THE TRIPLE-STORE EXPERIMENT: A FIRST SIMULTANEOUS TEST OF CLASSICAL AND QUANTUM PROBABILITIES IN CHOICE OVER MENUS

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The triple-store experiment: A first simultaneous test of classical and quantum probabilities in choice over menus.

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Abstract

Recently quantum probability theory started to be actively used in studies of human decision-making, in particular for the resolution of paradoxes (such as the Allais, Ellsberg, and Machina paradoxes). Previous studies were based on a cognitive metaphor of the quantum double-slit experiment - the basic quantum interference experiment. In this paper, we report on an economics experiment based on a three-slit experiment design, where the slits are menus of alternatives from which one can choose. The test of nonclassicality is based on the Sorkin equality (which was only recently tested in quantum physics). Each alternative is a voucher to buy products in one or more stores. The alternatives are obtained from all disjunctions including one, two or three stores. The participants have to reveal the amount for which they are willing to sell the chosen voucher. Interference terms are computed by comparing the willingness to sell a voucher built as a disjunction of stores and the willingness

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to sell the vouchers corresponding to the singleton stores. These willingness to sell amounts are used to estimate probabilities and to test both the law of total probabilities and the Born Rule. Results reject neither classical nor quantum probability. We discuss this initial experiment and our results and provide guidelines for future studies.

1 Introduction

Quantum decision theory is a field that has considerably expanded over the last fifteen years, supported by psychologists, physicists, mathematicians, economists and philosophers (see Busemeyer and Bruza, 2012; Haven and Khrennikov, 2013; Pothos and Busemeyer, 2013; Ashtiani and Azgomi, 2015; Bruza et al., 2015; Yearsley and Busemeyer, 2016, for reviews and explanations). In contrast to classical models of judgment, which are based on Bayesian probabilities and which state that agents exhibit defined preferences, quantum cognition models employ an innovative mathematical formalism, applied in quantum mechanics, in which individuals' preferences and beliefs are dependent on the context and the order of information. These models are mathematically sophisticated and can deal with numerous well-established paradoxes, including order effects (e.g., the answers to two questions depend on the order in which they are asked) and the conjunction fallacy (e.g., the fact that Linda is both a feminist and a bank employee is incorrectly considered more likely than just the fact that Linda is a bank employee). These theoretical models have grown rapidly and in many interdisciplinary fields (decision theory, game theory, cognitive biases and financial market prediction models). They are promising because they enable several paradoxical behaviours to be accounted for within the same mathematical framework, without the need to resort to ad-hoc models, thus paving the way for a more generic theory on human behaviour. For example, Wang and Busemeyer (2013) and Wang et al. (2014) have developed a quantum model for attitude questions in surveys. Conte et al. (2009) presented a model for mental states of visual perception, and Atmanspacher and Römer (2012) discussed non-commutativity. In addition, quantum models have been developed to account for the conjunction fallacy (Franco, 2009; Busemeyer et al., 2011, 2015; Busemeyer and Bruza, 2012; Pothos and Busemeyer, 2013; Yukalov and Sornette, 2010, 2011; Yearsley and Trueblood, 2018) and the violation of the sure-thing principle (Busemeyer et al., 2006; Busemeyer and Wang, 2007). Aerts and Sozzo (2011), al Nowaihi and Dhami (2017), Aerts et al. (2018), and Eichberger and Pirner (2018) modelled the famous Ellsberg paradox, while Khrennikov and Haven (2009),Yukalov and Sornette (2010), and Aerts et al. (2011) discussed the Allais paradox. In a more general framework, models of decision theory and bounded rationality have been introduced (Danilov and Lambert-Mogiliansky, 2008, 2010; Mogiliansky et al., 2009; Yukalov and Sornette, 2011; Asano et al., 2017; Basieva et al., 2018; Pisano and Sozzo, 2020). Finally, in game theory, quantum models that account for paradoxical strategies have also emerged (Piotrowski and Sładkowski, 2003; Landsburg, 2004; Pothos and Busemeyer, 2009; Brandenburger, 2010; Piotrowski and Sładkowski, 2017; Denolf et al., 2017; Khrennikov and Basieva, 2014a; Khrennikov, 2015).

Despite the explanatory power of these models, it is important to investigate the theoretical constraints underlying the quantum formalism and then compare them with empirical data. This is absolutely required to determine whether quantum cognition models can be justified and to evaluate to what extent they can be used as descriptive or predictive models. In this regard, Boyer-Kassem et al. (2016b) studied non-degenerate quantum models of order effects (Conte et al., 2009; Busemeyer and Bruza, 2012; Pothos and Busemeyer, 2013; Wang and Busemeyer, 2013; Wang et al., 2014), and derived mathematical constraints called Grand Reciprocity (GR) equations from the law of reciprocity. To be acceptable, models must respect these constraints. The authors then analyzed large empirical data sets and observed that the non-degenerate models did not satisfy these equations. Boyer-Kassem et al. (2016a) and Duchêne et al. (2017) investigated the empirical validity of a large proportion of the non-degenerate and degenerate quantum models of the conjunction fallacy (Franco, 2009; Busemeyer et al., 2011, 2015; Busemeyer and Bruza, 2012; Pothos and Busemeyer, 2013) and proposed several tests. The authors then conducted different types of experiments, with or without monetary incentives and on computer or on paper, in different universities and subsequently compared the experimental data with these tests. The results of Boyer-Kassem et al. (2016a) indicate that some of these models, exploiting order effects, do not meet these tests. The debate about whether these models fail to account for the conjunction fallacy is still an open issue with pros and cons arguments. In view of the limitations of standard quantum models, new promising directions have been proposed with generalized observables or a more general measurement theory; for example based on positive operator-valued measures (POVMs) and quantum instruments (Khrennikov and Basieva, 2014b; Khrennikov et al., 2014; Ozawa and Khrennikov, 2021; Busemeyer and Wang, 2019), or on the modification of the Born rule (Aerts and de Bianchi, 2017). In that direction, the Busemeyer et al. (2011) quantum model for the conjunction fallacy has been updated using POVM's, instead of projectors (Miyadera and Philips, 2012). Aerts (2009) also offered an interesting alternative approach — with respect to most of the models — and does not require any order effects. In this crucial debate on the extension of quantum models, it is therefore central for the scientific community to continue the exploration and evaluation of the empirical robustness of these models.

A particularly intriguing opportunity to test simultaneously the empirical validity of both the classical and quantum probabilistic frameworks comes from Sorkin's equalities (Sorkin, 1994). Those mathematical conditions are well-represented by a three-slit extension of Young's famous double-slit experiment studying the interference patterns of light waves. Sorkin theoretically derived, from the framework of quantum mechanics, that the interference pattern formed in a multi-slit experiment is a simple combination of patterns observed in two-slit experiments. Therefore, in an observed three-slit physical system, a violation of quantum mechanics can be identified if interference terms of a higher order than the two-slit term are measured experimentally. Similarly, classical probability (CP) is violated if interference terms of any order are measured. Indeed, the intriguing idea is to use a three-slit experiment and the derived Sorkin's equality to measure and interpret potential deviations from either Kolmogorov' axioms or, at a higher order, Born's rule and the superposition principle (Sinha et al., 2015; Skagerstam, 2018). In recent years, the multi-slit modelling framework has gained momentum in the physics literature. It has been exploited to run precise experiments for testing quantum properties of several physical systems such as photons, nuclear spins, nitrogen-vacancy centres in diamond, and large molecules and atoms (Sinha et al., 2010; Söllner et al., 2012; Park et al., 2012; Gagnon et al., 2014; Kauten et al., 2014, 2017; Cotter et al., 2017). Inspired by this literature, we conducted an experiment with human participants to investigate Sorkin's equality in the context of quantum cognition (Basieva and Khrennikov, 2017). As far as we know, this is the first scientific contribution that tests the validity of quantum probabilities (QP) in human decision-making by adopting the three-slit framework.

In the next section, we present the Sorkin's equality and its interpretations in physics and decision theory. Then, we propose and describe the triple store experiment in Section 3 and in Section 4 we detail its implementation. Section 5 highlights the main results and the statistical analysis. The last section concludes by discussing the results and potential further investigations.

2 The Sorkin Equality and its relevance in physics and decision theory

Let's consider three mutually exclusive events A_1, A_2 and A_3, and an event B. We define the following disjunctive events: $A_{1,2} = (A_1 \cup A_2)$, $A_{1,3} = (A_1 \cup A_3)$, $A_{2,3} = (A_2 \cup A_3)$, and $A_{1,2,3} = (A_1 \cup A_2 \cup A_3)$.

Usually, in physics, we consider a set of experiments where photons are projected onto a screen and the number of slits they can go through vary between one, two and three. In this framework, the random variable A represents the set of slits the photons went through (with probability $p(A)$), and the random variable B represents the final location of a photon on the screen (see Figure 1).

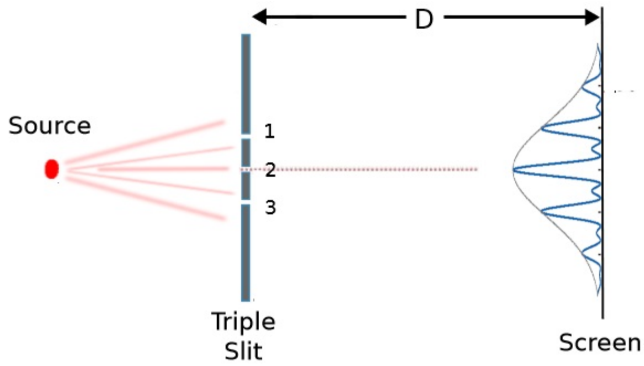


Figure 1: Illustration of three slits interference experiment. When all the slits are open, there is three possible path for the photon. (Qureshi, 2019)

In decision theory, the events A could represent the act of choosing some alternatives or menus from a choice set, and the event B could be any other decision following the first decision A (e.g. choosing a particular alternative within a selected menu A, choosing to sell the selected menu A for a certain amount, etc.).

In classical probability (CP), the law of total probability (LTP) states that:

$$p(A_{1,2})p(B|A_{1,2}) = p(A_1)p(B|A_1) + p(A_2)p(B|A_2) \tag{1}$$

$$p(A_{1,3})p(B|A_{1,3}) = p(A_1)p(B|A_1) + p(A_3)p(B|A_3) \tag{2}$$

$$p(A_{2,3})p(B|A_{2,3}) = p(A_2)p(B|A_2) + p(A_3)p(B|A_3) \tag{3}$$

$$p(A_{1,2,3})p(B|A_{1,2,3}) = p(A_1)p(B|A_1) + p(A_2)p(B|A_2) + p(A_3)p(B|A_3) \tag{4}$$

In physics, if photons were following the classical law of probability, the probability to observe a photon in location B, when the two slits are opened, should be equal to the sum of the probabilities of observing the photon in B after going through each slit, weighted by the probability that the photon went through the considered slit. We now address the question of the relevance of such constraints in decision theory, when events A and B represent choices. The LTP states that the probability to observe the choice B after observing the choice $A_{12} = A_1 \cup A_2$, when A_1 and A_2 are disjoint, equals the probability to observe the choice B after the choice A_1 has been observed plus the probability to observe the choice B after the choice A_2 has been observed. We shall make two observations. First, many papers studying random choice define *random choice rules* as a probability measure on the σ-algebra on the set of available alternatives (see e.g. Lu, 2016; Gul and Pesendorfer, 2006). In those conditions, the LTP implicitly holds by construction of the choice problem, above any consideration of rationalization.[1] In any case, as long as we assume that only singleton choices are meaningful and observable, and that the meaning of observing A_{12} (e.g. "the client has chosen coffee or tea") is reduced to observing A_1 ("the client has chosen coffee") or observing A_2 ("the client has chosen tea"), then the LTP will hold trivially.

On the opposite, there is a literature on choice over menus, that directly deals with an observable choice of a set of alternatives. The first paper on the topic (Kreps, 1979), studying deterministic menu choices, highlights that a rational decision maker should be indifferent between the menu A_{12} and the preferred menu between A_1 and A_2, if she knows perfectly her meal preferences and what her subsequent meal choice will be in any menu. On the other side, she should exhibit a preference for flexibility ($A_{12} \succ \max\{A_1, A_2\}$), as long as she is uncertain about her future tastes. Recently, Heydari (2020) extended Kreps' model to the stochastic case and modeled decision makers who choose the menu providing them the higher random utility, defined as the maximum (random) utility of its elements. It is thus straightforward that LTP holds at the aggregate level for heterogeneous decision makers without preferences for flexibility, or for an individual who makes decisions a la Heydari (2020). However, this may not be the case for decision makers with preferences for flexibility (Kreps, 1979; Ahn and Sarver, 2013), or with temptation and self-control issues (Gul and Pesendorfer, 2001).

Quantum probabilities (QP), as an extension of classical probabilities, allow for some violation of the LTP. In the physical 3-slit experiment, when more than one slit is open and the photon

[1] Other papers (see e.g. Manzini and Mariotti, 2014) define random choice rules over the set of alternatives as follows: $p : X \times \mathcal{D} \to [0, 1]$ where X is the set of available alternatives and $\mathcal{D}$ a domain of menus (i.e. $\mathcal{D}$ a set of subsets of X) with p(a,A)=0 if a $\notin A$ and $\sum_{a \in A} p(a, A) = 1$. In those conditions, random choice rules, while being interpreted as choice probability by the authors, are not genuine probability measures.

path in the opened slits is not measured before the final location, one could observed non-null "interference terms", as a consequence of QP's non-commutativity:

$$I_{12} = p(A_{1,2})p(B|A_{1,2}) - p(A_1)p(B|A_1) - p(A_2)p(B|A_2) \neq 0 \tag{5}$$

$$I_{13} = p(A_{1,3})p(B|A_{1,3}) - p(A_1)p(B|A_1) - p(A_3)p(B|A_3) \neq 0 \tag{6}$$

$$I_{23} = p(A_{2,3})p(B|A_{2,3}) - p(A_2)p(B|A_2) - p(A_3)p(B|A_3) \neq 0 \tag{7}$$

$$I_{123} = p(A_{1,2,3})p(B|A_{1,2,3}) - p(A_1)p(B|A_1) - p(A_2)p(B|A_2) - p(A_3)p(B|A_3) \neq 0 \tag{8}$$

Interference is a complex concept to interpret in physics (as quantum mechanics is in general). However we can provide different explanations of non-null interference terms in decision theory. In general, if we consider B as the certain event ($p(B) = 1$), a positive interference term implies that the likelihood of choosing a disjunction of two menus is higher than the sum of the likelihood of choosing each menu separately, exhibiting preferences for flexibility (Kreps, 1979). Conversely, a negative interference term means that decision makers prefer smaller menu, which can be the expression of self-control over temptation issues (Gul and Pesendorfer, 2001). If we consider the B as the choice of a final alternative within a previously chosen menu, interference terms can also encompass context effects, as the attraction or compromise effect. For example, let assume that A_1 and A_2 are menus for a conference diner that have to be chosen at the conference registration: A_1=$\{cake^*, banana\}$, A_2=$\{cake^-, apple\}$, where $B = \{cake^*\}$ corresponds to choosing ultimately cake* within the menu, finally $cake^-$ is (unequivocally) less good than $cake^*$. Assume that the decision maker always prefers A_1 to A_2 (and we have $p(A_{12}) = p(A_1) = 1$). Then positivity of I_{12} means that $p(cake^*, \{cake^*, cake^-, banane, apple\}) > p(cake^*, \{cake^*, banane\})$, which is an expression of the attraction effect (see e.g. Castillo, 2020, for a recent review on the attraction effect).

Therefore, QP allows naturally for violation of the LTP and offers a unified and single framework to explain a large set of documented behaviors. Nevertheless, QP remains a testable and refutable framework. In particular, if I_{123} can be non-null in QP, the Born Rule only holds if the following equality is satisfied:[2]

$$I_{123} = I_{12} + I_{13} + I_{23} \tag{9}$$

[2]See Basieva and Khrennikov (2017) for the demonstration.

That is the 3^{rd} order interference term should be equal to the sum of the second order interference terms. Therefore, testing this equality could provide information to assess if the observed behaviors can be modeled by quantum probabilities, or not. Therefore, in the future, this equation could potentially be used by decision theorists, as a novel technical axiom to characterize choices revealing “quantum-like” preferences.

3 The triple-store experiment

The aim of this paper is to propose an experiment that measures the interference terms I_{12}, I_{13}, I_{23} and I_{123}. For this purpose, one needs to conceptually define unambiguously “choosing a disjunction of mutually exclusive alternatives” — for example $(A_1 \cup A_2)$. We propose that such choice is the decision to renounce all the other available options except A_1 and A_2, while the uncertainty in the choice between those two options has not necessarily been resolved (both for the decision-maker and for the observer). In this line, Basieva and Khrennikov (2017) proposed a hypothetical framework where participants were asked to choose among “pairs of countries” to emigrate to (e.g., “I would prefer to emigrate to Brazil or Canada instead of Australia”), followed by a measurement B, consisting in asking whether the participant would consider changing profession if emigrating to the chosen country (or pair of countries). The use of the preposition “or” in this hypothetical design is convenient for the experimenter, who can easily create choices between any disjunction of mutually exclusive options, and presents the advantage that the uncertainty between those mutually exclusive options never needs to be resolved, since the decision does not need to be implemented. However, it is impossible to be sure that the participant is interpreting the “or” as a proper mathematical disjunction (e.g., the participant may think that she is choosing a lottery of countries with unknown probabilities). Moreover, from a practical point of view, rejecting CP or QP only with hypothetical choices is not fully satisfying. Indeed, we can not derive much knowledge of human behaviours if anomalies are only observed in hypothetical and unrealistic situations. Therefore, we propose to design an experimental protocol involving choices with real and incentive-compatible consequences, based on the disjunction of mutually exclusive options which are tangible and unambiguous.

As a typical tangible choice between the disjunction of mutually exclusive options, we propose a choice between vouchers that are defined by *the set of stores in which they are available to be used*. Ultimately, each voucher can be used only once, and in only one store. For example, if an individual has a voucher $A_{i,j} = \{S_i, S_j\}$, *“available in store S_i OR available in store S_j”* the participant can go either to store S_i or to store S_j to exchange this voucher for products from the store. In this condition, it is clear that voucher $A_{1,2}$ *"available in store S_1 OR available in store S_2"* is the disjunction of voucher A_1 *"available in store S_1"* and voucher A_2 *"available in store S_2"* ($A_{1,2} = A_1 \cup A_2$). Indeed, if the individual chooses voucher $A_{1,2}$ instead of a voucher A_3 *"available in the store S_3"*, the individual renounces the use of the voucher in store S_3, but does not renounce its use in S_1 or S_2. Therefore, she does not need to resolve the uncertainty between the stores before using the voucher. Until that moment, we are in a situation as if the decision generates a superposition of vouchers A_1 and A_2, as if the decision maker has in possession two vouchers but can use only one.

More precisely, to test the Sorkin Equality, we must define a set of stores $\mathcal{S} = \{S_1, S_2, S_3\}$, and a set of possible vouchers $\mathcal{A} = 2^{\mathcal{S}}$ which contains the three singleton vouchers $A_i = \{S_i\}$ *"available in store S_i"* ($A_1,\ A_2,\ A_3$), three double-store vouchers $A_{i,j} = \{S_i, S_j\}$ *"available in store S_i OR available in store S_j"* ($A_{1,2},\ A_{1,3},\ A_{2,3}$), a triple-store voucher $A_{1,2,3} = \{S_1, S_2, S_3\}$ *"available in store S_1, OR available in store S_2, OR available in store S_3"*, and an empty voucher $A_0 = \emptyset$ *"not available in any store"*. We propose 5 conditions:

1. a first condition where individuals choose a voucher A from among A_1, A_2, A_3, and A_0,
2. a second condition where individuals choose a voucher A from among $A_{1,2}$, A_3, and A_0,
3. a third condition where individuals choose a voucher A from among $A_{1,3}$, A_2, and A_0,
4. a fourth condition where individuals choose a voucher A from among $A_{2,3}$, A_1, and A_0,
5. a fifth condition where individuals choose a voucher A from among $A_{1,2,3}$, and A_0.

As a second measurement, we can measure the value of the chosen voucher, before its use, by asking participants to reveal the minimum amount of money they will accept to sell the chosen vouchers to the experimenter. In economics, this value is called willingness to sell (WTS). We employ this measure because i) it directly measures individuals' preferences, which is a core notion in economic theory, ii) it is easy to elicit in an "incentive-compatible" manner.[3]

[3]Since B is now a continuous random variable, the "interference terms" can be written in a functional form as follows:

$$I_{12} = p(A_{1,2}) \times f_{A_{1,2}} - p(A_1) \times f_{A_1} - p(A_2) \times f_{A_2} \tag{10}$$
$$I_{13} = p(A_{1,3}) \times f_{A_{1,3}} - p(A_1) \times f_{A_1} - p(A_3) \times f_{A_3} \tag{11}$$
$$I_{23} = p(A_{2,3}) \times f_{A_{2,3}} - p(A_2) \times f_{A_2} - p(A_3) \times f_{A_3} \tag{12}$$
$$I_{123} = p(A_{1,2,3}) \times f_{A_{1,2,3}} - p(A_1) \times f_{A_1} - p(A_2) \times f_{A_2} - p(A_3) \times f_{A_3}, \tag{13}$$

The "empty voucher" was introduced mainly for practical reasons as we considered it was important to elicit the voucher choice and WTS with the same experimental structure and instructions in all conditions. Indeed, the first measurement made in the experiment consisted in a choice among a set of vouchers. In the first four conditions, the inclusion of an empty voucher was not necessary to elicit a choice among several vouchers. However, in the fifth condition, to avoid participants to make a decision without any choice and just one possibility ($A_{1,2,3}$), we had to introduce a somehow irrelevant "outside" option. Finally, in order to keep a structure of choice as close as possible in all conditions, this outside option was introduced in all the groups. We expected that no participant would choose the empty voucher, but discuss possible interpretations for this behavior in the result section.

Moreover, in this experiment, if for each participant, the value of the voucher was defined by the value attributed to the preferred goods combination the individual can ultimately buy, then the distribution of the WTS should respect LTP. Indeed, since a voucher $A_{i,j} = \{S_i,\ S_j\}$ can be ultimately used in only one of the stores, the value of the disjunction of a voucher should equals he value of a voucher available in the preferred store (see Kreps, 1979):

$$\text{For any individual, and for any voucher } A \in \mathcal{A}\text{: } WTS(A) = \max_{S \in A} WTS(\{S\}), \tag{14}$$

a rational individual should choose the voucher available in a set of stores if and only if the set of stores contains her preferred store. Therefore, at the aggregated level, as long as the initial set of available stores $\mathcal{S}$ (in which an individual can ultimately use the chosen voucher) is the same, one must expect that the choice probability $p(A)$, for any voucher $A \in \mathcal{A}$, equals the sum of the probabilities $p(\{S\})$ to choose the singleton vouchers include in A:

$$\text{For any voucher } A \in \mathcal{A}\text{: } p(A) = \sum_{S \in A} p(\{S\}) \tag{15}$$

Therefore, rational choice theory predicts that the LTP should hold at the aggregate level for this particular decision problem.[4] The next section describes the implementation of this

where f_A denotes the density of B conditional on event A. Therefore to test Sorkin's equality, it is necessary and sufficient to measure:

(i) $f_1 = p(A_1) \times f_{A_1}$, $f_2 = p(A_2) \times f_{A_2}$, and $f_3 = p(A_3) \times f_{A_3}$,

(ii) $f_{12} = p(A_{1,2}) \times f_{A_{1,2}}$, $f_{13} = p(A_{1,3}) \times f_{A_{1,3}}$, and $f_{23} = p(A_{2,3}) \times f_{A_{2,3}}$

(iii) $f_{123} = p(A_{1,2,3}) \times f_{A_{1,2,3}}$.

[4]As discussed in the previous section, the LTP should also be valid at the individual level if individuals are random utility maximizers (Heydari, 2020).

triple-store experiment and how the choices were elicited.

4 Experimental Implementation

We recruited 175 participants at the *Laboratoire d'Economie Experimentale de Nice* (LEEN) using the ORSEE system (Greiner, 2015). The experiment took place on February 21 and 22, 2019, in 8 distinct experimental sessions lasting no more than one hour. Each participant participated in only one experimental session. In the experiment, choices were made using pen and paper and returned in envelopes. All participants received a show-up fee of €5 and had one chance in ten of having their choice implemented. Ten-sided dice were used for all randomization procedures.

As explained in the previous section, we randomized participants across 5 treatment conditions depending on the voucher choice set (see Table 1).

Condition	G1	G2	G3	G4	G5
Choice Set	$\{A_0, A_1, A_2, A_3\}$	$\{A_0, A_{1,2}, A_3\}$	$\{A_0, A_{1,3}, A_2\}$	$\{A_0, A_{2,3}, A_1\}$	$\{A_0, A_{1,2,3}\}$

Table 1: Choice set in each condition

In order to control for the number of products in each store and their prices, and to share a common knowledge about the available stores, we created our own stores for this study. The stores were constructed based on the results of an online pretest, to ensure that the proposed products were desirable for the participants (see Appendix A for details). We built three stores, each of them specializing in a different product category (S_1*: "Multimedia";* S_2 : *"Household appliances";* S_3*: "Sport, leisure and accessories"*). Each store offered 25 products with a similar distribution of prices among four different levels: €20 (×12 products), €40 (×6), €60 (×4) or €80 (×3). All the vouchers offered in the experiment had a value of €80. All the experimental material (in French), including stores' catalogues, answer forms, instructions, and guide for the experimenters are available on the following repository: https://osf.io/reh23/

The different steps of the experiment (**Step 0:** General instructions; **Step 1:** Catalogue inspection; **Step 2:** Voucher choice; **Step 3:** Willingness to sell elicitation task; **Step 4:** Dice roll) are presented in Figures 2 and 3.

(Step 0): General instructions — After entering the room and being randomly assigned to a separate box and an experimental condition, participants received general instructions on sheets of paper, that were also read aloud by the experimenter (see **??** for the instruction details).

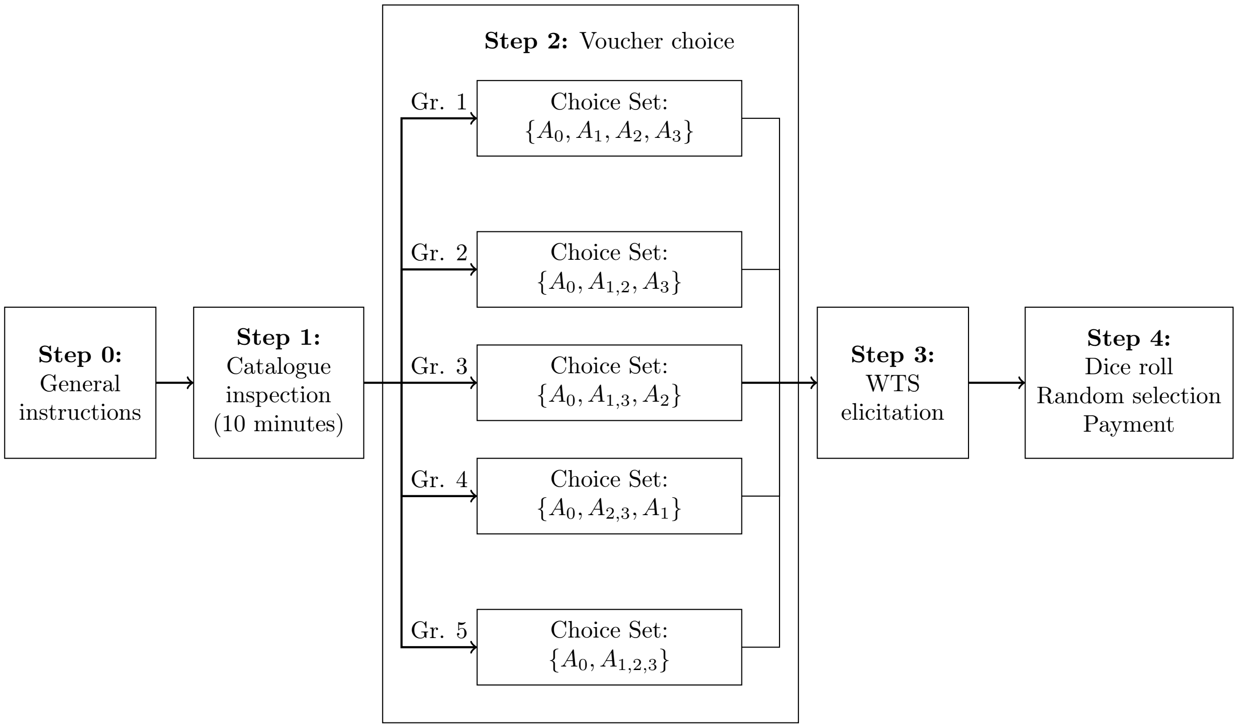


Figure 2: Protocol of the experimental session

Note: This figure shows the different steps of the experiment. ***Step 0:*** *The session started with general instructions: Participants were informed a) about the existence and the names of the 3 stores; b) that they would have exactly 10 minutes to inspect the stores' catalogues; c) that they would have to choose among a set of vouchers of €80 value; d) that they would have one chance in ten (dice roll) to win and thus for their choice to be funded and implemented; and e) that if they won they would have to come back to the laboratory the following week at their convenience (any working day between 10am and 6pm) in order to use the voucher. The WTS elicitation task was not mentioned yet.* ***Step 1:*** *Participants received the catalogues and were invited to inspect the stores. The catalogues included the descriptions and prices of products (see Supplementary Material).* ***Step 2:*** *The experimenter then took back the catalogues and distributed a) the voucher choice set with the vouchers' conditions and b) a response form and an envelope to make the decision. Participants had 5 minutes to complete and submit their response form.* ***Step 3:*** *Participants were informed about the option to sell back their chosen voucher. They received another response form, on which they indicated the minimum offer they would accept. Participants had 5 minutes to make their decision.* ***Step 4:*** *Participants rolled the dice and received the payment individually in a separate room (see Figure 3 for more details).*

More precisely, participants were told about the existence and the names of the three stores. Participants learned that each store would be detailed in a catalogue in Step 1 and that they would have to choose a voucher of €80 value in Step 2. Participants knew that their choice had one chance in ten of being paid.[5] However the terms for using the vouchers, the content of

[5]To ensure incentive compatibility, we were fully transparent with participants regarding the selection method. Each participant started the experiment with a 10-sided dice on their desk and was allowed to roll and test the dice. Participants knew that at the end of the experiment, they would have to correctly guess the dice roll in

the stores and the option to sell back the voucher in Step 3 were not communicated during the general instructions, only at the beginning of the relevant subsequent step.

(Step 1): Catalogue inspection — The three catalogues were provided on paper through 3×10-page documents. In each catalogue, products were classified from the most to the least expensive, with 3 products on each page (except for the last page containing only one product). Each product was described by a photograph, a name, a price, a short description and – when appropriate – a bullet list of characteristics. Participants had 10 minutes to examine the three catalogues.

(Step 2): Voucher choice — The stores' catalogues were collected and participants received the material for "Step 2" in an envelope containing: the list of available vouchers (see table 1) and the terms for using the vouchers, as well as a paper decision form and a small "Envelope A" to submit their decision. In particular, for the vouchers available in several stores, we insisted on the following points: 1) all the purchases should be made in a single store; 2) When they returned to the lab to use the voucher, they would have to choose the store in which they wanted to make their purchases **before** looking at the catalogue again. Participants have 5 minutes to choose their preferred voucher A and to submit their answer by completing the paper form and putting it in the envelope, which is collected by the experimenter.

(Step 3): WTS elicitation task — Participants were informed of the option to sell the chosen voucher, A, back to the experimenter at the end of the experiment. They were invited to open the "Step 3 Envelope", which contains the terms and the instructions for the WTS, as well as a response form and a small "Envelope B". In this task, participants had to indicate the minimum value X they would accept to sell the chosen voucher. We used a Becker–DeGroot–Marschak method to elicit participants' WTS (Becker et al., 1964). More precisely, participants were informed that at the end of the experiment, the experimenter would randomly select an offer $Y \in (0, 99)$, based on the result of two 10-sided dice rolls. The participant would then either receive €Y, if $X \leq Y$, or keep voucher A, if $X > Y$. Participants had 5 minutes to indicate their WTS by completing the paper form and putting it in "Envelope B", which was collected by the experimenter.

(Step 4): Dice roll — The experimenter called the participants individually and asked them to guess a one-digit number, then rolled a 10-sided dice (using the dice that was on the participant's desk). If the participant correctly guessed the result of the dice roll, the experimenter rolled two different 10-sided dice to obtain a number, Y, in order to formulate an offer.

front of the experimenter in order to be selected for choice implementation.

The experimenter then opened "Envelope B" to verify if the participant's reservation price exceed offer Y or not. If it did, the experimenter opened the first envelope to give the chosen voucher to the participant. Otherwise, the participant received the offer amount in cash and could leave the laboratory. Regardless of the results of the rolls, all participants received a €5 show-up fee.

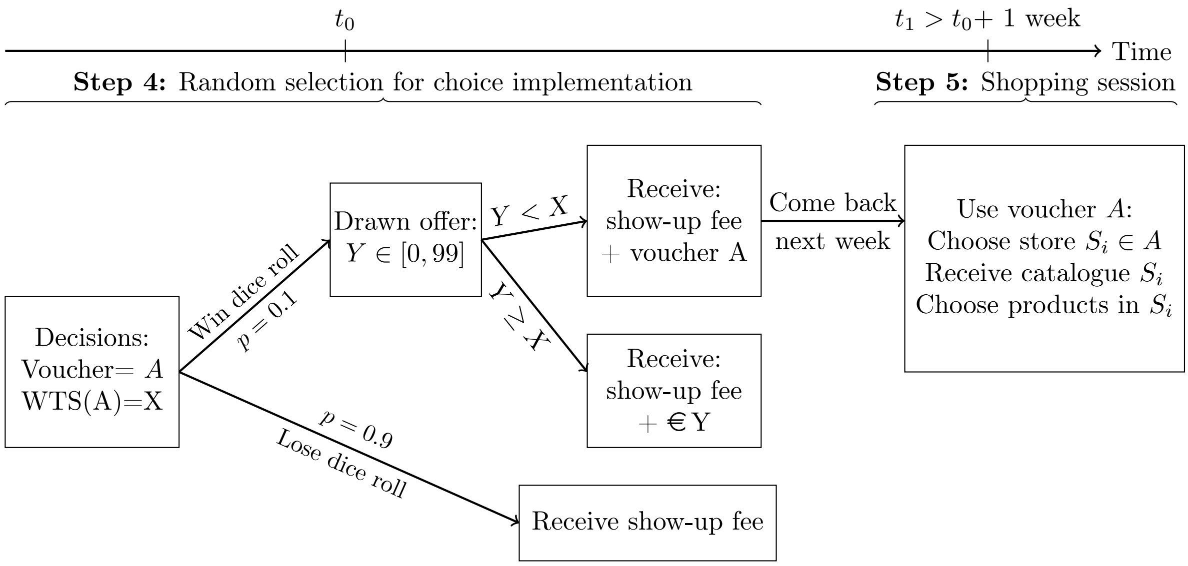


Figure 3: Choices' incentives.

Note: This figure shows how decisions were incentivized. During the experiment, the participant made two decisions: the choice of a voucher A (in Step 2) and the willingness to sell the voucher $WTS(A) = X$ (in Step 3). ***(Step 4)*** *Participants were called individually and had to guess the result of a 10-sided dice roll. If the participant guessed incorrectly, her choices were not implemented: she received a show-up fee of €5 and left the laboratory. In case of a correct guess, her choices were implemented. The experimenter randomly drew an offer, Y, from 0 to 99 and compared it with the participant's $WTS(A) = X$. If $Y \geq X$, then the participant "accepted the offer", and earned € Y plus the show-up fee of €5 and then left the laboratory. If $Y < X$, then the participant "refused the offer", she received a show-up fee of €5 and earned the voucher A.* ***(Step 5)*** *If the participant earned a voucher A, she could use it as follows. The participant must come back to the lab the following week for a shopping session. The shopping session started at the participant's convenience during the lab's opening hours (indicated in the general instructions and in the voucher's terms). If the chosen voucher, A, contained more than one store, at the beginning of the shopping session the participant had to choose one store $S \in A$ from which to purchase. Then the participant received the associated catalogue and was able to choose any products in the catalogue, as long as the sum of the prices did not exceed €80.*

(Step 5): Shopping session — The participants who earned a voucher could come back to the laboratory the following week for a shopping session. When the participant came back, if her voucher could be used in more than one store, then the experimenter first asked the participant to choose the store from which she would like to purchase. Only then did the experimenter give the participant the catalogue of the chosen store. There was no time limit to choose the products but the participants were not allowed to browse the internet or to use any communication device when making the purchase. The participant gave her list of products to the experimenter to order and was contacted when the products arrived.

5 Results

Data and statistical code are available on the following repository: https://osf.io/reh23/. Figure 4 shows the empirical density of WTS for each condition and Table 2 reports, for each condition (lines), the number of participants who chose each voucher (columns), as well as the average WTS of the chosen voucher and its standard deviation. We observe that 10 participants (5.71%) selected the "empty voucher". While this "empty voucher" was designed in order to be dominated by any of the vouchers, we propose two explanations for this particular choice. The first one consists in a simple "mistake" from participants who did not pay attention, did not understand the instructions, or did not respond to incentives. Another explanation would be that those participants are indifferent between the empty voucher and the other ones. This is possible whenever the participant knows for sure that she is not willing to return to the laboratory to use the voucher. This might occur if she is not available in the following weeks, or because she attributes to any voucher a value that does not exceed the costs of having to return to the laboratory and spending the voucher. In this latter case, all vouchers should have a null value for the participant. The distribution of the WTS of the empty voucher partly disentangles between the two explanations: 6 participants out of 10 are willing to sell the empty voucher for any positive amount of money, and only 2 participants revealed a WTS higher than 9. Therefore we consider that the proportion of participants who did not respond to our incentives is marginal.

We conduct a one way ANOVA and we find no effect of the condition on the WTS [$F(4, 170) = 0.91,\ p = 0.459$].

	Choice Set	A_0	A_1	A_2	A_3	$A_{1,2}$	$A_{1,3}$	$A_{2,3}$	$A_{1,2,3}$	Total
G1	(A_0, A_1, A_2, A_3)	2	20	9	4	*29*	*24*	*13*	*33*	*35*
		[5.00]	[58.20]	[37.22]	[45.00]	*[51.69]*	*[56.00]*	*[39.62]*	*[50.88]*	*[48.26]*
		(5.66)	(20.09)	(32.12)	(45.23)	*(25.81)*	*(24.09)*	*(33.57)*	*(27.34)*	*(28.65)*
G2	(A_0, $A_{1,2}$, A_3)	3			6	26			*32*	*35*
		[1.67]			[52.50]	[47.07]			*[48.09]*	*[44.11]*
		(2.89)			(16.04)	(28.34)			*(26.35)*	*(28.41)*
G3	(A_0, $A_{1,3}$, A_2)	1		15			19		*34*	*35*
		[80.00]		[45.93]			[59.11]		*[53.29]*	*[54.06]*
		0		(31.99)			(22.27)		*(27.36)*	*(27.33)*
G4	(A_0, $A_{2,3}$, A_1)	1	18					16	*34*	*35*
		[0.00]	[49.44]					[55.94]	*[52.50]*	*[51.00]*
		0	(31.97)					29.34	*(30.47)*	*(31.31)*
G5	(A_0, $A_{1,2,3}$)	3							32	*35*
		[13.33]							[58.90]	*[56.17]*
		(23.09)							(29.07)	*(31.66)*

Table 2: Choice frequency of the vouchers, [Mean] and (Standard deviation) of the Willingness to sell the chosen voucher.

Note: Values in italics are computed based on the aggregation of singleton vouchers.

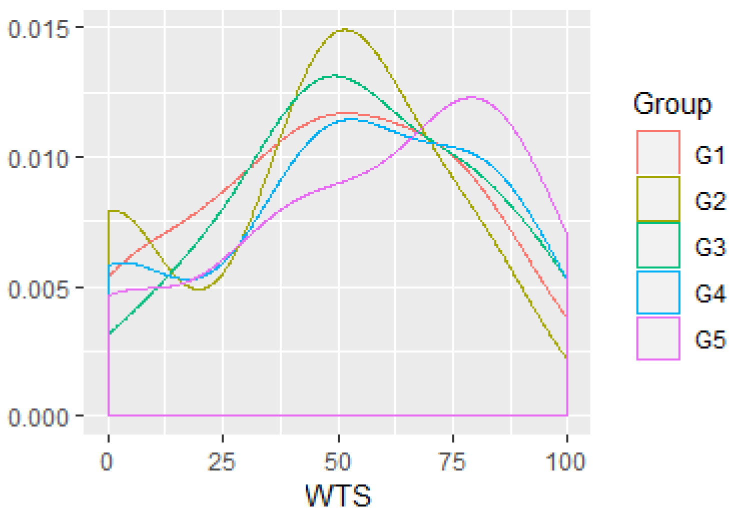
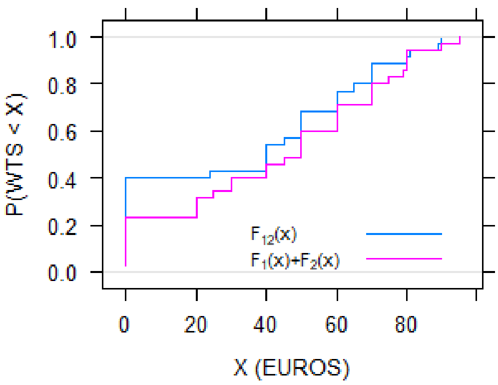


Figure 4: Density of WTS by condition

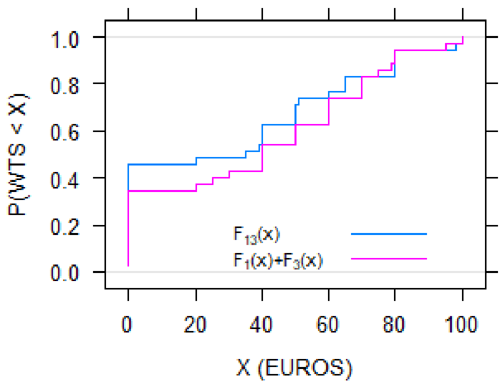
Note: G1 represents the group with only singleton vouchers, G2-4 represent the groups with double-store vouchers, G5 represents the group with a triple-store voucher.

Let $X_i = WTS(A_i)$ if the participant has chosen the voucher A_i and 0 otherwise. Let $X_{i,j} = WTS(A_{i,j})$ if the participant has chosen the voucher $A_{i,j}$ and 0 otherwise. Let $X_{123} = WTS(A_{1,2,3})$ if the participant has chosen the voucher $A_{1,2,3}$ and 0 otherwise.

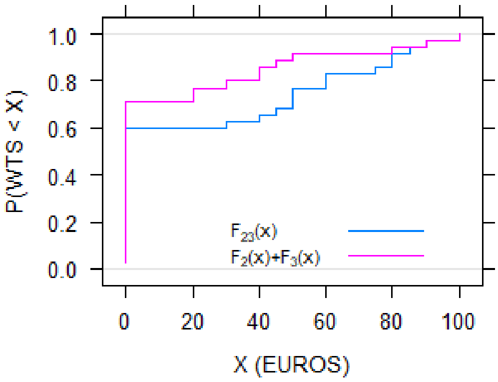
As explained in Section 2, a violation of classical probabilities (through a violation of the LTP) occurs when non-null interference terms are observed. The distributions of the interference terms are shown in Figure 5 by the distance between the two curves.



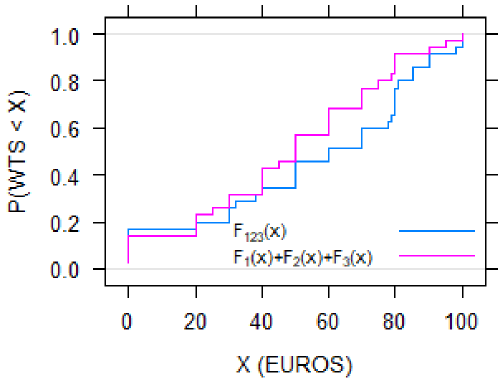
$I_{12} = F_{12}(X) - F_1(X) - F_2(X)$



$I_{13} = F_{13}(X) - F_1(X) - F_3(X)$



$I_{23} = F_{12}(X) - F_2(X) - F_3(X)$



$I_{123} = F_{123}(X) - F_1(X) - F_2(X) - F_3(X)$

Figure 5: Interference terms, difference in WTS cumulative distributions

Note: F_i denotes the cumulative distribution function of the willingness to sell the voucher A_i

The nullity of mean interference terms are tested with two-sample t-tests and the nullity in the distributions are tested with a bootstrap version of the Kolmogorov-Smirnov test, by comparing $X_{i,j}$ and $X_i + X_j$ (for 2nd-order interference) and by comparing $X_{1,2,3}$ with $X_1 + X_2 + X_3$ (for

3rd-order interference). We do not find any statistically significant interference terms, neither in the means, nor in the distributions (see Table 3).[6]

	M	t-test		(bootstrap) ks-test	
		t	p-value	D	p-value
Test of classical probabilities					
$I_{12} = X_{1,2} - X_1 - X_2$	-7.857	-1.048	0.298	0.171	0.487
$I_{13} = X_{1,3} - X_1 - X_3$	-6.314	-0.789	0.433	0.114	0.835
$I_{23} = X_{2,3} - X_2 - X_3$	10.857	1.453	0.151	0.200	0.179
$I_{123} = X_{1,2,3} - X_1 - X_2 - X_3$	7.057	0.950	0.346	0.200	0.354
Test of quantum probabilities					
$I_{123} - I_{12} - I_{13} - I_{23}$	10.371	1.423	0.154		

Table 3: Test of nullity of the interference terms.

A violation of the Born Rule and quantum probabilities occurs if $I_{123} = I_{12} + I_{13} + I_{23}$ does not hold. That is, the 3rd-order interference term is not equal to the sum of 2nd-order interference terms. To test this hypothesis, we construct a t-test-like statistic, to test if the mean 3rd-order interference term equals the sum of the 2nd-order mean interference terms. Under $H0 : I_{123} = I_{12} + I_{13} + I_{23}$, this statistic asymptotically approaches a normal distribution:

$$Z = \frac{\sqrt{N}}{\sqrt{(5)}} \times \left(\frac{\bar{X}_{123}}{\hat{\sigma}_{123}} - \frac{\bar{X}_{12}}{\hat{\sigma}_{12}} - \frac{\bar{X}_{13}}{\hat{\sigma}_{13}} - \frac{\bar{X}_{23}}{\hat{\sigma}_{23}} + \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{\hat{\sigma}} \right) \rightarrow \mathcal{N}(0,1)\,, \tag{16}$$

where N is the number of observations in each group, μ and $\hat{\sigma}$ are the expectation value and the sample standard deviation of $X_1 + X_2 + X_3$, and μ_i and $\hat{\sigma}_i$ are the expectation values and the sample standard deviations of X_i (for $i = \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$).[7]

[6] There are two advantages of this method when compared to the one-way ANOVA presented at the beginning of the result Section. First, it disentangles the different interference terms. Second, the test is not impacted by the distribution of the WTS revealed by the participants who have chosen the empty voucher (as stated previously, those participants are potentially not responding to incentives).

[7]**Proof** — Applying the Central Limit Theorem in each condition, we obtain: $\sqrt{N}\frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{\hat{\sigma}} \rightarrow \mathcal{N}(\mu,1)$ in the first condition, $\sqrt{N}\frac{\bar{X}_{12}}{\hat{\sigma}_{12}} \rightarrow \mathcal{N}(\mu_{12},1)$ in the second condition, $\sqrt{N}\frac{\bar{X}_{13}}{\hat{\sigma}_{13}} \rightarrow \mathcal{N}(\mu_{13},1)$ in the third condition, $\sqrt{N}\frac{\bar{X}_{23}}{\hat{\sigma}_{23}} \rightarrow \mathcal{N}(\mu_{23},1)$ in the fourth condition, and $\sqrt{N}\frac{\bar{X}_{123}}{\hat{\sigma}_{123}} \rightarrow \mathcal{N}(\mu_{123},1)$ in the fifth condition, where N is the number of observations in each group, μ and $\hat{\sigma}$ are the expectation value and the sample standard deviation of $X_1 + X_2 + X_3$, and μ_i and $\hat{\sigma}_i$ are the expectation values and the sample standard deviations of X_i (for $i = \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$).

Since the above empirical means in the five groups are independent, we have:

$$\sqrt{N} \times \left(\frac{\bar{X}_{123}}{\hat{\sigma}_{123}} - \frac{\bar{X}_{12}}{\hat{\sigma}_{12}} - \frac{\bar{X}_{13}}{\hat{\sigma}_{13}} - \frac{\bar{X}_{23}}{\hat{\sigma}_{23}} + \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{\hat{\sigma}} \right) \rightarrow \mathcal{N}(\mu_{123} - \mu_{12} - \mu_{13} - \mu_{23} + \mu, 5)$$

If the Sorkin Equality holds, we have $H0 : \mu_{123} - \mu_{12} - \mu_{13} - \mu_{23} + \mu = 0$. So under $H0$, the statistic

$$Z = \frac{\sqrt{N}}{\sqrt{5}} \times \left(\frac{\bar{X}_{123}}{\hat{\sigma}_{123}} - \frac{\bar{X}_{12}}{\hat{\sigma}_{12}} - \frac{\bar{X}_{13}}{\hat{\sigma}_{13}} - \frac{\bar{X}_{23}}{\hat{\sigma}_{23}} + \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{\hat{\sigma}} \right) \rightarrow \mathcal{N}(0,1)\,,$$

Based on this test, we cannot reject the Born Rule, so we can reject neither classical nor quantum probabilities (see Table 3).

6 Discussion

To the best of our knowledge, this experiment conducts the first empirical test of the Sorkin Equality using an incentivized experiment in Human Decision-Making. Although we were not able to statistically reject classical probabilities, this study is a further step toward testing quantum-like theories of human decision-making. Indeed, we highlight that it is possible to have a tangible definition of the disjunction of mutually exclusive events, using vouchers, and to calculate interference terms at any possible order. Being able to identify and detect positive or negative interference between goods, services or features could help marketers or policy makers to identify incompatible alternatives or features that are likely to produce contextual effects, temptation, or other behaviors, which are not taken into account by classical random utility models, but can be explained by quantum-like models. Moreover, one benefit of our method is to quantify those interference terms with a monetary measure, thanks to the second measurement that consists in a WTS elicitation. Expressing interference in monetary terms would allow researchers or practitioners to estimate the cost or the benefit (for the society or for a company) of "non-classical" behaviors.

We propose several interpretations for the absence or the triviality of the interference terms. First, it is possible that no interference occurs in our experiment and that participants' choices respect classical probabilities. Indeed, quantum probabilities allow but did not predict non-null interference (and therefore classical probabilities are a special case of quantum probabilities).[8] It is also possible that only a fraction of participants exhibited quantum interference or that the interference was too small to be detected by our experiment. Future experiments with more participants thus have to be considered. Moreover, a reason that might explain why no interference occurred — for all or for a fraction of the participants — could be that most of the participants, when choosing a voucher $A_{1,2}$, available in store S_1 or available in store S_2 had already resolved the uncertainty between their preferred stores. This is possible if most

asymptotically approaches a normal distribution. □

[8]In quantum theory, nontrivial interference terms can appear only for incompatible observables that are mathematically represented by non-commutative operators. In fact, the situation is even more complicated and the presence (absence) of the interference term can be state dependent. In human decision making, the situation is complicated by the absence of the canonical quantization procedure. Therefore, mathematics does not help for understanding whether observables are compatible or not. Experimental tests, such as those performed in this article, can be considered as tests of incompatibility.

of the participants had strong, deterministic, and stable preferences. Furthermore, it is likely that individuals were able to perform a "preference measurement" themselves, even when it was not explicitly required by the experimenter. This last point is an important distinction between quantum physics and human cognition, since particles are not able to "measure their states". One solution to investigate this issue could be to decrease the time limits to investigate the stores, choose the voucher and elicit willingness to sell.

References

Aerts, D. (2009). Quantum structure in cognition. Journal of Mathematical Psychology 53(5), 314–348.

Aerts, D., J. Broekaert, M. Czachor, and B. D'Hooghe (2011). A quantum-conceptual explanation of violations of expected utility in economics. In International Symposium on Quantum Interaction, pp. 192–198. Springer.

Aerts, D. and M. S. de Bianchi (2017). Beyond-quantum modeling of question order effects and response replicability in psychological measurements. Journal of Mathematical Psychology 79, 104–120.

Aerts, D., S. Geriente, C. Moreira, and S. Sozzo (2018). Testing ambiguity and machina preferences within a quantum-theoretic framework for decision-making. Journal of Mathematical Economics 78, 176–185.

Aerts, D. and S. Sozzo (2011). A contextual risk model for the ellsberg paradox. Journal of Engineering Science and Technology Review 4(3), 246–250.

Ahn, D. S. and T. Sarver (2013). Preference for flexibility and random choice. Econometrica 81(1), 341–361.

al Nowaihi, A. and S. Dhami (2017). The ellsberg paradox: A challenge to quantum decision theory? Journal of Mathematical Psychology 78, 40–50.

Asano, M., I. Basieva, A. Khrennikov, M. Ohya, and Y. Tanaka (2017). A quantum-like model of selection behavior. Journal of Mathematical Psychology 78, 2–12.

Ashtiani, M. and M. A. Azgomi (2015). A survey of quantum-like approaches to decision making and cognition. Mathematical Social Sciences 75, 49–80.

Atmanspacher, H. and H. Römer (2012). Order effects in sequential measurements of non-commuting psychological observables. Journal of Mathematical Psychology 56(4), 274–280.

Basieva, I. and A. Khrennikov (2017). Testing boundaries of applicability of quantum probabilistic formalism to modeling of cognition: Metaphors of two and three slit experiments. Lecture Notes in Computer Science 10106, 49–56.

Basieva, I., P. Khrennikova, E. M. Pothos, M. Asano, and A. Khrennikov (2018). Quantum-like model of subjective expected utility. Journal of Mathematical Economics 78, 150–162.

Becker, G. M., M. H. DeGroot, and J. Marschak (1964). Measuring utility by a single-response sequential method. Behavioral science 9(3), 226–232.

Boyer-Kassem, T., S. Duchêne, and E. Guerci (2016a). Quantum-like models cannot account for the conjunction fallacy. Theory and Decision 81(4), 479–510.

Boyer-Kassem, T., S. Duchêne, and E. Guerci (2016b). Testing quantum-like models of judgment for question order effect. Mathematical Social Sciences 80, 33–46.

Brandenburger, A. (2010). The relationship between quantum and classical correlation in games. Games and Economic behavior 69(1), 175–183.

Bruza, P. D., Z. Wang, and J. R. Busemeyer (2015). Quantum cognition: a new theoretical approach to psychology. Trends in cognitive sciences 19(7), 383–393.

Busemeyer, J. and Z. Wang (2019). Hilbert space multidimensional modelling of continuous measurements. Philosophical Transactions of the Royal Society A 377(2157), 20190142.

Busemeyer, J., Z. Wang, E. Pothos, and J. Trueblood (2015). The conjunction fallacy, confirmation, and quantum theory: Comment on tentori, crupi, and russo (2013). Journal of Experimental Psychology: General 144(1), 236–243.

Busemeyer, J. R. and P. D. Bruza (2012). Quantum models of cognition and decision. Cambridge University Press.

Busemeyer, J. R., M. R. Matthew, and Z. Wang (2006). A quantum information processing explanation of disjunction effects. In Proceedings of the annual meeting of the cognitive science society, Volume 28.

Busemeyer, J. R., E. M. Pothos, R. Franco, and J. S. Trueblood (2011). A quantum theoretical explanation for probability judgment errors. Psychological review 118(2), 193.

Busemeyer, J. R. and Z. Wang (2007). Quantum information processing explanation for interactions between inferences and decisions. In AAAI Spring Symposium: Quantum Interaction, pp. 91–97.

Castillo, G. (2020). The attraction effect and its explanations. Games and Economic Behavior 119, 123–147.

Conte, E., A. Y. Khrennikov, O. Todarello, A. Federici, L. Mendolicchio, and J. P. Zbilut (2009). Mental states follow quantum mechanics during perception and cognition of ambiguous figures. Open Systems & Information Dynamics 16(01), 85–100.

Cotter, J. P., C. Brand, C. Knobloch, Y. Lilach, O. Cheshnovsky, and M. Arndt (2017). In search of multipath interference using large molecules. Science advances 3(8), e1602478.

Danilov, V. I. and A. Lambert-Mogiliansky (2008). Measurable systems and behavioral sciences. Mathematical Social Sciences 55(3), 315–340.

Danilov, V. I. and A. Lambert-Mogiliansky (2010). Expected utility theory under non-classical uncertainty. Theory and decision 68(1-2), 25–47.

Denolf, J., I. Martínez-Martínez, H. Josephy, and A. Barque-Duran (2017). A quantum-like model for complementarity of preferences and beliefs in dilemma games. Journal of Mathematical Psychology 78, 96–106.

Duchêne, S., T. Boyer-Kassem, and E. Guerci (2017). A new experimental approach to test quantum-like models for the conjunction fallacy. Revue economique 5(5), 757–771.

Eichberger, J. and H. J. Pirner (2018). Decision theory with a state of mind represented by an element of a hilbert space: The ellsberg paradox. Journal of Mathematical Economics 78, 131–141.

Franco, R. (2009). The conjunction fallacy and interference effects. Journal of Mathematical Psychology 53(5), 415–422.

Gagnon, E., C. D. Brown, and A. L. Lytle (2014). Effects of detector size and position on a test of born's rule using a three-slit experiment. Physical Review A 90(1), 013832.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. Journal of the Economic Science Association 1(1), 114–125.

Gul, F. and W. Pesendorfer (2001). Temptation and self-control. Econometrica 69(6), 1403–1435.

Gul, F. and W. Pesendorfer (2006). Random expected utility. Econometrica 74(1), 121–146.

Haven, E. and A. Khrennikov (2013). Quantum social science. Cambridge University Press.

Heydari, P. (2020). Stochastic choice over menus. Theory and Decision 88(2), 257–268.

Kauten, T., R. Keil, T. Kaufmann, B. Pressl, Č. Brukner, and G. Weihs (2017). Obtaining tight bounds on higher-order interferences with a 5-path interferometer. New Journal of Physics 19(3), 033017.

Kauten, T., B. Pressl, T. Kaufmann, and G. Weihs (2014). Measurement and modeling of the nonlinearity of photovoltaic and geiger-mode photodiodes. Review of Scientific Instruments 85(6), 063102.

Khrennikov, A. (2015). Quantum version of aumann's approach to common knowledge: Sufficient conditions of impossibility to agree on disagree. Journal of Mathematical Economics 60, 89–104.

Khrennikov, A. and I. Basieva (2014a). Possibility to agree on disagree from quantum information and decision making. Journal of Mathematical Psychology 62, 1–15.

Khrennikov, A. and I. Basieva (2014b). Quantum model for psychological measurements: from the projection postulate to interference of mental observables represented as positive operator valued measures. NeuroQuantology 12(3), 324–337.

Khrennikov, A., I. Basieva, E. N. Dzhafarov, and J. R. Busemeyer (2014). Quantum models for psychological measurements: an unsolved problem. PloS one 9(10), e110909.

Khrennikov, A. Y. and E. Haven (2009). Quantum mechanics and violations of the sure-thing principle: The use of probability interference and other concepts. Journal of Mathematical Psychology 53(5), 378–388.

Kreps, D. M. (1979). A representation theorem for" preference for flexibility". Econometrica: Journal of the Econometric Society, 565–577.

Landsburg, S. E. (2004). Quantum game theory. Notices of the AMS 51(4), 394–399.

Lu, J. (2016). Random choice and private information. Econometrica 84(6), 1983–2027.

Manzini, P. and M. Mariotti (2014). Stochastic choice and consideration sets. Econometrica 82(3), 1153–1176.

Miyadera, T. and S. Philips (2012). A quantum probability-theoretic account of human judgment using positive-operator-valued measures. In Proceedings of the Annual Meeting of the Cognitive Science Society, Volume 34.

Mogiliansky, A. L., S. Zamir, and H. Zwirn (2009). Type indeterminacy: A model of the kt (kahneman–tversky)-man. Journal of Mathematical Psychology 53(5), 349–361.

Ozawa, M. and A. Khrennikov (2021). Modeling combination of question order effect, response replicability effect, and qq-equality with quantum instruments. Journal of Mathematical Psychology 100, 102491.

Park, D. K., O. Moussa, and R. Laflamme (2012). Three path interference using nuclear magnetic resonance: a test of the consistency of born's rule. New Journal of Physics 14(11), 113025.

Piotrowski, E. W. and J. Sładkowski (2003). An invitation to quantum game theory. International Journal of Theoretical Physics 42(5), 1089–1099.

Piotrowski, E. W. and J. Sładkowski (2017). Quantum game theoretical frameworks in economics. In The Palgrave Handbook of Quantum Models in Social Science, pp. 39–57. Springer.

Pisano, R. and S. Sozzo (2020). A unified theory of human judgements and decision-making under uncertainty. Entropy 22(7), 738.

Pothos, E. M. and J. R. Busemeyer (2009). A quantum probability explanation for violations of 'rational'decision theory. Proceedings of the Royal Society B: Biological Sciences 276(1665), 2171–2178.

Pothos, E. M. and J. R. Busemeyer (2013). Can quantum probability provide a new direction for cognitive modeling? Behavioral and Brain Sciences 36(3), 255–274.

Qureshi, T. (2019). Coherence, interference and visibility. arXiv preprint arXiv:1905.00917.

Sinha, A., A. H. Vijay, and U. Sinha (2015). On the superposition principle in interference experiments. Scientific reports 5(1), 1–9.

Sinha, U., C. Couteau, T. Jennewein, R. Laflamme, and G. Weihs (2010). Ruling out multi-order interference in quantum mechanics. Science 329(5990), 418–421.

Skagerstam, B.-S. K. (2018). On the three-slit experiment and quantum mechanics. Journal of Physics Communications 2(12), 125014.

Söllner, I., B. Gschösser, P. Mai, B. Pressl, Z. Vörös, and G. Weihs (2012). Testing born's rule in quantum mechanics for three mutually exclusive events. Foundations of Physics 42(6), 742–751.

Sorkin, R. D. (1994). Quantum mechanics as quantum measure theory. Modern Physics Letters A 9(33), 3119–3127.

Wang, Z. and J. R. Busemeyer (2013). A quantum question order model supported by empirical tests of an a priori and precise prediction. Topics in Cognitive Science 5(4), 689–710.

Wang, Z., T. Solloway, R. M. Shiffrin, and J. R. Busemeyer (2014). Context effects produced by question orders reveal quantum nature of human judgments. Proceedings of the National Academy of Sciences 111(26), 9431–9436.

Yearsley, J. M. and J. R. Busemeyer (2016). Quantum cognition and decision theories: A tutorial. Journal of Mathematical Psychology 74, 99–116.

Yearsley, J. M. and J. S. Trueblood (2018). A quantum theory account of order effects and conjunction fallacies in political judgments. Psychonomic bulletin & review 25(4), 1517–1525.

Yukalov, V. I. and D. Sornette (2010). Mathematical structure of quantum decision theory. Advances in Complex Systems 13(05), 659–698.

Yukalov, V. I. and D. Sornette (2011). Decision theory with prospect interference and entanglement. Theory and Decision 70(3), 283–328.

Appendices

A Construction of stores

We decided to construct our own stores to control the number of products and the distribution of prices within the stores, as well as participant's knowledge of the stores. To ensure that the stores contained products that would be attractive to our participants, we decided to run a preliminary online experiment with the same subject pool as for the main experiment.

100 participants were recruited in November 2018 to participate in a web-based experiment. We sent participants instructions and an excel form by email.

Participants were asked to choose freely on 3 websites (cdiscount.com, galerielafayette.com and decathlon.fr) a total of 18 products in 3 different categories ("Multimedia", "Household appliances" and "Sport, leisure and accessories") $\times$ 4 price ranges ($[0, 20]$(x2),$]20, 40]$(x2),$]40, 60]$ and $]60, 80]$ euros). Participants were told that 6 out of the completed responses (knowing that there were 100 participants) would be randomly selected and receive one of the products they had chosen.

We received a total of 48 completed forms for a total of 864 products. We then constructed one store per category by randomly selecting:

- 3 products in the range $]60, 80]$;
- 4 products in the range $]40, 60]$;
- 6 products in the range $]20, 40]$;
- 12 products in the range $]0, 20]$.

We then made the price of each product equal to the upper bound of its range.

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