out of the non-permanent members. Then, this hypergraph counts a total of  $\binom{10}{4} = 210$  hyperlinks. A proper coloring for this hypergraph can be achieved by assigning the red color to one of the permanent members, and the blue color to all other nodes. Thus,

the chromatic number of this hypergraph is equal to 2, and the fragmentation is equal to 1. Assigning the red color to one of the permanent members highlights the corresponding veto power: this member can be designed as representative of any winning coalition. Notably, this is not the only possible proper coloring. Since the permanent members are fully symmetric within the formal rules hypergraph, any of their corresponding nodes can be assigned alone the red color while still maintaining a proper coloring. More proper colorings which assign the red color to more nodes are also admitted. Assigning the red color to solely a non-permanent member, instead, would not provide a proper coloring.

In Figure 2-(b), we represent the structure of a hyperlink in the UNSC formal rules hypergraph when voting on procedural matters, where permanent members do not have a veto right. In this case, a hyperlink consists of nine nodes (as the number of the members necessary for the majority), for a total of  $\binom{15}{9} = 5005$  hyperlinks. A proper coloring can be achieved by coloring in red any seven nodes, without distinction between non-permanent and permanent members, and in blue the other eight nodes, thus ensuring that any group of nine members includes nodes of two different colors. Thus, the chromatic number of this hypergraph is equal to 2, and the fragmentation is equal to 7.

Both for non-procedural and for procedural matters, the chromatic number of the corresponding formal rule hypergraphs is 2. However, the higher number of nodes that need to be colored red in the hypergraph representing procedural decisions—i.e., the higher fragmentation of the corresponding hypergraph—is a clear indication of the variety in the structure of the hyperlinks and their intersections. This difference in fragmentation illustrates a key structural contrast between the two types of decision-making processes within the UNSC. In the hypergraph representing non-procedural matters, fragmentation is minimal because every winning coalition must include at least one permanent member, reflecting the strict constraint imposed by the veto power. This creates a centralized and highly cohesive structure. In contrast, the procedural hypergraph allows for much greater flexibility: any combination of nine members may form a winning coalition, regardless of permanent or non-permanent status. This results in higher fragmentation, as a larger number of nodes must be distinguished to ensure proper coloring. In essence, the lower fragmentation of the first hypergraph reflects a rigid, hierarchical system centered on veto power, while the higher fragmentation of the second captures the openness and inclusiveness of procedural voting, where all members stand on equal footing.

## 4.2 Voting behavior hypergraphs

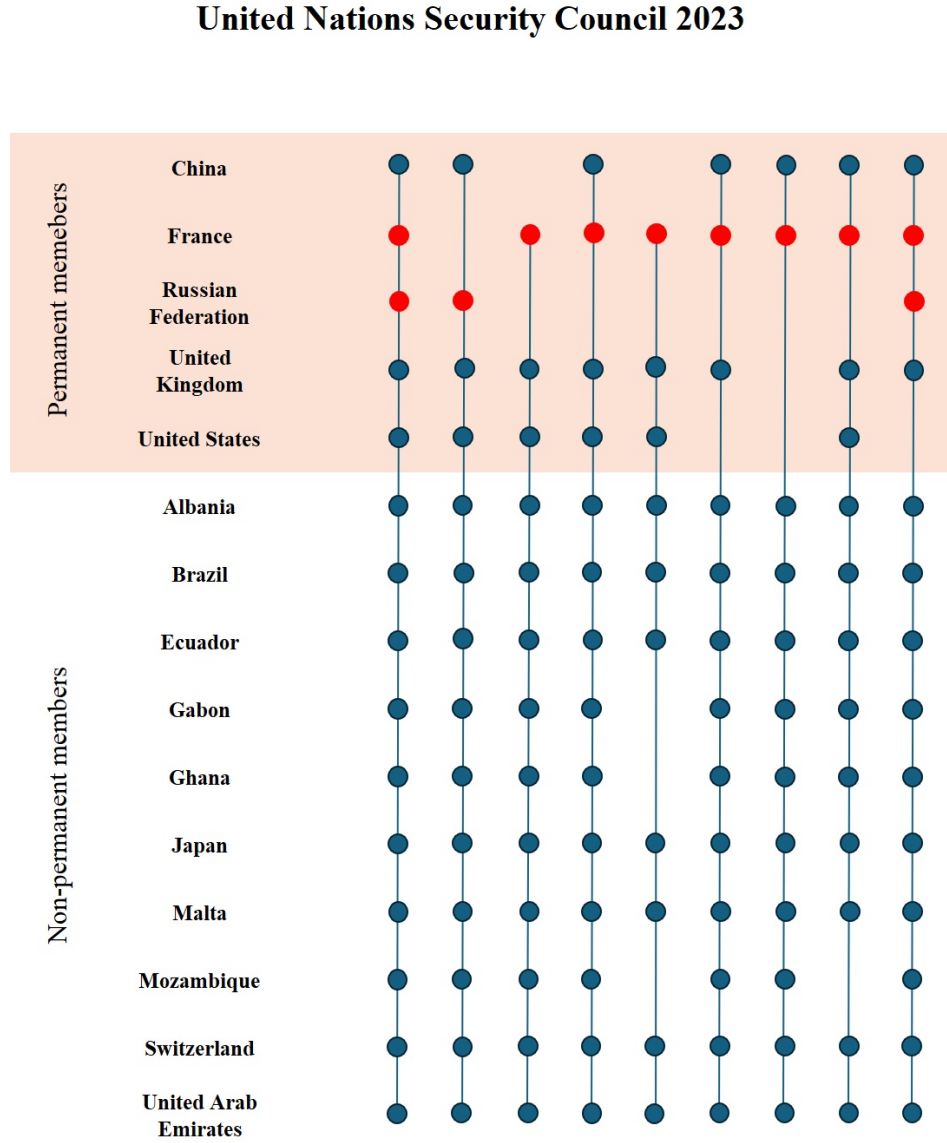


Figure 3: 2023 UNSC composition, voting behavior hypergraph and proper coloring.

Figures 2, 3, and 4 illustrate all hyperlinks in the voting behavior hypergraphs for 2023, 2024, and 2025 (up to March 17th), respectively. It is possible to observe how, for these hypergraphs, the number of hyperlinks

## United Nations Security Council 2024

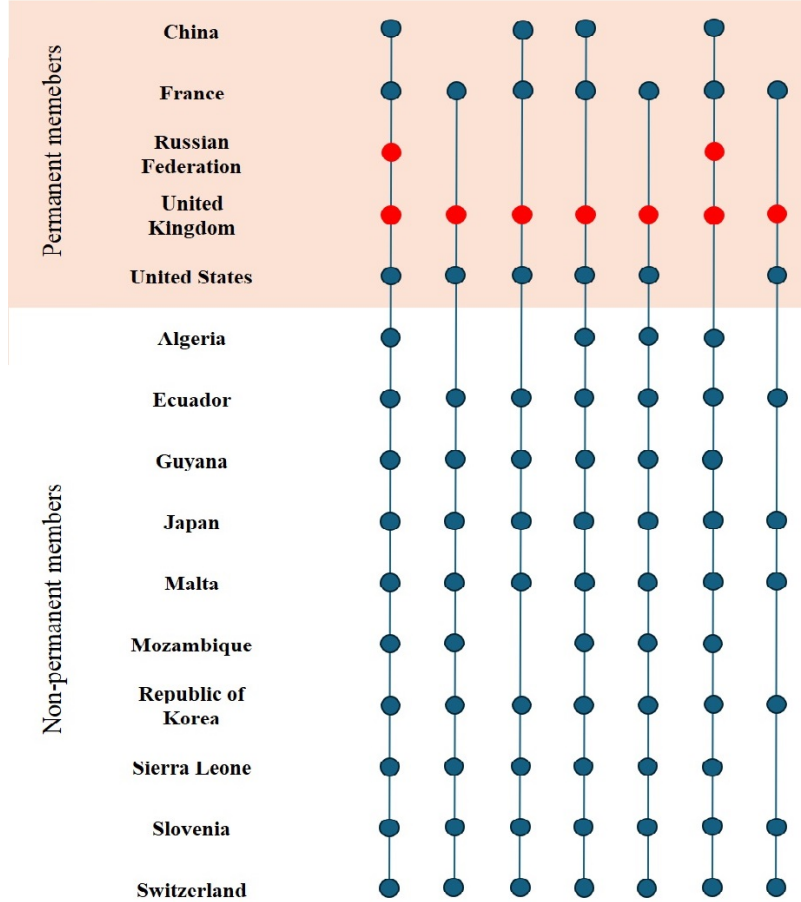


Figure 4: 2024 UNSC composition, voting behavior hypergraph and proper coloring.

is reduced to fewer than ten. A hyperlink is given by the nodes in a vertical line. Each hyperlink represents at least one recorded voting result in the corresponding year (more voting results may correspond to the same hyperlink). The nodes within a hyperlink correspond to members who voted in favor, while missing nodes represent members who either voted “no” or abstained. Observe that in the UNSC, abstention of permanent members for non-procedural matters is often adopted for signaling disapproval without

the diplomatic cost of an explicit veto. But abstention plays a crucial role in UNSC diplomacy both for permanent and non-permanent members, offering an intermediate option that allows resolutions to advance while documenting concerns and preserving diplomatic relationships. Both non-procedural and procedural voting results are presented together. Notably, our analysis focuses solely on voting behavior, without considering the final outcome of each session (i.e., whether the resolution was adopted or not).

Figures 2, 3, and 4 also visualize a proper coloring, ensuring that each hyperlink contains nodes of at least two different colors. In all three examples, it is possible to achieve a proper coloring using only two colors, meaning the chromatic number is 2. For example, in 2023, France and the Russian Federation are assigned the red color, while all other members are assigned the blue color. Since all voting results include at least one of France or the Russian Federation, as well as another member, in favor of the corresponding proposal, all hyperlinks contain both colors, making this a proper coloring. In voting behavior hypergraphs, our chosen proper coloring prioritize assigning the red color to permanent members, and it aims at assigning the red color to a minimum number of nodes. Thus, we can observe that all three hypergraphs have fragmentation equal to 2. To achieve a proper coloring, we consistently select one non-Western permanent member state (in all cases, the Russian Federation) and one Western permanent member state (either France or the United Kingdom). While alternative proper colorings exist, we observe that the structure between permanent members is no longer symmetric, as was the case in the formal rules hypergraph. In particular, selecting any two permanent members at random would not necessarily result in a valid proper coloring.

More broadly, in our examples, a proper coloring highlights representatives of specific partitions—countries that consistently vote in alignment with a particular bloc of states. The Non-Western member represents the Non-Western bloc, and the Western member represents Western countries. By assigning the red color to these two representatives, the proper coloring reveals the number of different blocs that emerge in the voting procedure, thereby emphasizing structural voting patterns and alliances within the UNSC. Specifically, two colored nodes in a proper coloring correspond to two distinct blocs. The proper colorings show that the UNSC’s voting behavior preserves a fundamental binary structure. The coloring reveals in fact a persistent ideological divide between Western and Non-Western member states: each voting result systematically sees the favor of either the Western red representative, the Non-Western red representative, or both. In fact, there is never a situation where both are absent.

## United Nations Security Council 2025

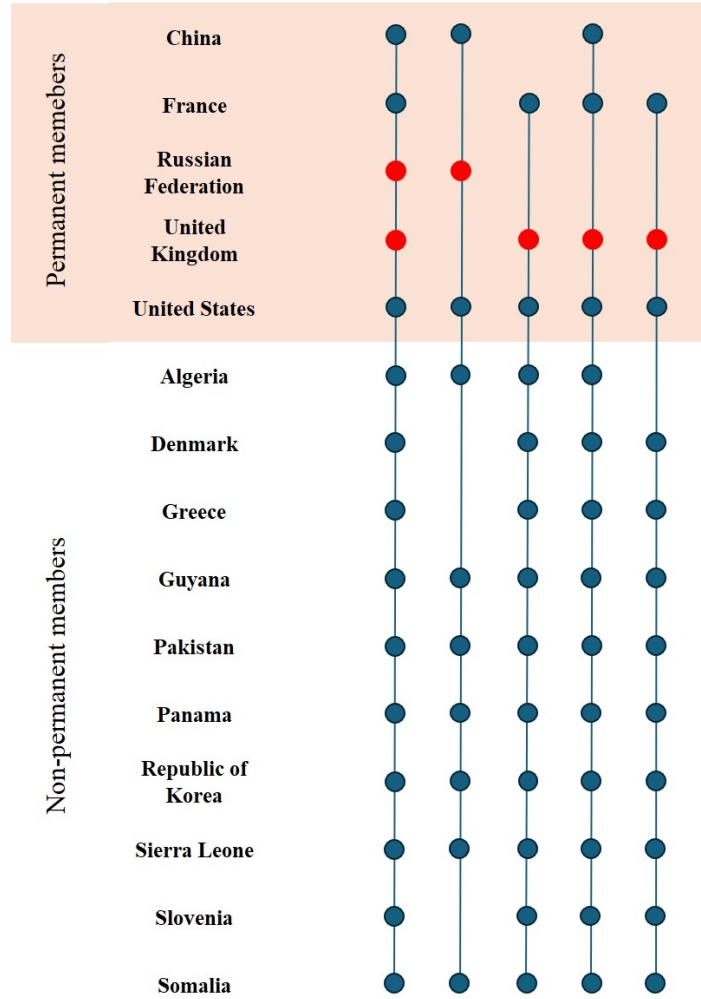


Figure 5: 2025 UNSC composition, voting behavior hypergraph and proper coloring. Available data till March 15, 2025.

As a final observation, it is interesting to note that in the three selected years, China could be colored in place of Russia, and this would have no effect. This interchangeability suggests a stable alignment between these two countries. In contrast, regarding the representative of the Western countries, it was not possible to consistently select the same representative among

the permanent members. Specifically, when merging data across the three years—and more generally, when analyzing voting behavior over a longer time span—we observe that achieving a proper coloring requires assigning the red color to three nodes: one among China and Russia, and two among the Western permanent members. As a result, the fragmentation of this merged hypergraph is equal to 3. This finding highlights a greater level of cohesion among non-Western countries, contrasted with a persistent lack of unity within the Western group.

Looking ahead, fragmentation may increase further. For instance, developments in 2025 or in the coming years could lead to a situation in which four or more nodes must be assigned the red color to maintain a proper coloring. Such an outcome would be a clear indication of growing discrepancies within the Western bloc, and as such, it constitutes a trend worth monitoring closely in the near future.

## 5 Conclusion

In this paper, we present an innovative methodological approach that apply hypergraph theory to analyze voting systems. By applying this mathematical framework to the United Nations Security Council (UNSC), we demonstrate how formal concepts from graph theory can illuminate complex voting dynamics that would be difficult to capture using traditional methods. Our methodology stands out because, while some of the voting dynamics we have discussed are already known through empirical observation, the theory of proper coloring and chromatic number provides a rigorous formal framework that enables us to: (1) Precisely quantify the degree of fragmentation within a council; (2) Identify emerging voting blocs not immediately evident through conventional analysis; (3) Track the temporal evolution of coalitions through consistent mathematical metrics; (4) Predict potential future configurations based on structural constraints of the system.

This mathematical framework allows for objective comparisons across different historical periods and between different voting systems. The implications of this approach extend well beyond descriptive analysis. The concept of chromatic number and the newly defined concept of fragmentation, when systematically applied to historical UNSC data from its foundation to the present, can reveal evolutionary patterns in international relations that are difficult to identify using other methods. This time-based perspective is particularly valuable in light of recent geopolitical events, which have significantly altered established equilibria in the international system.

In the longer term, the systematic integration between cooperative game theory and weighted hypergraphs represents a particularly promising research frontier. Exploring the interface between these disciplines offers the opportunity to develop more sophisticated analytical tools for understanding collective decision-making processes in strategic contexts. This interdisciplinary approach could not only advance the theoretical understanding of voting systems but also inform the design of more effective and representative decision-making mechanisms in contemporary international institutions.



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