AN EXPERIMENT ON THE NASH PROGRAM: COMPARING TWO MECHANISMS IMPLEMENTING THE SHAPLEY VALUE

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An experiment on the Nash program: Comparing two mechanisms implementing the Shapley value

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Abstract

We experimentally compare two well-known mechanisms inducing the Shapley value as an *ex ante* equilibrium outcome of a noncooperative bargaining procedure: the demand-based Winter's demand commitment bargaining mechanism and the offer-based Hart and Mas-Colell bidding procedure. Our results suggest that, on the one hand, the offer-based Hart and Mas-Colell mechanism better induces players to cooperate and to agree on an efficient outcome; on the other hand, the demand-based Winter mechanism better implements allocations that reflect players' effective bargaining power.

JEL code: C71, C72, C90, D82

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1 Introduction

Bridging the gap between the *noncooperative* models, in which the primitives are the sets of possible actions of individual players, and the *cooperative* ones, in which they are the sets of possible joint actions of groups of players, has been recognized as a fundamental issue of game theory. The very first attempt of this so-called *Nash program* dates back to almost 70 years ago by Nash himself (Nash, 1953). His idea was to provide a noncooperative foundation of cooperative solution concepts, and he first started implementing it by designing a noncooperative game that sustained as equilibrium the Nash solution of his cooperative *bargaining problem* (Nash, 1950). Since then, many different theoretical mechanisms have been designed, with the aim of implementing cooperative solution concepts via a strategic interaction of the players. This is the case, for example, of the pillar work of Harsanyi (1974), who reinterpreted the von Neumann–Morgenstern solution as an equilibrium of a noncooperative bargaining mechanism, or of the many works sustaining the most famous axiomatic solution concept by Shapley (1953), the Shapley value (see, among others, Gul, 1989; Winter, 1994; Hart and Mas-Colell, 1996).

Despite a large body of existing literature, the Nash program *"is not ready for retirement yet"*; on the contrary, it is *"still full of energy"* and *"waiting for good papers to be written"* (Serrano, 2020). In this work, we aim to contribute to this research agenda by providing new insights gained from a controlled laboratory experiment.

Two main issues arise with most noncooperative bargaining models as observed by Fréchette et al. (2005) in an experimental work implementing some well-known legislative bargaining processes. First, the theoretical predictions they propose are very sensitive to variations in the rules of the game, for example, whether a demand-based or an offer-based mechanism is considered. However, experiments show that actual bargaining behavior is sometimes not as sensitive to the different bargaining rules as the theory suggests. Second, the equilibrium solution may require an unrealistic degree of rationality on the part of the players, such that the experimental evidence is substantially different from the theoretical prediction.

In this paper, we experimentally compare two well-known mechanisms inducing the Shapley value as the *ex ante* equilibrium outcome of a noncooperative bargaining procedure. We chose two mechanisms that are based on opposite approaches (demand vs offer) but which remain, in our opinion, similar in implementation and ease of understanding for the participants in a laboratory experiment.[1] The first one is the *Winter's demand commitment bargaining mechanism* (Winter, 1994, referred to as the *Winter mechanism* below). The second one is the *Hart and Mas-Colell bidding procedure* (Hart and Mas-Colell, 1996, referred to as the *H-MC mechanism* below).

Both procedures are described as sequential, perfect information games, where at each stage a player becomes a proposer. In the first one, which is defined for cooperative games with increasing returns to scale for cooperation (strictly convex games), the proposer makes a demand for herself, of the payoff she is willing to receive from a possible collaboration. In the second one, which is defined for monotonic games (a much weaker assumption), the proposer makes a proposal to each of the other players, of the payoff she is willing to offer them.

The difference between a demand-based versus an offer-based mechanism has been argued to be less relevant when considering two-player games, such as in Rubinstein (1982) for the divide-the-dollar game (see, Fréchette et al., 2005). Instead, it may become crucial when considering groups with at least three members. In particular, offer-based mechanisms are comparable with a voting procedure in which all the other players either accept or reject the proposed utility share made by the proposer. As such, they are theoretically expected to show a high degree of asymmetry between the proposer and all the other players. In contrast, however, offer-based mechanisms may be seen as n-player ultimatum bargaining games, in which the existing experimental results are often not in agreement with the theoretical prediction. As many experiments of the classical two-player ultimatum bargaining game (Güth et al., 1982) show, the intrinsic asymmetry often allows a fairer division (see, e.g., Roth et al., 1991; Oosterbeek et al., 2004).[2] In our case, both mechanisms are expected to show some form of proposer advantage. In fact, for both mechanisms, the *ex post* predicted solution strongly depends on the selected proposer, while, in particular, for the Winter mechanism, it depends on the complete ordering.

Our analysis mainly focuses on: *(i)* analyzing whether these mechanisms lead to for-

[1]Comparison between an offer-based versus a demand-based mechanism has been done experimentally for voting games by Fréchette et al. (2005), as well as empirically by, for example, Warwick and Druckman (2001) and Ansolabehere et al. (2005) employing field data.

[2]Andersen et al. (2011) report, however, that when the stake size is very large, unfair divisions are more frequently observed.

mation of the grand coalition; *(ii)* testing the convergence, in expected value and as predicted by the theory, to the Shapley value; *(iii)* testing the axioms that are, historically, the most relevant for characterizing the Shapley value. In particular, we investigate to what extent the properties of efficiency, symmetry, additivity, homogeneity, null player, strong monotonicity, and fairness are satisfied. By doing so, we aim to provide insights regarding situations in which a demand-based mechanism is more appropriate than an offer-based mechanism, and vice versa.

Our results show that the H-MC mechanism results in a higher frequency of grand coalition formation and a higher efficiency than the Winter mechanism. The Winter mechanism, on the contrary, satisfies most of the axioms, and better implements the Shapley value as the average payoff share. Our results, therefore, suggest that an offer-based H-MC mechanism better induces players to cooperate and to agree on an efficient outcome, while a demand-based Winter mechanism better implements allocations that reflect players' effective bargaining power.

The remainder of the paper is organized as follows. Section 2 reviews existing studies that are most relevant to our work. Section 3 presents the general definition and the properties of a cooperative transferable utility (TU) game, as well as the Shapley value and the equal division solution together with their axiomatizations. Section 4 presents the two mechanisms we investigate, namely the Winter and the H-MC mechanisms. Section 5 describes the setting of our experiment and presents our hypotheses. The results are presented in Section 6, and Section 7 concludes.

2 Related work

For a relevant and extensive review of the literature on the Nash program, we refer to the recent survey by Serrano (2020). In this section, we focus on the studies that are most relevant to our analysis.

The literature devoted to testing cooperative game theory through experiments has up to now focused mainly on three different research streams. The first stream provides a normative interpretation, as in De Clippel and Rozen (2013), in which subjects designated as decision-makers express their view on what is fair for others, by recommending a payoff allocation. De Clippel and Rozen (2013) show that the decision-maker's choices can be described as a convex combination of the Shapley value and the equal division solution.

The second stream investigates how an unstructured interaction affects the final agreement. An example is Kalisch et al. (1954) in which groups of players were asked to freely discuss the formation of coalitions and reach agreement on how to split the related values. The authors identified many different factors influencing the final outcome of such a procedure, such as the personality differences or the geometrical arrangement of players around the table. Similarly, but more focused on voting games, Montero et al. (2008) suggest an unstructured bargaining protocol in which participants propose and vote on how to distribute a fixed budget among themselves. The paper provides experimental evidence of the so-called *paradox of new members*, according to which enlargement of a voting body (i.e., addition of a new voter) can increase the voting power of an existing member. Guerci et al. (2014) study the impact of variations in the experimental protocol of Montero et al. (2008) on the formation of the so-called minimal winning coalitions, i.e., coalitions for which each player is crucial.

Most of the experimental papers in the literature on the topic, however, follow a third stream, which studies the outcome when a more formal (or structured) bargaining protocol is imposed. Our paper broadens this last stream of research.

On the one hand, formal bargaining protocols have been implemented to tackle different aspects of the cooperative inclination of the players under different settings. For example, Murnighan and Roth (1977) investigate the effect of some different communication/information conditions on the final outcome, in a specific game played by a monopolist and two weaker players. They show how the results over the entire set of conditions closely approximate the Shapley value, although they often report a clear tendency for an equal split of the pie. In the same vein, Murnighan and Roth (1982) introduce bargaining models to investigate the influence of information shared by subjects about the games (payoffs, etc.) on the final outcome. They show that the quality of the information has an impact on the final outcome and that the Nash bargaining solution has good predictive performance in many cases. Bolton et al. (2003) also investigate how the communication configuration affects coalition negotiation, and show how players with weaker alternatives would benefit from a more constrained structure, especially if they can be the conduit of communication, while those endowed with stronger alternatives would do well to work within a more public communication structure that promotes competitive bidding. Other papers are more specifically oriented on the coalition formation process, such as Nash et al. (2012) and Shinoda and Funaki (2019). In the former, the authors implement finitely repeated three-person coalition formation games, showing how efficiency requires people's willingness to accept the agency of others, such as political leaders. The latter is a follow-up, in which the authors maintain the same value of the coalitions as in Nash et al. (2012), but implement a different bargaining protocol. They report a rare formation of the grand coalition, which can be better induced by some external factors, such as the presence of a chat window.

On the other hand, formal bargaining protocols are most often based on the implementation of theoretical mechanisms, which are shown to converge to some specific well-known solutions. This is the case, for example, in Nash (1953) and Harsanyi (1974), which we have referred to already, or in the bargaining mechanism proposed by Raiffa (1953) to implement the Raiffa solution (as opposed to the Nash solution) to the Nash cooperative bargaining problem. Some experimental implementations have been proposed, with the final goal of testing Nash axioms, or of comparing the Nash and Raiffa solutions (see, e.g., Nydegger and Owen, 1975; Rapoport et al., 1977). We also cite the large literature devoted to the study of a specific class of bidding mechanisms. Bidding mechanisms were introduced by Demange (1984) and Moulin (1984) and studied by Moulin and Jackson (1992) in economic environments, and then developed by Perez-Castrillo and Wettstein (2001) and Ju and Wettstein (2009) to implement solution concepts in the framework of cooperative TU games.

In particular, many different theoretical mechanisms have been designed specifically with the objective of implementing the best-known cooperative solution, the Shapley value (see Shapley, 1953). Because this solution is applied primarily in many economic problems, it is considered important to justify it using a strategic explanation. Among others, we refer to Harsanyi (1981), Gul (1989), Hart and Moore (1990), Winter (1994), Hart and Mas-Colell (1996), and Krishna and Serrano (1995), who deepen the study of the set of subgame perfect equilibria associated with the bargaining mechanism proposed by Hart and Mas-Colell (1996).

In this paper, we propose an experimental implementation of two of these mechanisms: one by Winter (1994) and another by Hart and Mas-Colell (1996). For the former, we consider a simplified one-period version which was also previously used by Bennett and van Damme (1991) in Apex games, a type of weighted majority game. For the latter, we consider a particular case where a proposer whose proposal is rejected leaves the game with probability 1. Our paper is similar to Fréchette et al. (2005) who experimentally compare the offer-based model of Baron and Ferejohn (1989) with the demand-based model of Morelli (1999) in weighted majority voting games. The earlier experimental studies of the Baron–Ferejohn model are Fréchette et al. (2003, 2005b), while the experimental study of demand bargaining is Fréchette et al. (2005a).[3] Fréchette et al. (2005) is, however, the first paper to directly compare the two within an experimental framework. Their results show that proposers have some first mover advantage in both the demand and offer games, but their power does not differ nearly as much between the two mechanisms as theory predicts.

3 Theoretical model

3.1 Cooperative TU games and solutions

Let $N = \{1, \ldots, n\}$ be a finite set of *players*. Each subset $S \subseteq N$ is called a *coalition*, and N is called the *grand coalition*. A *cooperative TU game* (from now on, *cooperative game*) consists of a couple (N, v), where N is the set of players and $v : 2^N \to \mathbb{R}$ is the *characteristic function*, which assigns to each coalition $S \subseteq N$ the *worth* $v(S)$, with the normalization condition $v(\emptyset) = 0$. The worth of a coalition represents the value that members of S can achieve by agreeing to cooperate. To simplify the notation and if no ambiguity appears, we consider the set of players N fixed and we write v instead of (N, v). We denote with $\mathcal{G}^N$ the set of all games with player set N.

A game $v \in \mathcal{G}^N$ is said to be

- *monotonic* if $v(S) \leq v(T)$ for each $S \subseteq T \subseteq N$,
- *superadditive* if $v(S) + v(T) \leq v(S \cup T)$ whenever $S \cap T = \emptyset$, with $S, T \subseteq N$,
- *convex* if $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$, for each $S, T \subseteq N$, and *strictly convex* if the inequality holds strictly.

We may observe that convexity $\Rightarrow$ superadditivity $\Rightarrow$ monotonicity. In (strictly) convex games, cooperation becomes increasingly appealing, and a so-called "snowball effect"

[3] Fiorina and Plott (1978) also propose multiple experiments on committee decision-making under majority rules to test a wide range of solution concepts of noncooperative games.

is expected, leading to the formation of the grand coalition. Another equivalent definition for convexity can be stated as $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$, for each $S \subseteq T \subseteq N \setminus \{i\}$.

Given a game $v \in \mathcal{G}^N$, an *allocation* is an n-dimensional vector $(x_1, \ldots, x_n) \in \mathbb{R}^N$ assigning to player i the amount $x_i \in \mathbb{R}$. For each $S \subseteq N$, we denote $x(S) = \sum_{i \in S} x_i$. The *imputation set* is defined by

$$I(v) = \{x \in \mathbb{R}^n | x(N) = v(N) \text{ and } x_i \geq v(\{i\}) \ \forall i \in N\},$$

i.e., it contains all the allocations that are *efficient* ($x(N) = v(N)$) and *individually rational* ($x_i \geq v(\{i\}) \forall i \in N$).

The core is the set of imputations that are also *coalitionally rational*, i.e.,

$$C(v) = \{x \in I(v) | x(S) \geq v(S) \ \forall S \subseteq N\}.$$

An element of the core is stable in the sense that if such a vector is proposed as an allocation for the grand coalition, no coalition will have an incentive to split off and cooperate on its own. Intuitively, the idea behind the core is analogous to that behind a (strong) Nash equilibrium of a noncooperative game: an outcome is stable if no deviation is profitable. For the Nash equilibrium, the possible deviation is for a single player, while in the core we speak about deviations of groups of players.

A *solution* is a function $\psi : \mathcal{G}^N \to \mathbb{R}^N$ that assigns an allocation $\psi(v)$ to every game $v \in \mathcal{G}^N$. The *Shapley value* is the best-known solution concept, which is widely applied in economic models, and is defined as

$$\phi_i(v) = \sum_{S \subseteq N, i \in S} \frac{(|S| - 1)!(|N| - |S|)!}{|N|!} (v(S) - v(S \setminus \{i\})) \ \forall i \in N.$$

The Shapley value assigns to every player its expected marginal contribution to the coalition of players that enter before him, given that every order of entrance has equal probability. This solution concept has been defined as respecting some notion of fairness (see Section 3.2 for more discussion about its properties), but it is not, on the contrary, necessarily stable. However, if the game is superadditive, the Shapley value is an imputation, and if the game is convex, it belongs to the core (in particular, it is its barycenter).

Another solution concept, the *equal division solution*, distributes the worth $v(N)$ of the grand coalition equally among all players in any game, it is then defined as

$$ED_i(v) = \frac{v(N)}{n} \; \forall i \in N.$$

3.2 Axiomatizations of the game theoretical solutions

We provide two more definitions, which are used in the following.

Players i and j are *symmetric* in $v \in \mathcal{G}^N$, if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$. Player i is a *null player* in $v \in \mathcal{G}^N$ if $v(S) = v(S \setminus \{i\})$ for all $S \subseteq N$.

In the literature, we can find various axiomatic characterizations of the cooperative solutions and, in particular, of the Shapley value. Given a solution $\psi : \mathcal{G}^N \to \mathbb{R}^N$, we list here some of the most commonly used axioms to provide a characterization.

Axiom 1 (Efficiency): for every v in $\mathcal{G}^N$, $\sum_{i \in N} \psi_i(v) = v(N)$.

Axiom 2 (Symmetry): if i and j are symmetric players in game $v \in \mathcal{G}^N$, then $\psi_i(v) = \psi_j(v)$.

Axiom 3 (Additivity): for all $v, w \in \mathcal{G}^N$, $\psi(v + w) = \psi(v) + \psi(w)$.

Axiom 4 (Homogeneity): for all $v \in \mathcal{G}^N$ and $a \in \mathbb{R}$, $\psi(av) = a\psi(v)$.

Axiom 5 (Null player property): if i is a null player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$.

Axiom 6 (Strong monotonicity): if $i \in N$ is such that $v(S \cup \{i\}) - v(S) \leq w(S \cup \{i\}) - w(S)$ for each $S \subseteq N$, then $\psi_i(v) \leq \psi_i(w)$.

Axiom 7 (Fairness): if i, j are symmetric in $w \in \mathcal{G}^N$, then $\psi_i(v + w) - \psi_i(v) = \psi_j(v + w) - \psi_j(v)$ for all $v \in \mathcal{G}^N$.
Fairness states that if to a game $v \in \mathcal{G}^N$ we add a game $w \in \mathcal{G}^N$ in which players i and j are symmetric, then the payoffs of players i and j change by the same amount.

In particular, among the many others, the axiomatization of Shapley (1953), which is the most classical one, involves axioms 1, 2, 3, and 5. That of Young (1985) involves axioms 1, 2, and 6, while that of van den Brink (2002) involves axioms 1, 5, and 7. Note that axiom 4, even if not directly involved in any of these axiomatizations, is crucial, together with axiom 3, as it guarantees the linearity of the solution.[4]

4 Two mechanisms

In this section, we present the demand-based Winter mechanism (Section 4.1) and the offer-based H-MC mechanism (Section 4.2) in more detail. Section 4.3 presents an example of the implementation of the two mechanisms.

4.1 The Winter mechanism

Winter (1994) presents a bargaining model based on sequential demands for strictly convex cooperative games. We recall that in such games, cooperation becomes increasingly appealing, and a snowball effect is expected, leading to the formation of the grand coalition. Moreover, in convex games, the Shapley value is a central point in the core, which is always nonempty.

In this model, players in turn announce publicly their demands, meaning "I am willing to join any coalition yielding me a payoff of ..." and wait for these demands to be met by other players. The bargaining starts with a randomly chosen player from N, say player i. This player announces publicly her demand d_i and then chooses a second player who has to give her demand. Then, the game proceeds by having each player pointing at a new player to take her turn after introducing a demand herself. If and when at some point a compatible demand is introduced, which means that there exists a coalition S for which the total demand for players in S does not exceed $v(S)$, then the first player with such a demand selects a compatible coalition S. The players in S get

[4]The equal division solution satisfies 1, 2, and 3, but it does not satisfy the null player property in 5. However, it satisfies a similar property when null players are replaced with nullifying players. Player i is a *nullifying player* if $v(S) = 0$ for each $S \subseteq N$ such that $i \in S$. Then, we can state the following additional axiom that can be called the Nullifying player property: if i is a nullifying player in game $v \in \mathcal{G}^N$, then $\psi_i(v) = 0$. Replacing the null player property in the axiomatization of the Shapley value in Shapley (1953) with the nullifying player property characterizes the equal division solution (see van den Brink, 2006).

their demands, leave the game, and the bargaining continues with the rest of the players using the same rule on v restricted on $N \setminus S$.

We present here more formally the one-period Winter mechanism. This is a simplified version of the more general mechanism in Winter (1994), which allows for more periods and includes a discount factor. A decision point position at time t of the one-period demand commitment game is given by the vector $(S_1^t, S_2^t, d_{S_2^t}, j)$, where:

$S_1^t \subseteq N$ is the set of players remaining in the game,

$S_2^t \subset S_1^t$ is the set of players who have submitted demands that are not yet met,

$d_{S_2^t} = (d_i)_{i \in S_2^t}$ is the vector of demands submitted by players in S_2^t, $(0 \leq d_i \leq \max_{S \subseteq N} v(S))$, and

$j \in S_1^t \setminus S_2^t$ is the player taking the decision by introducing a demand d_j. Her demand d_j is said to be *compatible* if there exists some $S \subseteq S_2^t$ with $v(S \cup \{j\}) - \sum_{i \in S} d_i \geq d_j$. Otherwise, d_j is not compatible.

With j's decision, the game proceeds now in the following way.

1) If d_j is compatible, then j specifies a compatible coalition S, each player $i \in S \cup \{j\}$ is paid d_i and nature chooses randomly a player $k \neq j$ from $S_1^t \setminus S_2^t$. The new position is now given by $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t \setminus (S \cup \{j\})$ and $S_2^{t+1} = S_2^t \setminus (S \cup \{j\})$.

2) If d_j is noncompatible, then two cases are distinguished:

2_a) if $S_2^t = S_1^t \setminus \{j\}$ (j is the last player to demand), then each player $i \in S_1^t$ (j included) gets her individual payoff $v(\{i\})$, and the game ends;

2_b) if $S_2^t \subset S_1^t \setminus \{j\}$, then j specifies a new player $k \neq j$ in $S_1^t \setminus S_2^t$ and the new position is $(S_1^{t+1}, S_2^{t+1}, d_{S_2^{t+1}}, k)$, with $S_1^{t+1} = S_1^t$ and $S_2^{t+1} = S_2^t \cup \{j\}$.

The game starts with the random selection of a player $j \in N$. Then, the initial position is set to be $(N, \emptyset, d_\emptyset, j)$. It terminates either when there are no more players in the game (see point 1 above), or when $S_1^t \cup \{j\} = S_2^t$ (see point 2_a above).

As shown by Winter for the more generic case, this mechanism has a unique subgame perfect equilibrium that assigns equal probabilities at indifference. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. Moreover, given a specific ordering of the players, the *a posteriori* equilibrium payoff of each player depends only on the order of the set of her successors, and not on the way these players are ordered, as each player demands the marginal contribution to the set of her successors.

4.2 The Hart and Mas-Colell mechanism

Hart and Mas-Colell (1996) propose a bargaining procedure for monotonic cooperative games. We may observe that this is a much weaker assumption compared with the strict convexity required by the Winter mechanism. Thus, the H-MC procedure is applicable for a larger set of cooperative games.

In this mechanism, the bargaining starts with a randomly chosen proposer making an offer to the other players, meaning "If you wish to form a coalition with me, I will give you...". Then, the other players may either accept or reject the proposal. The requirement for agreement is unanimity. The key modeling issue is the specification of what happens if there is no agreement and, as a consequence, the game moves to the next stage. In our implementation, if the proposal is rejected, the proposer leaves the game with her individual value and the bargaining continues with the rest of the players, with a new player randomly chosen as the new proposer.

We present here a more formal description of the H-MC mechanism. A decision point position at time t is simply given by the vector (S^t, j), where:

$S^t \subseteq N$ is the set of players remaining in the game,

$j \in S^t$ is the player making an offer to the remaining players $(t_i)_{i\in S^t\setminus\{j\}}$ such that $\sum_{i\in S^t\setminus\{j\}} t_i \leq v(S^t)$.

With j's proposal, the game proceeds now in the following way.

1) If each $i \in S^t \setminus \{j\}$ independently and simultaneously accepts the proposal, then players in $S^t \setminus \{j\}$ are paid $(t_i)_{i\in S^t\setminus\{j\}}$, player j is paid $v(S^t) - \sum_{i\in S^t\setminus\{j\}} t_i$, and the game ends;

$2)$ If at least one player $i \in S^t \setminus \{j\}$ refuses the offer, then two cases are distinguished:

$2a)$ if $|S^t| = 2$ (only one more player is left, together with j), then they both get their individual value $v(\{i\})$ for each $i \in S^t$, and the game ends;

$2b)$ if $|S^t| > 2$, then player i is removed from the game, she gets her individual payoff $v(\{i\})$, a new proposer $k \in S^{t+1} = S^t \setminus \{j\}$ is randomly selected, and the new position is (S^{t+1}, k).

The game starts with a randomly chosen player $j \in N$. Then the initial position is set to be (N, j). It terminates either when there are no more players in the game (see point $2a$ above), or when the proposal is unanimously accepted (see point 1 above).

Hart and Mas-Colell (1996) show that this game has a unique subgame perfect equilibrium. At this equilibrium, the grand coalition forms and the *a priori* expected equilibrium payoff coincides with the Shapley value. Differently from the Winter mechanism, given a specific initial proposer $j \in N$ (in the previous mechanism, it was necessary to specify the order of all the players, while in this case only one player having the role of proposer needs to be specified at an equilibrium), the *a posteriori* equilibrium payoff assigns to each other player her Shapley value in the cooperative game reduced to the set of players $N \setminus \{j\}$, and to the proposer, the marginal contribution to the grand coalition $v(N) - v(N \setminus \{j\})$.

4.3 A comparison between the Winter and the H-MC mechanisms

We illustrate the two mechanisms using the strictly convex three-player game shown in Table 1. Although our experiment is based on four-player games, a three-player game example is of particular interest because it allows us to graphically represent the imputation set, the core, and the different solutions, as will be illustrated in Figure 1.

Assume that player 1 is chosen randomly as the first proposer in both the mechanisms. Independent of the order of the following players in the Winter mechanism, she will get an *a posteriori* equilibrium payoff equal to 40 under both mechanisms, which corresponds to her marginal contribution to the grand coalition $v(N) - v(N \setminus \{1\})$. We may see how both the mechanisms show a proposer advantage, as $40 > \frac{170}{6}$, meaning that, as the first proposer, player 1 can get more than her Shapley value.

Table 1: A three-player game.

S	1	2	3	1,2	1,3	2,3	N
$v(S)$	20	20	30	45	55	60	100

Assume that the total ordering of the players in the Winter mechanism is given by 1, 2, and 3. The *a posteriori* equilibrium payoff of the Winter mechanism is given by the vector $SOL_W(v) = (40, 30, 30)$, in which player 2 demands her marginal contribution $v(\{2, 3\}) - v(\{3\})$, and player 3 her individual value $v(\{3\})$.

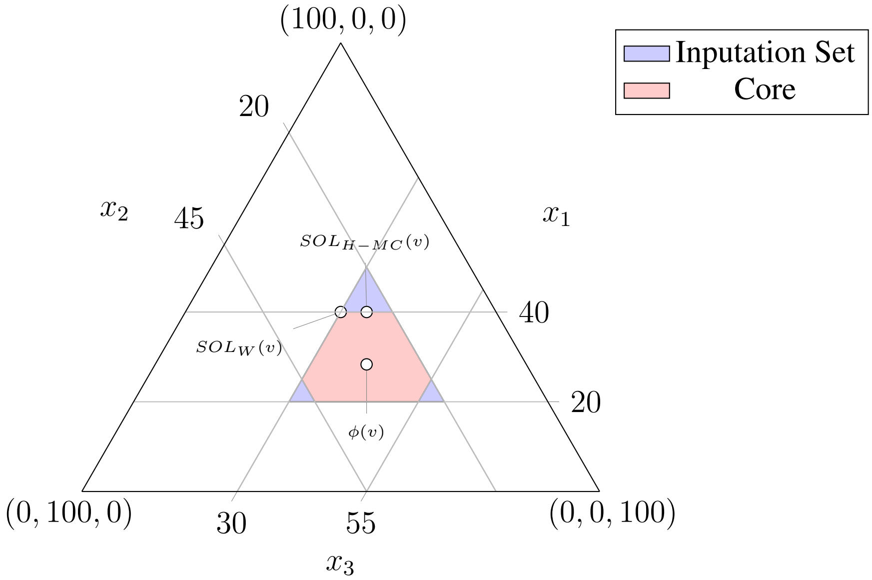
In the H-MC mechanism, on the other hand, the proposer offers the Shapley value of the reduced game to players 2 and 3. Thus, the *a posteriori* equilibrium payoff is given by the vector $SOL_{H-MC}(v) = (40, 25, 35)$. Even with the disadvantage of not being the first mover, player 2, as the second mover, manages to get more under the Winter mechanism than under the H-MC mechanism, even if, in both cases, she gets less than her Shapley value.

As we have already observed, the convexity assumption implies monotonicity. Thus, the game satisfies the assumptions of both the Winter and the H-MC mechanisms. The Shapley value of this game is given by the vector $\phi(v) = \left(\frac{170}{6}, \frac{185}{6}, \frac{245}{6}\right) = (28.33, 30.83, 40.83)$, which corresponds to the *a priori* equilibrium payoff for both the Winter and the H-MC mechanisms.

Figure 1 shows the imputation set $I(v) = co\,\langle(20, 50, 30), (50, 20, 30), (20, 20, 60)\rangle$, the core $C(v) = co\,\langle(40, 30, 30), (40, 20, 40), (25, 20, 55), (20, 25, 55), (20, 45, 35), (25, 45, 30)\rangle$, the Shapley value $\phi(v)$, and the two *a posteriori* solutions $SOL_W(v)$ and $SOL_{H-MC}(v)$ for the specific ordering 123 in a simplex. A point in the simplex corresponds to an allocation (x_1, x_2, x_3). For example, the height of a point from the edge that is opposite from the apex labeled $(100, 0, 0)$ represents the payoff allocated to player 1, thus a point on the bottom edge represents an observed allocation that gives 0 payoff to player 1. Similarly, the height of a point from the edge that is opposite from the apex labeled $(0, 0, 100)$ represents the payoff allocated to player 3.

We make the following two observations to conclude this example and the comparison between the two mechanisms.

Figure 1: The core of the three-player game.



Observation 1. *The core is always a convex polyhedron. The a posteriori equilibrium when implementing the Winter mechanism always coincides with a vertex of this polyhedron. The a posteriori equilibrium when implementing the H-MC mechanism always provides a vector on a face of this polyhedron.*

Observation 2. *In the H-MC mechanism, the proposer is forced to offer feasible demands, i.e., if S is the set of players remaining in the game, she has to propose a total distribution of payoff not bigger than $v(S)$. In the Winter mechanism, on the other hand, the players, speaking one after the other, may make unfeasible demands. Then, the formation of a coalition, in the H-MC mechanism, is simply given by the choice of the players to accept or not the proposal, while for the Winter mechanism, it can be blocked by some unfeasibility conditions.*

Table 2: The games.

S	1	2	3	4	1,2	1,3	1,4	2,3	2,4	3,4	1,2,3	1,2,4	1,3,4	2,3,4	N
$v_1(S)$	0	5	5	10	20	20	25	20	25	25	50	60	60	60	100
$v_2(S)$	0	20	20	30	20	20	30	45	55	60	45	55	60	100	100
$v_3(S)$								$= v_1(S) + v_2(S)$							
$v_4(S)$								$= 2v_1(S)$							

5 The experimental setting

5.1 The games

For our analysis, we implemented the four four-player games shown in Table 2. Note that:

- games 1, 3, and 4 are strictly convex, while game 2 is only convex. All four games are, by consequence, monotonic. Therefore, all four games respect the assumptions for the implementation of the H-MC mechanism, while all but game 2 respect the assumption for the implementation of the Winter mechanism. With game 2 being at least convex, however, we consider that "strict convexity" could be relaxed and the mechanism could still be implemented in such a case;

- in games 1 and 4, players 2 and 3 are symmetric. They will be used to test the **symmetry** axiom;

- in game 2, player 1 is a null player. This is why the game is only convex, but not strictly convex, as the presence of a null player does not allow, by definition, the possibility of having a strictly increasing marginal contribution for such a player. It will be used to test the **null player** axiom;

- game 3 is defined as the sum of games 1 and 2. It will be used to test the **additivity** and the **fairness** axioms;

- game 4 is defined as twice game 1 and it preserves the symmetry of players 2 and 3. It will be used to test the **symmetry** and the **homogeneity** axioms;

Table 3: The Shapley value of games 1, 2, 3, and 4.

	$\phi_1(v)$	$\phi_2(v)$	$\phi_3(v)$	$\phi_4(v)$
Game 1	22.08	23.75	23.75	30,42
Game 2	0	28.33	30.83	40.83
Game 3	22.08	52.08	54.58	71.25
Game 4	44.16	47.5	47.5	60.83

- the marginal contributions of player 1 are always higher in game 1 than in game 2, and also higher in game 4 than in game 3. Then, the payoffs of player 1 in the four games will be used to check the **strong monotonicity** axiom.

The Shapley values of the four games are presented in Table 3. The equal division payoff vector is simply equal to $ED(v_k) = (25, 25, 25, 25)$ when $k = 1, 2$, and $ED(v_k) = (50, 50, 50, 50)$ when $k = 3, 4$.

5.2 The hypotheses

Our hypotheses rely on the theoretical predictions of the implementation of the Winter and the H-MC mechanisms, which we presented in Sections 4.1 and 4.2, on the properties of the Shapley value solution, which we presented in Sections 3.1 and 3.2, and on some behavioral assumptions.

Our first hypothesis (**H1**) investigates the capability of the players to cooperate together and form the grand coalition. Recall that if the players play according to the equilibrium, with both mechanisms, they should form the grand coalition 100% of the times. However, in Observation 2, we noted that the H-MC mechanism forces feasible offers, while the Winter mechanism may see unfeasible demands. Because of this, we expect the subjects to succeed in forming the grand coalition in the H-MC mechanism more often than in the Winter mechanism. As a result, we expect that efficiency is higher for the H-MC mechanism than the Winter mechanism.

H1 *(a) The proportion of times the grand coalition forms is higher under the H-MC mechanism than under the Winter mechanism. As a result, (b) efficiency is higher under the H-MC mechanism than under the Winter mechanism.*

Our second hypothesis (**H2**) is about the convergence of our mechanisms to the expected *ex ante* equilibrium prediction, the Shapley value. Note, however, if the frequencies of the grand coalition formation are low, one should not expect the average payoffs to follow the Shapley value. We, therefore, investigate whether the average payoff shares, instead of payoffs themselves, follow the Shapley value. Because the theory suggests both mechanisms implement the Shapley value, we expect both mechanisms to do so equally.

> **H2** *The H-MC and Winter mechanisms equally implement on average the Shapley value solution in terms of payoff shares.*

In contrast, when it comes to *ex post* equilibrium prediction, the two mechanisms differ. As noted in Observation 1, the Winter mechanism provides solutions on the vertexes of the core, while the H-MC mechanism does so on the faces of the polyhedron. Thus, the latter is closer to the barycenter of the core (i.e., for convex games, to the Shapley value) than the former. This leads to our third hypothesis (**H3**).

> **H3** *The H-MC mechanism provides ex post payoff shares that are closer to the Shapley value than the Winter mechanism.*

Our fourth hypothesis (**H4**) is a behavioral hypothesis, and is based on the literature according to which an offer-based mechanism should provide some payoff allocations that are closer to the equal division solution.

> **H4** *The payoff shares resulting from the H-MC mechanism are closer to an equal division solution than those resulting from the Winter mechanism.*

Our last hypothesis (**H5**) is whether the axioms we presented in Section 3.2 are satisfied or not, at least in terms of payoff shares (except for efficiency (Axiom 1), which is about resulting payoffs). As noted above in stating **H2**, we expect both mechanisms to equally satisfy (at least in terms of payoff shares) various axioms.

> **H5** *The payoff shares resulting from the implementation of the H-MC and the Winter mechanisms equally satisfy the axioms 2–7 that characterize the Shapley value.*

6 Results

The experiment was conducted at the Institute of Social and Economic Research (ISER), Osaka University, between January and December 2019. A total of 180 students, who had never participated in similar experiments before, were recruited as subjects of the experiment, 96 playing the Winter mechanism and 84 playing the H-MC mechanism.[5] The experiment was computerized with z-Tree (Fischbacher, 2007) and participants were recruited using ORSEE (Greiner, 2015).

To control for potential ordering effects, each participant played all four games twice in one of the following four orderings: 1234, 2143, 3412, and 4321. Between each play of a game (called a round), players were randomly rematched into groups of four players, and participants were randomly assigned a new role within the newly created group. At the end of the experiment, two rounds (one from the first four rounds and another from the last four rounds) were randomly selected for payments. Participants received cash rewards based on the points they earned in these two selected rounds with an exchange rate of 20 JPY = 1 point in addition to the 1500 JPY participation fee. The experiment lasted on average 100 min for Winter and 90 min for H-MC including the instructions (15 min for Winter and 11 min for H-MC), a comprehension quiz (5 min), and payment.[6] Average earnings were 2650 JPY for Winter and 2780 JPY for H-MC.

6.1 Grand coalition formation and efficiency

Figure 2 presents the results of the grand coalition formation, in the H-MC mechanism and in the Winter mechanism, for the four games.[7]

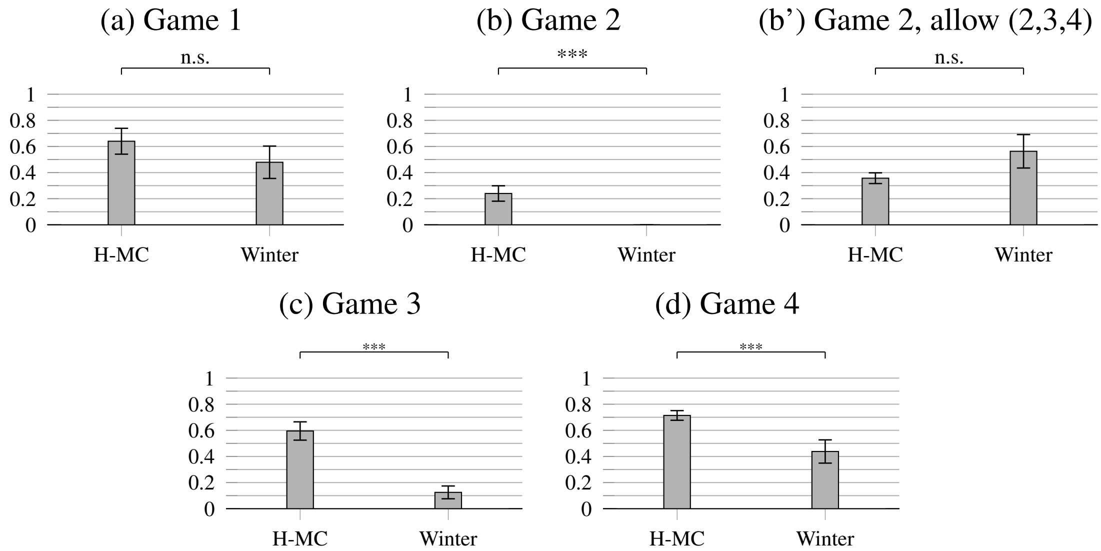
For game 2 under the Winter mechanism, the grand coalition never forms (because player 1 is a null player and, consequently, the game is only convex and not strictly

[5]The difference in the number of participants between the two mechanisms is a result of variations in the show-up rate among experimental sessions.

[6]Participants received a copy of the instruction slides, and prerecorded instruction movies were played. See Appendix D for English translations of the instruction slides and the comprehension quiz.

[7]The figure is created based on the estimated coefficients of the following linear regressions: $gc_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$ where gc_i is a dummy variable that takes the value of 1 if the grand coalition is formed, and zero otherwise, in group i, HMC_i ($Winter_i$) is a dummy variable that takes the value of 1 if the H-MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

Figure 2: H-MC and Winter mechanisms: proportion of times the grand coalition formed.



Note: Error bars show one standard error range. *** indicates the proportion of times the grand coalition formation was significantly higher for the H-MC implementation than the Winter implementation at the 1% significance level (Wald test).

convex[8]). Therefore, for game 2, we also consider the partition $\{1\}$, $\{2,3,4\}$ as a realization of the grand coalition for both the H-MC and the Winter mechanisms.

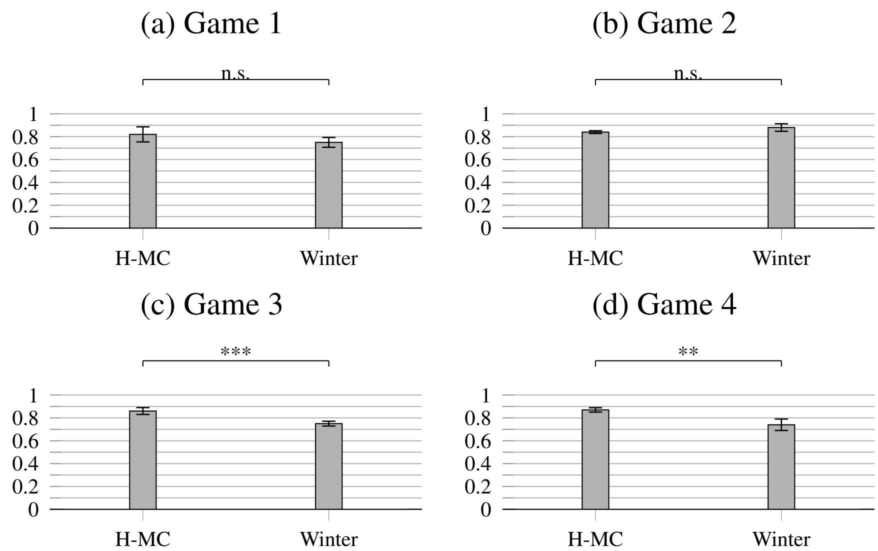
The H-MC mechanism manages to enhance complete cooperation (in the case of game 2, and either the grand coalition or the $\{2,3,4\}$ coalition) in 60.4% of the cases. The Winter mechanism, however, manages to enhance complete cooperation in only 41.1% of the cases. Although the grand coalition is not formed frequently, we observe that it is formed more frequently under the H-MC mechanism in three of the four games, compared with the Winter mechanism. In particular, for games 3 and 4, this difference is significant at the 1% level.

As a direct consequence of the low frequencies of grand coalition formation, we note that both mechanisms fail to achieve full efficiency. Efficiency is computed as the fraction of the sum of the payoffs obtained by the four players compared with the value

[8]Recall that the Winter mechanism is theoretically defined for strictly convex games. Player 1, in this game, always has a zero marginal contribution and, as such, can be left out of any coalition at no cost to either him/her or the other players

of the grand coalition of the considered game (100 for games 1 and 2 and 200 for games 3 and 4). However, as Figure 3 shows, efficiency is higher under the H-MC mechanism than under the Winter mechanism in games 1, 3, and 4. In particular, these differences are significant at the 1% level for game 3, and at the 5% level for game 4.[9]

Figure 3: Efficiency.



(a) Game 1 (b) Game 2 (c) Game 3 (d) Game 4

Note: Error bars show the one standard error range. *** and ** indicate the proportion of times that verification of the efficiency axiom was significantly higher for the H-MC implementation than the Winter implementation at the 1 and 5% significance levels (Wald test).

Therefore, we conclude the following.

Result 1. *Although the grand coalition is not formed frequently under the two mechanisms, it is more frequently formed under the H-MC mechanism than under the Winter mechanism. Consequently, efficiency is higher under the H-MC mechanism than under the Winter mechanism. Thus,* ***H1*** *is verified.*

[9]The figure is created based on the estimated coefficients of the following linear regressions: $Eff_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$ where $Eff_i \equiv \frac{\sum_i \pi_i}{v(N)}$ is the efficiency measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes the value of 1 if the H-MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of two treatment dummies.

6.2 Payoff shares

We denote by $\pi^{H-MC}(v_k)$ a vector of payoffs obtained by the players while implementing the H-MC mechanism on game k, with $k = 1, 2, 3, 4$. Analogously, we denote by $\pi^W(v_k)$ a vector of payoffs obtained by the players while implementing the Winter mechanism. The *ex ante* theoretical prediction of both mechanisms states that the mean of such vectors when implementing either the H-MC or the Winter mechanism many times with different orderings of the players should converge to the Shapley value. In the first part of this section, we test such a hypothesis.

As a consequence of the players often failing to form the grand coalition, as observed in Section 6.1, the total share of utility of a payoff vector is often smaller than the value of the grand coalition (see Figure 3). As a result, the average realized payoff vectors are significantly different from the Shapley value as shown in Figure 6 of Appendix A. Therefore, we focus on analyzing the mean of the normalized (to the value of the grand coalition) payoff vectors, instead of the realized payoff vectors themselves. This aims to investigate whether the proportion of the power share, in lieu of the absolute payoffs, converges to the Shapley value. For this reason, in this and in the following sections, we consider the normalized vectors of payoffs with components $\widetilde{\pi}_i^W(v_k) = \frac{\pi_i^W(v_k)}{\sum_{j\in N}\pi_j^W(v_k)} \times v_k(N)$ and $\widetilde{\pi}_i^{H-MC}(v_k) = \frac{\pi_i^{H-MC}(v_k)}{\sum_{j\in N}\pi_j^{H-MC}(v_k)} \times v_k(N)$ for each $i = 1, 2, 3, 4$ (recall that the value of the grand coalition is equal to 100 for games 1 and 2 and to 200 for games 3 and 4).[10]

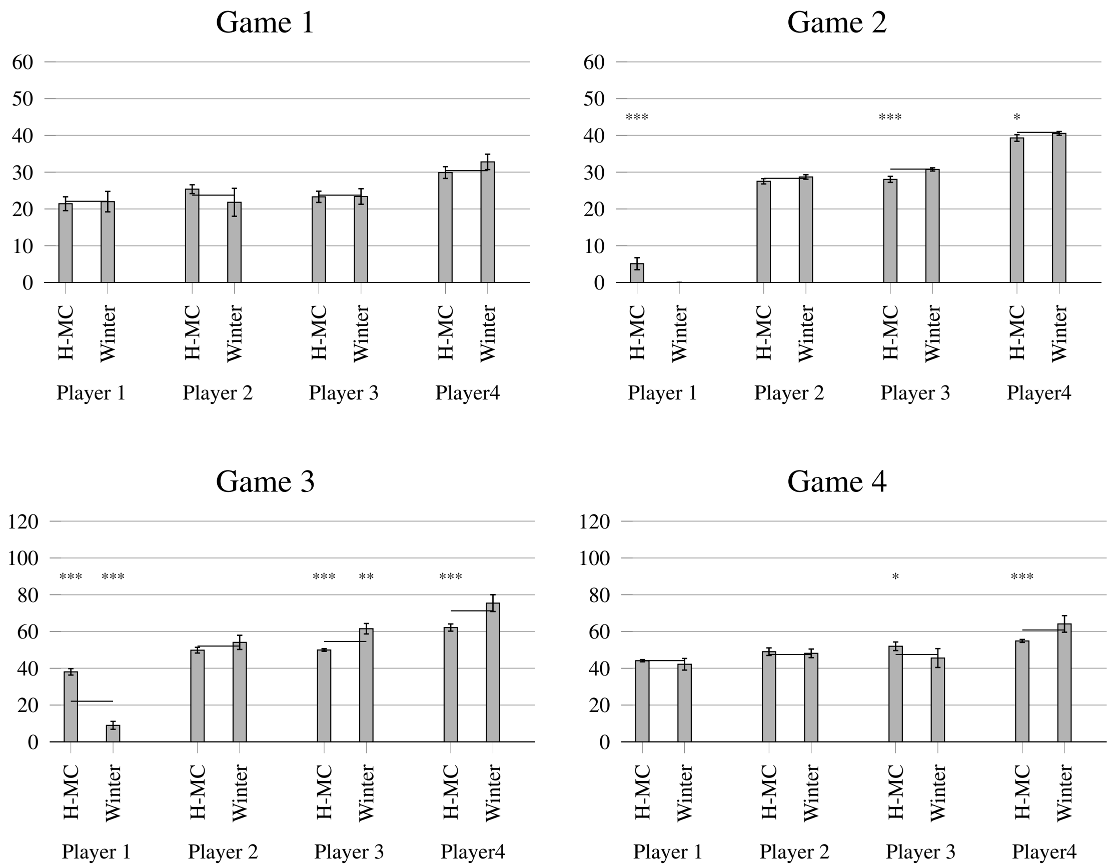
Figure 4 shows the mean of the normalized payoffs in the four games; the horizontal lines indicate the Shapley values for each game.[11] One can observe that, while for the Winter mechanism, the average normalized payoffs are not significantly different from the Shapley value for all four players in games 1, 2, and 4, for the H-MC mechanism, the average normalized payoffs for all four players differ from the Shapley value only for game 1.

Result 2. *The average payoff shares follow the Shapley value more closely under the*

[10]In Appendix C, we report the results based only on the realized payoff allocations when grand coalitions are formed. The results are qualitatively the same.

[11]The error bars are based on the standard errors that are corrected for within-session clustering effects. These standard errors are obtained by estimating the system of linear regressions described in Section 6.3. The standard errors are corrected for session-level clustering effects. The statistical tests are based on these regressions.

Figure 4: Mean of the normalized payoffs.



Note: The horizontal lines indicate the Shapley values. Error bars show the one standard error range. ***, **, and * indicate the average normalized payoff being significantly different from the Shapley value at the 1, 5, and 10% significance levels (Wald test).

Winter mechanism than under the H-MC mechanism. Thus, we reject ***H2*** *that states that both mechanisms implement the Shapley value equally.*

Furthermore, we observe from Figure 4 that the **null player** property **(Axiom 5)** is always verified for the Winter mechanism (100% of the time), as the null player 1, in game 2, always gets 0 as a payoff. On the contrary, this property is not verified for the H-MC mechanism, in which the same player in the same game gets on average a payoff equal to 5.44, and, in particular, she gets a payoff of 0 in only 28 out of 48 implementations of the game (58.33% of the time). We interpret this result as a consequence of the offer-based nature of the H-MC mechanism because the players may feel uncomfortable offering a 0 payoff, even to a player who is recognized to not have a useful role in the cooperation, for fear of the proposal being rejected.

We now turn to investigate **H3** and **H4**, which relate to *ex post* payoffs. We measure the distance between the realized normalized payoff vectors and the Shapley value, as well as the equal division solution, for the four games by the Euclidean distances between the two. Namely, we compute, $Dis2_\phi = \sqrt{\sum_i(\widetilde{\pi}_i - \phi_i)^2}$ and $Dis2_{ED} = \sqrt{\sum_i(\widetilde{\pi}_i - ED_i)^2}$, respectively. In the formula, for the sake of simplicity, we omit the specifications about the considered mechanism and the game. Figure 5 shows the mean $Dis2_\phi$ and the mean $Dis2_{ED}$ of such distances for the two mechanisms in the four games.[12]

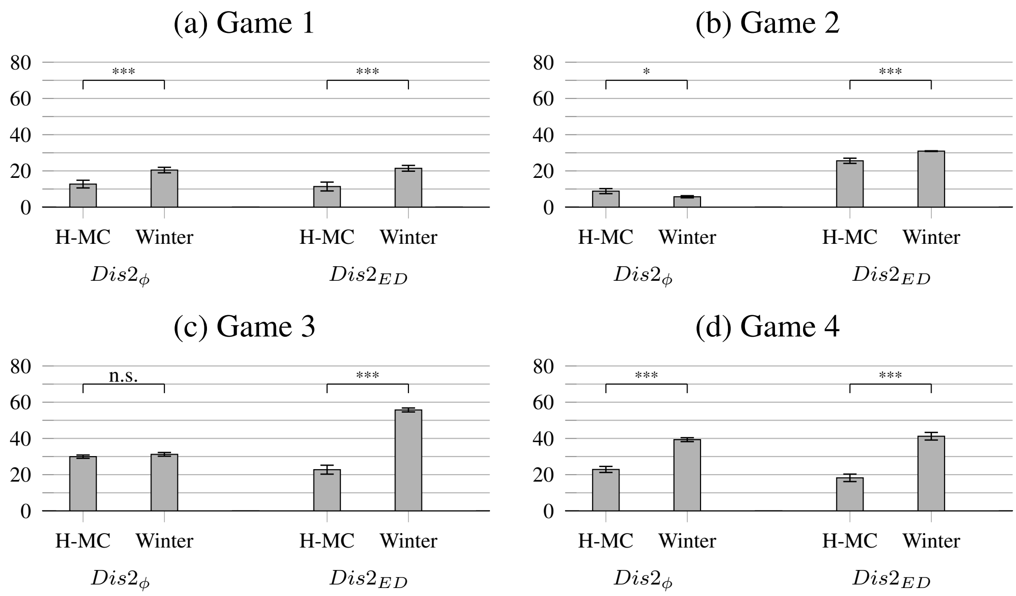
The distance to the Shapley value is smaller for the H-MC mechanism than the Winter mechanism in games 1, 3, and 4, and significantly so for games 1 and 4, at the 1% level. The only exception is game 2, in which the distance is smaller for the Winter mechanism than the H-MC mechanism at the 10% level. Furthermore, the distance to the equal division solution is always significantly lower (at the 1% level) for the H-MC mechanism than for the Winter mechanism.

Result 3. *The H-MC mechanism results in payoff shares that are more equal and also closer to the Shapley value than the Winter mechanism. Thus, both **H3** and **H4** are verified.*

Such results reflect the very nature of the two mechanisms that we have already indicated when formulating the hypotheses. In fact, the *ex post* equilibrium predictions state that the Winter mechanism should provide solutions on the vertexes of the core, while the H-MC mechanism provides solutions on the faces of the polyhedron, and are thus closer to its barycenter (i.e., for convex games, to the Shapley value).

[12]The figure is created based on the estimated coefficients of the following linear regressions: $Dis_i = \beta_1 HMC_i + \beta_2 Winter_i + \mu_i$ where Dis_i is the relevant distance measure for group i, HMC_i ($Winter_i$) is a dummy variable that takes the value of 1 if the H-MC (Winter) mechanism is used, and zero otherwise. The standard errors are corrected for within-session clustering effects. The statistical tests are based on the Wald test for the equality of the estimated coefficients of the two treatment dummies.

Figure 5: Mean of the distances of the normalized payoff vectors from the Shapley value and the equal division solutions.



Note: Error bars show the one standard error range. ***, **, and * indicate the distance of the normalized payoff vectors from the Shapley value or from the equal division solution was significantly different between the H-MC and the Winter implementation, at the 1, 5, and 10% significance levels (Wald test).

6.3 Testing the axioms

We have already noted that both mechanisms fail to satisfy efficiency **(Axiom 1)**, and while the Winter mechanism satisfies the **null player** property **(Axiom 5)**, the H-MC mechanism fails to do so. We now test the remaining axioms.

To test **symmetry (Axiom 2)**, **additivity (Axiom 3)**, **homogeneity (Axiom 4)**, **strong monotonicity (Axiom 6)**, and **fairness (Axiom 7)**, we estimate a set of OLS regressions for the following system of equations, with dependent variables being the average normalized payoffs for player i, $\widetilde{\pi}_i$, and the independent variables being g_1, g_2,

Table 4: Wald tests of the H-MC and Winter mechanisms for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms.

Axiom	H_0	H-MC χ^2	H-MC p-value	Winter χ^2	Winter p-value
Symmetry	$b_1 = c_1$	1.01	0.314	0.08	0.781
	$b_4 = c_4$	0.47	0.492	0.14	0.712
Additivity	$a_3 = a_1 + a_2$	36.91	0.000	7.25	0.007
	$b_3 = b_1 + b_2$	1.11	0.292	0.65	0.422
	$c_3 = c_1 + c_2$	0.53	0.466	2.54	0.111
	$d_3 = d_1 + d_2$	4.78	0.0288	0.35	0.555
Homogeneity	$a_4 = 2a_1$	0.16	0.689	0.06	0.805
	$b_4 = 2b_1$	0.28	0.598	0.37	0.542
	$c_4 = 2c_1$	5.90	0.015	0.02	0.892
	$d_4 = 2d_1$	3.23	0.072	0.35	0.552
Strong monotonicity	$a_1 = a_2$	23.87	0.000	62.74	0.000
	$a_4 = a_3$	11.55	0.001	147.12	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	0.15	0.694	7.53	0.006

g_3, g_4 and without a constant:

$$\begin{aligned}
\widetilde{\pi}_1 &= a_1 g_1 + a_2 g_2 + a_3 g_3 + a_4 g_4 + u_1 \\
\widetilde{\pi}_2 &= b_1 g_1 + b_2 g_2 + b_3 g_3 + b_4 g_4 + u_2 \\
\widetilde{\pi}_3 &= c_1 g_1 + c_2 g_2 + c_3 g_3 + c_4 g_4 + u_3 \\
\widetilde{\pi}_4 &= d_1 g_1 + d_2 g_2 + d_3 g_3 + d_4 g_4 + u_4
\end{aligned}$$

where g_k is a dummy variable that equals 1 if game k is played, and zero otherwise. Various axioms are tested based on the estimated coefficients of these regressions.[13] Symmetry requires $b_1 = c_1$ and $b_4 = c_4$. Additivity and homogeneity require $x_3 = x_1 + x_2$ and $x_4 = 2x_1$ for $x \in \{a, b, c, d\}$, respectively. Strong monotonicity requires $a_1 > a_2$ and $a_4 > a_3$. Finally, fairness requires, $b_3 - b_2 = c_3 - c_2$. In Table 4, we present the results of the Wald test of the verification of these axioms, together with the null hypothesis (H_0).

[13] Table 6 in Appendix B reports the results of these regressions, H-MC in the left panel and Winter in the right panel. The standard errors are corrected for session-level clustering effects.

Table 5: H-MC and Winter mechanisms: axioms.

Axiom	H-MC	Winter
Efficiency	+	–
Symmetry	+	+
Additivity	–	+
Homogeneity	–	+
Null player property	–	+
Strong monotonicity	+	+
Fairness	+	–

Note that the symmetry (according to which H_0 should not be rejected) is always confirmed, both for the H-MC and the Winter mechanisms. Additivity and homogeneity (according to which H_0 should not be rejected) is almost always confirmed for the Winter mechanism, but is confirmed only half the time for the H-MC mechanism. Strong monotonicity (according to which H_0 *should be rejected*) is confirmed. Fairness (according to which H_0 should not be rejected) is rejected for the Winter mechanism, but confirmed for the H-MC mechanism. Table 5 summarizes whether each axiom is satisfied on average (+) or not (–) for the two mechanisms. We can state these results as follows.

Result 4. *In terms of payoff shares, the Winter mechanism satisfies the axioms that characterize the Shapley value better than the H-MC mechanism. Thus, we reject* ***H5****, which states that the two mechanisms satisfy these axioms equally.*

7 Conclusions

We have compared experimentally two of the best-known bargaining procedures in the Nash program: the H-MC and Winter mechanisms. Our main rationale in this choice has been simplicity, which represents a main desideratum when considering possible applicability in the real world. These two mechanisms are simple and similar in terms of implementation, and are thus suitable for direct comparison. The two mechanisms differ in the way they implement bargaining: based on offers, for H-MC, and on demands, for Winter.

Previous experimental analyses on legislative bargainings find a certain similarity between a demand-based and an offer-based mechanisms (see Fréchette et al., 2005), despite their sharply different theoretical predictions. Our findings partially contradict these results, showing how two very similar mechanisms can behave differently, again, despite their similar theoretical predictions. In particular, the H-MC mechanism resulted in higher frequency of grand coalition formation and higher efficiency than the Winter mechanism. In contrast, the Winter mechanism resulted in average payoff shares that are closer to the Shapley value and better satisfy various axioms. Therefore, our results suggest that the offer-based H-MC mechanism better induces players to cooperate and to agree on an efficient outcome, while the demand-based Winter mechanism better implements allocations that reflect players' effective bargaining power.

Enhancing the understanding of axiomatic (cooperative game) solutions by providing noncooperative foundations was a main objective of the Nash program. Given our results, we cannot confirm the superiority of one of the axiomatizations in Section 3.2 from a behavioral perspective. However, we can provide some insight into which axioms are the most likely to be respected in various bargaining situations and thus, which axioms are the ones that should be imposed by an alternative solution. For example, players seem to be capable of getting the same payoffs when two individuals have equal bargaining weight (symmetry axiom), or acknowledge the superiority of one particular player in terms of marginal contribution when comparing two different situations (strong monotonicity axiom). On the one hand, requiring efficiency of a solution seems utopian in many cases. Players find it difficult to offer no reward when facing a null player. On the other hand, they do not have a problem refusing a nonzero demand by the same null player. We believe that a more rigorous verification of various axioms underling cooperative game solutions from a behavioral point of view represents the first important direction for future work.

Our findings suggest that the choice of a given mechanism may have some unexpected nudging effects, regardless of the theoretical prediction. Furthermore, this should be taken into account when making such a choice in various applications. In fact, different bargaining mechanisms, even when equivalent from a theoretical point of view, favor different properties, which are reflected by the resulting allocations. A deeper analysis of this issue is our second suggestion for future work in this domain.

In addition, many potentially important complementary questions can be addressed in future research. Among these is an analysis of the more complex versions of our proposed mechanisms (e.g., the Winter mechanism with more periods and a discount factor, or the H-MC mechanism where the proposer who sees her offer refused leaves the game with a probability strictly smaller than one), which can be compared with our actual results.

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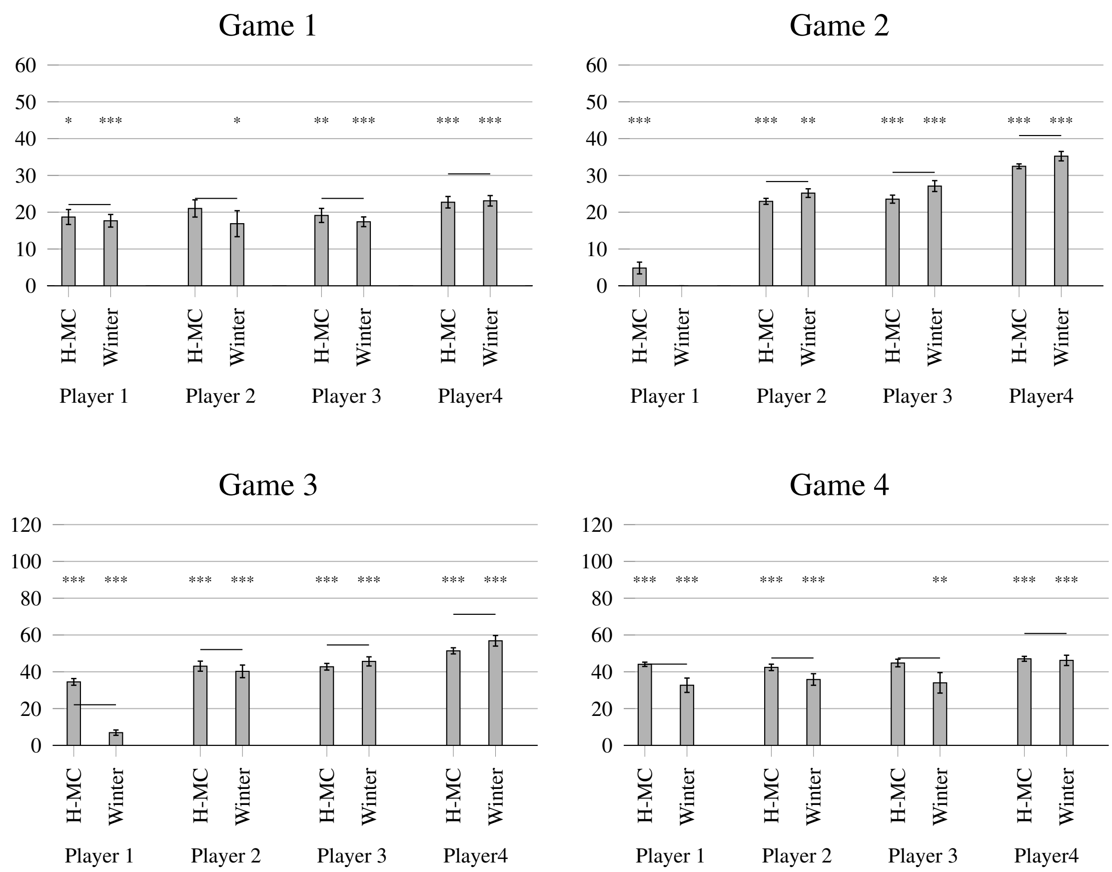
A The average payoffs

Figure 6 provides the mean of the nonnormalized payoffs in the four games, the horizontal lines indicating the Shapley values for each game.[14]

Figure 6: Mean payoffs.



Note: Error bars show the one standard error range. ***, **, and * indicate the average payoff being significantly different from the Shapley value at the 1, 5, and 10% significance levels (Wald test)

[14]Just as in Figure 4, the mean and the standard errors are obtained by estimating the system of linear regressions that takes each player's payoff as a dependent variable along with four-game dummy variables without a constant (similar to the one described in Section 6.3). The standard errors are corrected for session-level clustering effects. The statistical tests are based on these regressions.

B Results of linear regression for normalized payoffs

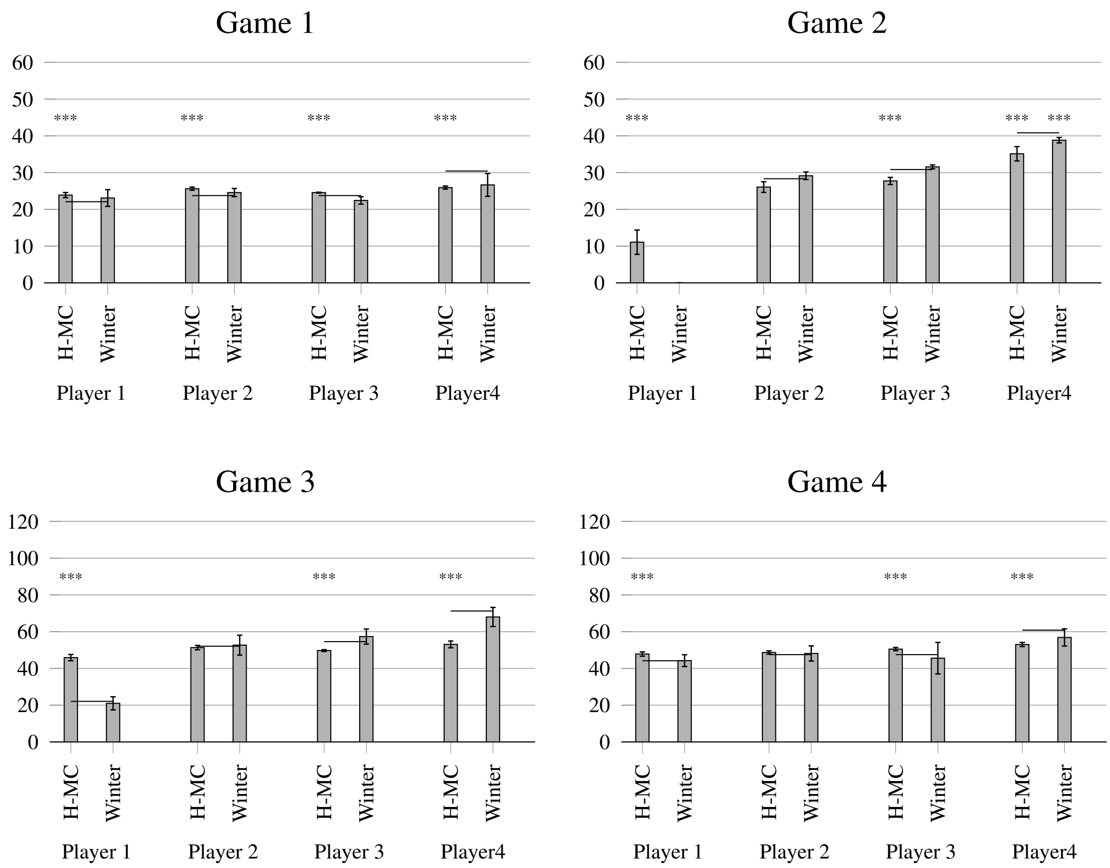
Table 6: Results of linear regression for normalized payoffs.

H-MC					Winter				
	$\widetilde{\pi_1}$	$\widetilde{\pi_2}$	$\widetilde{\pi_3}$	$\widetilde{\pi_4}$		$\widetilde{\pi_1}$	$\widetilde{\pi_2}$	$\widetilde{\pi_3}$	$\widetilde{\pi_4}$
g1	21.43	25.38	23.30	29.90	g1	21.99	21.91	23.39	32.80
	(1.88)	(1.20)	(1.53)	(1.60)		(2.78)	(3.80)	(2.12)	(2.09)
g2	5.12	27.52	28.04	39.31	g2	0.0	28.70	30.75	40.55
	(1.63)	(0.72)	(0.83)	(0.92)		-	(0.61)	(0.44)	(0.51)
g3	38.06	49.84	49.94	62.15	g3	8.95	54.09	61.52	75.55
	(1.75)	(1.56)	(0.70)	(1.96)		(2.16)	(3.86)	(2.84)	(4.57)
g4	44.13	49.03	51.97	54.87	g4	42.18	48.13	45.57	64.13
	(0.56)	(2.06)	(2.33)	(0.81)		(3.15)	(2.34)	(5.14)	(4.51)
R^2	0.83	0.96	0.96	0.96	R^2	0.73	0.90	0.92	0.93
Obs.	168	168	168	168	Obs.	192	192	192	192

C Additional results based only on the groups that formed a grand coalition

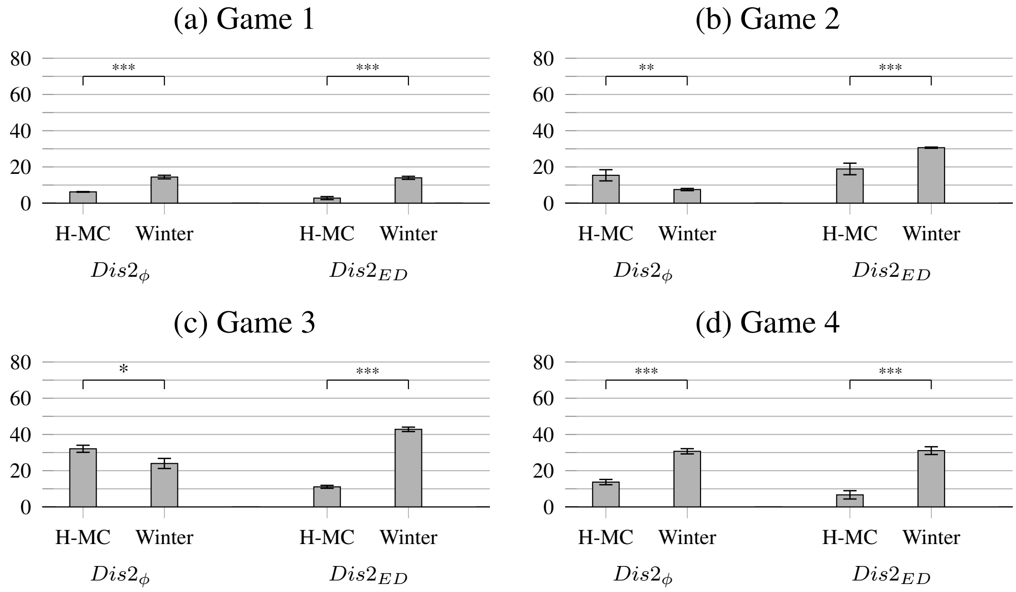
Here, we redo the analyses testing H3 and H7 reported in the main text based only on those groups that formed a grand coalition. The qualitative results, however, remain the same.

Figure 7: Mean payoffs based only on the groups that formed a grand coalition.



Note: the horizontal lines indicate the Shapley values. Error bars show the one standard error range. ***, **, and * indicate the average normalized payoff being significantly different from the Shapley value at the 1, 5, and 10% significance levels (Wald test).

Figure 8: Mean of the distances of the payoff vectors from the Shapley value and the equal division solutions for the groups that formed a grand coalition.



Note: Error bars show the one standard error range. ***, **, and * indicate the distance of the normalized payoff vectors from the Shapley value or from the equal division solution is significantly different between the H-MC and the Winter implementation, at the 1, 5, and 10% significance levels (Wald test).

Table 7: Results of linear regression based only on the groups that formed a grand coalition.

H-MC

	π_1	π_2	π_3	π_4
g1	23.88	25.63	24.56	25.93
	(0.70)	(0.43)	(0.11)	(0.42)
g2	11.07	26.07	27.73	35.13
	(3.31)	(1.45)	(0.97)	(1.95)
g3	45.88	51.32	49.72	53.08
	(1.70)	(1.19)	(0.50)	(1.84)
g4	47.83	48.67	50.5	53.00
	(1.17)	(0.89)	(0.87)	(1.10)
R^2	0.96	0.99	0.99	0.97
Obs.	97	97	97	97

Winter

	π_1	π_2	π_3	π_4
g1	23.09	24.57	22.43	26.65
	(2.28)	(1.11)	(1.01)	(3.11)
g2	0.0	29.15	31.56	38.81
	-	(1.03)	(0.54)	(0.73)
g3	21.00	52.67	57.33	68.00
	(3.57)	(5.42)	(4.11)	(5.19)
g4	44.24	48.14	45.57	56.86
	(3.22)	(4.12)	(8.58)	(4.68)
R^2	0.90	0.93	0.91	0.95
Obs.	77	77	77	77

Table 8: Results of Wald tests for the verification of the symmetry, additivity, homogeneity, strong monotonicity, and fairness axioms (based only on the groups that formed a grand coalition).

		H-MC		Winter	
Axiom	H_0	χ^2	p-value	χ^2	p-value
Symmetry	$b_1 = c_1$	5.07	0.024	1.85	0.174
	$b_4 = c_4$	1.11	0.293	0.06	0.8110
Additivity	$a_3 = a_1 + a_2$	4.84	0.028	0.13	0.721
	$b_3 = b_1 + b_2$	0.03	0.861	0.02	0.878
	$c_3 = c_1 + c_2$	14.99	0.000	0.47	0.492
	$d_3 = d_1 + d_2$	11.10	0.001	0.78	0.376
Homogeneity	$a_4 = 2a_1$	0.00	0.983	0.10	0.749
	$b_4 = 2b_1$	13.12	0.000	0.11	0.745
	$c_4 = 2c_1$	2.25	0.134	0.00	0.947
	$d_4 = 2d_1$	0.43	0.513	0.82	0.365
Strong monotonicity	$a_1 = a_2$	215.83	0.000	102.24	0.000
	$a_4 = a_3$	0.67	0.411	26,84	0.000
Fairness	$b_3 - b_2 = c_3 - c_2$	3.02	0.082	0.74	0.391

Table 9: Tests of axioms (based only on the groups that formed a grand coalition).

Axiom	H-MC	Winter
Efficiency	+	+
Symmetry	–	+
Additivity	–	+
Homogeneity	+	+
Null player property	–	+
Strong monotonicity	–	–
Fairness	–	+

D Translated instructions and comprehension quiz

D.1 Winter mechanism instructions

An English translation of the handout can be downloaded from

https://bit.ly/33IzgMM

D.2 H-MC mechanism instructions

An English translation of the handout can be downloaded from

https://bit.ly/34FnIck

D.3 Comprehension quiz

An English translation of the comprehension quiz can be downloaded from

https://bit.ly/3mFlCjW

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