MONETARY AND PRUDENTIAL POLICY COORDINATION: IMPACT ON BANK'S RISK-TAKING

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Monetary and prudential policy coordination: impact on bank's risk-taking

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Abstract

This paper models monetary policy's transmission to bank risk in presence of a capital requirement ratio. We show that the impact of a change in monetary policy rate on bank's risk level is not independent from the strength of the capital requirement ratio. A monetary easing, as well as a monetary contraction, may lead bank to take more risk according to some effecs related to the risk sensitivity of its intermediation margin and risk sensitivity of the prudential tool. We show that the combination of monetary policy with prudential policy has different outcomes in terms of financial stability and expected cost of bank failure.

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1 Introduction

While central banks had a long time ago ignored the impact of interest rates on agents' investment behaviour, there is a growing literature that focus on the impact of monetary policy on the behaviour of financial agents. The pioneers of this theme are Bernanke and Blinder (1988) and Bernanke and Gertler (1989) who identify the channel of risk-taking. The recent financial crisis has pushed up the interest of researchers for the risk-taking channel of monetary policy. Backed by the fairly serene climate in the financial markets, financial players have increased their tolerance threshold to risk and their reliance on the robustness of the financial system. Galati and Moessner (2013) pointed out that the exaggerated reliance in the self-adjustment ability of the financial system drove the underestimation of the explosion of credit and asset prices (particularly in the real estate sector) that resulted in the accumulation of asset stocks and leverage. The pre-crisis macroeconomic conditions seem to have changed the determinants of agents' risk behaviour, perception and risk tolerance (Borio and Zhu, 2012). Excessive softening of lending standards due to too low levels of short-term (monetary policy) and long-term (government bond) interest rates, a concurrent widespread use of financial innovation resulting in high securitization activity, and weak supervision standards are found as major factors of the crisis (Rajan, 2005; Jimenez et al., 2008; Gambacorta, 2009; Altunbas et al. 2010). These authors find that monetary easing induce bank risk-taking. And that extended period of low interest rate exacerbate banks risk-taking.

However, it is possible to imagine that prudential policy could counter this source of financial instability. By mitigating banks' risk-taking incentives, prudential policy should offset the negative effects of an accommodative monetary policy (Gertler and Karadi, 2011). Nevertheless, two problems appear immediately. The first is that prudential policy can be constrained by the nature of monetary policy (Agur and Demertzis, 2012). For example, bank capital standards are less stringent in times of economic growth fostered by a low interest rate policy. The second is that prudential policy can also affect the real economy and may conflict with monetary policy objectives. Thus, the increase in constraints on bank capital (microprudential approach), the introduction of maximum leverage or a counter-cyclical capital buffer (macroprudential approach) can limit the volume of credit, increase financing constraints and impact the economic activity and the level of inflation.

These opposite results shed light on the necessity to further study the impact of monetary policy on banks risk-taking behaviour and its interaction with prudential policy. We try to address this question in this paper.

We model the choice of the optimal risk level of a bank that seeks to maximize its profit under a regulatory capital constraint, assuming a risk sensitive capital requirement ratio. Through this modelling, we assess the impact of a change in monetary policy, designed as a change in risk-free rates, on the optimal risk choice of the bank and examine how it interferes with micro- and macroprudential policy.

Our findings are as following. First, the bank's risk-taking behavior following a change in monetary interest rate is driven by two opposing effects: the expected net marginal intermediation gain and the additional capital provisioning. Hence, we find that a decrease in the risk-free interest rate could lead either to an increase or to a decrease in the bank's optimal risk level. If the expected net marginal intermediation gain following an increase in risk is enough to cover the rise in additional capital provisioning, a monetary easing pushes up the bank's equilibrium risk level. Conversely, if the expected net marginal intermediation gain following an increase in risk is lower than the additional capital provisioning, then the bank finds it optimal to decrease its risk level following monetary easing. Thus, the effects of monetary policy on bank's risk mostly depend on the sensitivity of the capital requirement ratio to risk. We evidence the existence of a couple (risk sensitivity of the bank's net intermediation margin; risk sensitivity of the regulatory capital ratio) that encourages risk-taking in the event of a fall in interest rates. When the risk sensitivity of the capital requirement is lower than the risk sensitivity of the bank's net intermediation margin, then the well-known bank risk channel of monetary policy works. That is, decreased interest rate whets bank's risk appetite and leads to an increase in the bank's optimal risk level. In fact, in that case the bank could take more risk without facing prohibitive additional capital charge. However, when the capital requirement is more sensitive to risk than the bank's intermediation margin, then the bank's risk-taking channel of monetary policy reverses. A decreased interest rate lead to a fall in the bank's optimal risk level. A prudential tool that is more risk sensitive than the bank's intermediation margin makes it most costly to take more risk. Then, the bank find it profitable to reduce risk since lowering risk level induces lower capital provisioning (and higher leverage) that will offset the loss in interest margin (decrease in the risk premium). So, the effects of monetary policy on bank's risk-taking behavior are not independent from the "strength" of the microprudential policy.

Second, we establish that changes in monetary policy have implications also in terms of the probability and cost of bank failure. The presence of a deposit insurance makes the bank free of reimbursing deposits in case of failure. Since the bank could take more risk following any change in monetary rate, on the one hand, it decreases the probability of success (decrease in financial stability) and increases the probability of default (and thus the probability for the whole economy to bear the cost of a crisis). On the other hand, higher risk induces lower leverage (less projects are financed). Consequently, the resulting effect on the expected cost of bank failure is not unique. The final impact of a change in monetary policy on the expected cost of a bank failure depends on the bargaining forces of four effects: the price effect, the risk-taking effect, the fragilization effect and the leverage effect. Hence, any change in monetary policy is followed by either an increase or a decrease in the expected social cost of bank failure. It appears that in some cases, the single microprudential is sufficient to ensure both a more stable financial system and a lower resolution cost in case of bankruptcy. However, in other cases, the increase in the resolution cost of a bank failure is mainly driven by the leverage effect. Then, a macroprudential policy, such as a leverage ratio must be coupled to microprudential policy to achieve these two goals. In these latter cases, it can be possible for the regulator to attenuate the increase in the resolution cost by imposing a leverage ratio which may limit the increase in the bank's leverage.

These findings raise the question of the determination of the "optimal" risk sensitivity of the microprudential tool, i.e the risk sensitivity of the capital requirement ratio that will offset the incentive to risk-taking. In fact, if it is easy to prove the existence of such risk sensitivity level, its implementation is not obvious. That is the reason why a proper coordination between monetary policy and microprudential policy appears necessary.

The features of our model are close to those existing in the literature on banking regulation (Dell'Ariccia and Marquez, 2006; Van den Heuvel, 2008; Covas and Fujita, 2010; Agur and Demertzis, 2019). Our model adapts features of the model developed by Dell'Ariccia and Marquez (2006), where they analyze the incentives for independent bank regulators with financially integrated jurisdictions to form a regulatory union. Contrary to them, we analyze the interaction of monetary and prudential policy in a single country framework. The question of coordination in a country has been tackled by Jeanne and Korinek (2013). In their model, monetary policy has a resolution objective that triggers more ex-ante risk-taking. Setting an ex-ante prudential policy allows to solve the time inconsistency problem of monetary authority. In our model, we consider monetary and prudential policy as exogenous, and the two policies play at each period contrary to them where monetary and prudential policies are assumed to play separately only in each period. Another model that is closer to ours is that of Agur and Demertzis (2019) which model the transmission of monetary policy to bank's risk-taking and its interaction with prudential policy. They show that a change in the monetary policy rate affects the regulator's entire trade-off in an ambiguous way depending on two countervailing effects: the profit effect and the leverage effect. According to the profit effect, a higher rate increases the bank's funding costs, hence, reduces its profitability. Due to deposit insurance, the bank has less to lose from a risky strategy and then take more risk. The leverage effect induces bank's risk to drop following monetary contraction since a higher policy rate makes debt more expensive. Then, the bank opts to deleverage, and has more "skin in-the-game" leading to invest in projects with lower default risk. They also show that the regulator allows interest rate changes to partly "pass through" to bank soundness by not neutralizing the bank risk channel of monetary policy. We complement their analysis by taking into account the capital cost and evidence that the effects of change in monetary rate on the bank's risk also depends on the risk sensitivity of the microprudential tool.

The rest of the paper is as follows. Section 2 describes the model by presenting the agents and the assumptions. The bank's problem is examined in section 3 while the bank risk channel of monetary policy and its interaction with prudential policy are analysed in section 4. Section 4 analyses the implication of monetary policy in terms of expected cost of bank failure. Section 6 gives an overview of the outcomes of different combination in terms of risk-taking and expected cost of bank failure. And section 7 concludes.

2 The structure of the model

We propose a model where a bank chooses its optimal asset risk level under various banking regulation and monetary policy. There are 5 types of agents in the economy: savers that are assumed to provide banks with an inelastic deposits offer; borrowers that are assumed to have inelastic credit demand in order to finance projects with different risk profiles; a representative bank, a monetary and a prudential authority.

The bank chooses the optimal risk level of projects it may finance to maximize its profit under given monetary and prudential policy. There is no information asymmetry as each agent acts knowing perfectly what the others do. Monetary policy and prudential policy authorities know perfectly how their actions affect the bank's optimal decisions, whereas the bank optimally reacts to the features of monetary and prudential policies.

2.1 Monetary and Prudential policies

We assume that monetary and prudential policies are exogenous and that the two policies are independent and act in an uncoordinated manner.

In general, monetary policy's aim is to regulate inflation and output gap by setting interest rates. As established by preceding studies, monetary policy also affects the financial system (Bernanke and Gertler, 1995; Bernanke et al., 1996; Smith, 2002; Rajan, 2005; Adrian and Shin, 2009; De Nicolo et al. 2010). Since we develop a partial equilibrium model, we do not consider the macroeconomic objectives of monetary policy but assume these objectives as exogenous to our model. We focus only on the impact of change in interest rate on the financial system and seek to determine how a change in interest rate impacts the bank's optimal risk choice. Hence, changes in the risk-free interest rate δ are used as proxy for monetary policy.

Prudential policy, for its part, deals with micro- and macroprudential policy. In our model, we focus on the microprudential side by considering that the prudential regulator acts through a capital requirement ratio (microprudential tool) and aims at strengthening bank soundness. The capital requirement ratio has been largely considered as efficient to deter the bank's risk-taking (mitigating the risk-taking behaviour that stems from limited liability and deposit insurance). In this model, we model the microprudential policy through a risk sensitive capital requirement ratio, following the Basel III logic. The microprudential policy in our model allows the regulator to set the risk sensitivity of the capital requirement ratio.

Besides the objective of financial stability, we assume that the regulator also care about the expected cost of a crisis resolution for the society. The expected cost of bank failure can be declined into the frequency and severity of a crisis. On the one side, bank's risk-taking does not only threats the financial stability, but also increases the probability of default which translates into higher probability of crisis (higher frequency of crisis). On the other side, the higher bank's leverage the more severe the crisis since the deposit insurance (which increases the moral hazard and risk-taking by the banks) leads the government (and so the whole society) to bear the cost of the resolution once the crisis materializes. In this sense, we consider that the regulator also aims at reducing the expected cost of bank failure.

For consistency in the sequence of actions, we assume that monetary policy and prudential regulator set respectively interest rates and the prudential tool at the same time.

2.2 The representative bank

The representative bank is financed by capital and deposits. The bank collects deposits from households at a cost δ which is also the risk-free interest rate set by the monetary policy. We assume that deposits are covered by a deposit insurance and that the supply of deposit is inelastic to the risk-free interest rate. It means that the bank can collect an indefinite amount of deposit paying δ. The bank is also endowed with a fix amount of capital $\overline{K}$ that is a more costly form of financing that deposits (Gorton and Winton (2002) and Repullo (2004)), and we define ρ as the cost of capital, with $\rho > \delta$.[1]

The bank faces a continuum of risky projects that it can finance thanks to capital and deposit. We assume that there is an inelastic demand of financing for each level of risk. Moreover, the return of risky projects is assumed to be an increasing function of their level of risk whereas their probability of success is decreasing with their risk level.

A project's risk level is measured by $\alpha \in [0, 1]$, $R(\alpha, \delta)$ and $P(\alpha)$ are respectively the gross return and the probability of success of a project according to its level of risk α. We make the following assumptions on these two functions.

H1. The safe project ($\alpha = 0$) yields the risk-free interest rate ($R(0, \delta) = \delta$) with a probability of success $P(0) = 1$, whereas the higher risky project ($\alpha = 1$) yields the maximum gross return of $R(1, \delta) = \overline{R}$ with a probability of success $P(1) = 0$.

H2. The gross return is an increasing concave function of the projects' level of risk with $\frac{\partial R(\alpha,\delta)}{\partial \alpha} = R'_\alpha(\alpha, \delta) > 0$, $\frac{\partial^2 R(\alpha,\delta)}{\partial \alpha^2} = R''_\alpha(\alpha, \delta) \leq 0$, $\lim_{\alpha \to 0} R'_\alpha(\alpha, \delta) \to +\infty$ and $\lim_{\alpha \to 1} R'_\alpha(\alpha, \delta) \to 0$.

H3. The probability of success is a decrease concave function of the projects' level of risk with $\frac{\partial P(\alpha)}{\partial \alpha} = P'_\alpha(\alpha) < 0$, $\frac{\partial^2 P(\alpha)}{\partial \alpha^2} = P''_\alpha(\alpha) \leq 0$, $\lim_{\alpha \to 0} P'_\alpha(\alpha) \to 0$ and $\lim_{\alpha \to 1} P'_\alpha(\alpha) \to -\infty$.

H4. The gross return of a project is positively related to the risk-free interest rate and $\frac{\partial R(\alpha,\delta)}{\partial \delta} = R'_\delta(\alpha, \delta) > 0$.

Lemma: The maximum expected return is obtained for the level of risk $0 < \overline{\alpha} < 1$.

Proof: see Appendix 1.

The bank is confronted with a risk-sensitive prudential regulation and must hold an amount of capital in line with the risk level of the asset it finances. We define L as the total amount of risky assets financed by the bank and the minimum level of regulatory capital the bank must hold is given by $k(\alpha)L$ with $k(\alpha)$ a measure of the risk-sensitive regulatory capital ratio. In accordance with the Basel III banking regulation, we assume that $k(\alpha)$ is an increasing concave function of the risk of the asset with $\frac{\partial k(\alpha)}{\partial \alpha} = k'_\alpha(\alpha) > 0$,

[1] The amount of capital $\overline{K}$ is given and its cost ρ is fixed since we assume that the bank has already raise capital at a cost which includes risk for investor and remains constant.

$\frac{\partial^2 k(\alpha)}{\partial \alpha^2} = k''_\alpha(\alpha) \leq 0$, $k(0) = \underline{k} > 0$ and $k(1) = \overline{k} < 1$. It means that a minimum level of capital is required even for financing a safe asset and that the level of capital for financing the riskier asset as a maximum value.

Finally, the objective of the bank is to maximize its profit by choosing the assets risk level it finances under the constraint given by the available amount of capital, which means:

$$\begin{aligned} \max_{\alpha} \pi(\alpha, \delta) =& LR(\alpha, \delta)P(\alpha) - (L - \overline{K})\delta P(\alpha) - \overline{K}\rho \\ s.t. \quad & \overline{K} \geq k(\alpha)L \end{aligned} \tag{1}$$

The first part of the equation $(LR(\alpha, \delta)P(\alpha))$ is the expected revenue of the bank. $(L - \overline{K})$ is the total amount of deposit that is required to finance the amount L of assets. The cost of these deposits is equal to δ and due to limited liability and deposit insurance, the bank payback deposits only in case of success, with probability $P(\alpha)$. Thus, $(L - \overline{K})\delta P(\alpha)$ is the cost of deposits in case of bank's success. The last term of the profit function, $\overline{K}\rho$ is a measure of the capital cost paid even in case of failure.[2]

As capital is costly, the bank doesn't hold excess capital over the regulatory level (Repullo and Suarez (2004)) and we have $\overline{K} = k(\alpha)L$.[3]

Consequently, the bank maximizes the following profit function:

$$\begin{aligned} \max_{\alpha} \pi(\alpha, \delta) =& LR(\alpha, \delta)P(\alpha) - (L - k(\alpha)L)\delta P(\alpha) - k(\alpha)L\rho \\ \max_{\alpha} \pi(\alpha, \delta) =& L[R(\alpha, \delta)P(\alpha) - (1 - k(\alpha))\delta P(\alpha) - k(\alpha)\rho] \end{aligned} \tag{2}$$

Finally, we assume that the bank makes its choice knowing the monetary and the prudential policies.

3 The bank's problem

The representative bank chooses the risk level α^* that maximizes its profit given the regulatory standards and the value of the risk-free interest rate fixed by monetary policy.

$$\max_{\alpha} \pi(\alpha, \delta) = L[R(\alpha, \delta)P(\alpha) - (1 - k(\alpha))\delta P(\alpha) - k(\alpha)\rho] \tag{2'}$$

[2]We follow Van den Heuvel (2008) and assume that the objective of the bank is to maximize shareholder value, net of the initial equity investment. The bank compares the projects' net return (net of deposit cost) to the cost of capital and invests if the projects' net return is greater than the cost of capital.

[3]The bank saturates the capital constraint because capital is more expensive than deposits and must absolutely be remunerated. Therefore, it is not optimal for the bank to hold excess capital over the regulatory level.

Proposition 1

i) There is a unique risk level $\alpha^* \in]0;1[$ that maximizes the bank's profit and $\alpha^* < \overline{\alpha}$;

ii) For α^*, there is a credit volume $L^* = \frac{\overline{K}}{k(\alpha^*)}$ that determines the bank's equilibrium leverage ratio.

Proof: see Appendix 2.

The rationale of proposition 1 is that the bank's choice results from the interaction of two effects: the profit effect and the regulatory effect.

The profit effect is as follows. Recall that the bank's primary objective is to maximize its profit. For a given risk-free interest rate, the bank increases its revenue via risk premium by investing in risky projects. Albeit increasing risk level increases the bank's revenue, it decreases the probability of success of the projects as well. Then, *ceteris paribus*, the profit effect results in the trade-off between increasing revenue (via more risk) and decreasing probability of getting this revenue. According to Lemma 1, we know that this profit effect is maximum for $\overline{\alpha}$.

The regulatory effect stems from the change in the regulatory capital following a change in the risk level chosen by the bank. When the bank increases its risk level, the regulatory capital increases, reducing deposit financing. However, since the cost of capital is higher than the deposits cost, the marginal funding cost increases with the risk. Then, the bank's profit decreases.

So, when the expected marginal gain of additional risk $(R'_\alpha(\alpha,\delta)P(\alpha) + R(\alpha,\delta)P'_\alpha(\alpha))$ is higher than the marginal funding cost $((1-k(\alpha))\delta P'_\alpha(\alpha) + k'_\alpha(\alpha)(\rho - \delta P(\alpha))$, the bank profit is increasing with α $(\frac{d\pi(\alpha,\delta)}{d\alpha} > 0)$. Conversely, the bank's profit decreases $(\frac{d\pi(\alpha,\delta)}{d\alpha} < 0)$ when taking additional risk triggers a marginal funding cost higher than the expected marginal gain.

Thus, there exists a unique risk level $\alpha^* \in]0;1[$ with $\alpha^* < \overline{\alpha}$ that maximizes the bank profit, i.e. there is a unique risk level that equals the expected marginal gain and the marginal funding cost.

After the bank has determined its optimal risk level α^*, it is then possible to determine $k(\alpha^*)$. Since the capital is costly and assumed to be fixed at level $\overline{K}$, the bank will provision exactly the capital amount required to comply with the regulation. That is, for the risk level α^*, the bank will finance an amount L^* of projects such that the credit volume $L^* = \frac{\overline{K}}{k(\alpha^*)}$. Hence, the bank's leverage is endogenously determined.

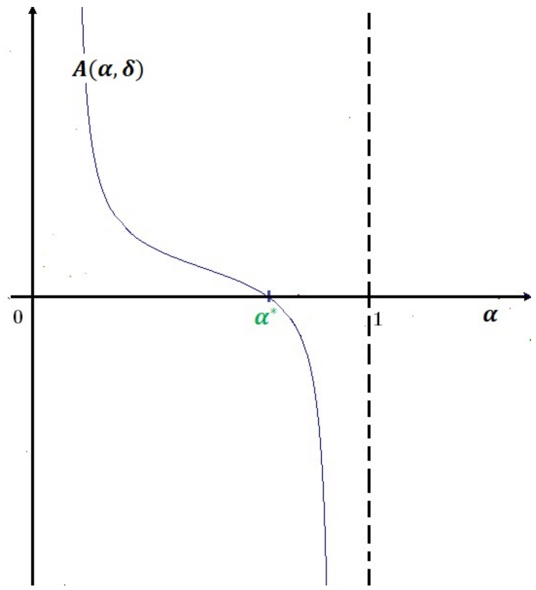
We can give a graphical illustration of proposition 1.

Let's denote $A(\alpha,\delta) = \frac{d\pi(\alpha,\delta)}{d\alpha}$,

$$A(\alpha,\delta) = R'_\alpha(\alpha,\delta)P(\alpha) + R(\alpha,\delta)P'_\alpha(\alpha) + k'_\alpha(\alpha)\delta P(\alpha) - [1-k(\alpha)]\delta P'_\alpha(\alpha) - k'_\alpha(\alpha) * \rho$$

The equilibrium risk level chosen by the bank is such that $A(\alpha^*, \delta) = 0$. Figure 1 gives an illustration of the $A(\alpha, \delta)$ curve.

Figure 1: Illustrative curve of $A(\alpha, \delta)$



According to Appendix 2, $A(\alpha)$ is a continuous decreasing function on]0;1[with $\lim_{\alpha\to 1} A(\alpha, \delta) \to +\infty$ and $\lim_{\alpha\to 1} A(\alpha, \delta) \to -\infty$. That there is a unique $\alpha^* \in]0; 1[$ so that $A(\alpha^*, \delta) = 0$.

It is important here to emphasize that the bank's optimal risk level depends on the risk-free interest rate (δ) and the prudential policy that determines the sensitivity of the regulatory capital ($k(\alpha)$) to the level of risk. This raises the question of the impact of monetary policy on the bank's risk-taking behaviour. We address this point in the following section.

4 Impact of monetary policy on bank risk-taking

According to the existing literature on banking regulation, monetary accommodation has been proven to whet bank's risk appetite (Bernanke and Gertler, 1995; Rajan, 2005; Altunbas et al., 2010). However, restrictive monetary policy could also impair bank soundness (Smith, 2002; Gan, 2004; Ngambou Djatche, 2019). In this section, we show that changes in monetary policy may lead either to an increase or to a decrease in the bank's risk level according to the strength of the prudential tool.

Let's examine the effects that can have a change in the monetary interest rate on the bank's optimal risk level in presence of a microprudential policy. Remind that a change in monetary policy is captured by a change in the risk-free interest rate, δ.

We have proven that the optimal level of risk chosen by the bank is such that $A(\alpha^*, \delta) =$

0. Consequently, the impact of a change on the risk-free interest on the bank's optimal risk level depends on the sensitivity of function $A(\alpha^*, \delta)$ to δ.

Proposition 2:

Under the assumption that $R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)] \geq 1$, we have:

i) If $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)]) > k'_\alpha(\alpha^*)$, a decrease in the risk-free interest rate leads to a rise in the equilibrium level of risk chosen by the bank;

ii) If $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)]) < k'_\alpha(\alpha^*)$, a decrease in the risk-free interest rate leads to a fall in the equilibrium level of risk chosen by the bank;

Proof. See Appendix 3.

$R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)] \geq 1$ means that the gross return of a risky project must be sufficiently sensitive to a change in the risk-free interest rate. In that case, although a decrease in the risk-free interest rate ($\Delta\delta < 0$) leads to a decrease in the deposits costs (since $0 < \frac{\partial[1-k(\alpha^*)]\delta}{\partial\delta} = [1 - k(\alpha^*)] < 1$), it produces a deeper decrease in the projects' return rate as well. Put it differently, the change in monetary rate, by changing the intermediation revenue, will lead the equilibrium to move.

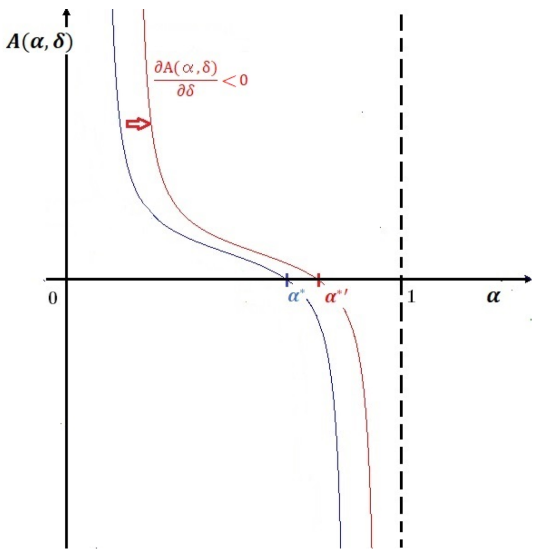
From proposition 2, $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)}$ denotes the percentage change in the probability that a project succeeds following a change in the risk level. $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * R'_\delta(\alpha^*, \delta)$ refers to the marginal impact on the project revenue of change in the risk level caused by a change in the risk-free interest rate. $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * [1 - k(\alpha^*)]$ is the marginal impact on the deposits funding of change in the risk level induced by a change in the risk-free interest rate. Thus, $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)])$ measures the net (net of deposits cost) marginal impact on the bank return of change in the risk level produced by a change in the risk-free interest rate. Given that $R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)] \geq 1$, we have $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)]) > 0$. Finally, $k'(\alpha^*)$ represents the risk sensibility of the capital requirement ratio and measures the additional capital provisioning following a change in the bank's risk level.

The bank's risk-taking behaviour will then be the result of a trade-off between two effects: the net marginal gain of additional unit of risk and the additional capital provisioning following an additional unit of risk. The first effect can be interpreted as a price effect whereas the second effect is close to a quantity effect. Given that the bank's total capital is give, this additional capital provision leads to a decrease in the total level of bank's financing.

If the net marginal gain of additional risk is greater than the additional capital provisioning (price effect higher than quantity effect), i.e. if $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)]) > k'_\alpha(\alpha^*)$, following monetary easing, the bank will find it optimal to take additional risk to increase its profit without fear of facing higher additional capital provisioning. In other words, following a decrease in monetary rate, the bank will take more risk as the expected net marginal intermediation gain is enough to cover the decrease in leverage. Here, the rationale is that a decrease in risk-free interest rate depletes the bank's profit. Since the risk sensitivity of the capital requirement ratio is lower than the expected marginal intermediation gain, the bank will take more risk to restore its benefit. A prudential tool that is less sensitive to risk than the bank's intermediation margin allows the bank to increase its revenue via more risk-taking without facing higher additional capital provision.

Figure 2 illustrates the impact of change of monetary policy on the bank's equilibrium risk level when the net marginal gain of additional risk is greater than the additional capital provisioning.

Figure 2: Negative impact of monetary easing on bank's risk



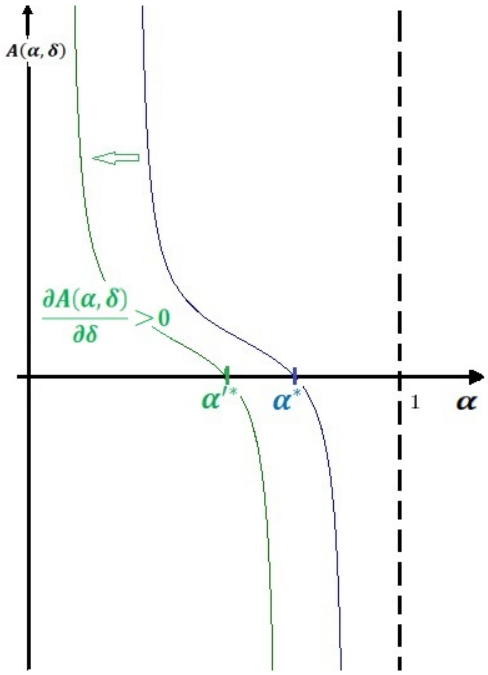
As depicted in Figure 2, the blue line represents the derivative of the bank's profit that determines its initial optimal risk level α^* given prudential and monetary policy. Then, when the net marginal gain of additional risk is greater than the additional capital provisioning, a monetary easing leads to an increase of the bank's optimal risk level.

However, if the net marginal gain of additional risk is lower than the additional capital provisioning (price effect lower than quantity effect), i.e. if $\frac{|P'_\alpha(\alpha^*)|}{P(\alpha^*)} * (R'_\delta(\alpha^*, \delta) - [1 - k(\alpha^*)]) < k'_\alpha(\alpha^*)$, the bank will find it optimal to decrease its risk level following a monetary easing. In fact, the bank deters its revenue by decreasing its risk level. But at the same time, since the capital requirement is more risk sensitive than the bank's intermediation margin, the bank benefits from a rise in leverage (via less capital provisioning). Hence, the bank will decrease its risk level until these two effects are balanced.

Figures 3 gives a bird's eye on how the bank's optimal risk level moves following change in monetary policy when the net marginal gain of additional risk is lower than the additional capital provisioning.

As depicted in Figure 3, the blue line represents the derivative of the bank's profit that determines its initial optimal risk level α^* given prudential and monetary policy. When the net marginal gain of additional risk is lower than the additional capital provisioning,

Figure 3: Positive impact of monetary easing on bank's risk



a monetary easing triggers a decrease (green line) of the bank's optimal risk level. That is, a monetary contraction may lead the bank to increase its risk level.

In a nutshell, monetary tightening and monetary easing may lead the bank to take more risk depending on the risk sensitivity of both the bank's intermediation margin and the capital requirement ratio.

Given these findings, it appears that monetary policy effects on bank's risk is not independent from the risk sensitivity of the microprudential tool. Moreover, we can wonder whether more risk-taking necessarily translates into more important damage to the economy in the event of a crisis. Now, let's examine the question of the coordination of monetary and prudential policies regarding not only the preventive side of prudential policy but also its resolution side.

5 The bank's risk and the social cost of bankruptcy

As shown in the preceding section, monetary policy, seeking to reach its objectives of price and GDP growth stability, may impact the banking soundness. In addition to the bank fragility, there is a social cost of bank failure which stems from the existence of the deposit insurance. The latter leads the bank not to internalize the cost of a bankruptcy since the bank repays deposits only in case of success. So, in case of failure the entire deposit repayment is at the charge of the government. In this sense, we assume that the objective of the prudential regulator is to mitigate both the bank's risk and the expected cost of banking failure.

Here, we analyse how the micro- and macroprudential policy may interfere with the monetary policy and derive the results in terms of financial stability (impact on bank's optimal risk level) and social cost of banking failure. Recall that in line with Basel III, it is assumed that the regulator care about the soundness of the individual bank (microprudential side). In addition, we assume that the regulator is also concerned about the implications of a banking crisis for the economy (macroprudential side), precisely in terms of the expected cost of bank failure (frequency and severity of crises). Hence, the regulator is also supposed to act such as to reduce the cost of a crisis resolution.

5.1 Monetary policy, microprudential policy and the expected cost of bank failure

In this subsection, we focus on the impact of a change in monetary rate on the expected cost of bank failure.

As said before, the expected cost of bank failure refers to the possibility that the bank fails and that the deposit repayment remains at the charge of the government. In fact, when the bank takes risk, it induces a probability of failure $(1 - P(\alpha))$ which also refers to the frequency of crises. In case of banking failure, the deposit insurance leads the government to support the deposits cost $([L - \overline{K})\delta])$ which refers to the cost of the crisis resolution and captures the severity of the crisis.

The expected social cost of the financial crisis is then given by:

$$C(\alpha, \delta) = (1 - P(\alpha))(L - \overline{K})\delta \tag{3}$$

Since $L = \frac{\overline{K}}{(k(\alpha))}$, we can write equation 3 as follows:

$$C(\alpha, \delta) = (1 - P(\alpha))(\frac{1}{(k(\alpha))} - 1)\delta\overline{K} \tag{3'}$$

As we can see, the expected social cost of banking failure clearly depends on the risk-free interest rate, δ, and the risk level, α, which itself determines the probability of failure, $1 - P(\alpha)$, and the capital requirement ratio, $k(\alpha)$.

Hence, the total differential of the social cost is given by:

$$dC = \frac{\partial C}{\partial \delta}.d\delta + \frac{\partial C}{\partial \alpha}.d\alpha$$

We can then recompose this equation and write:

$$dC = \left[\frac{\partial C}{\partial \delta} + \frac{\partial C}{\partial \alpha}.\frac{d\alpha}{d\delta}\right].d\delta \tag{4}$$

Let's examine the impact of the risk-free interest rate on the cost of bank failure.

From equation 4, it is noticeable that the change in the cost of the bank failure following a change in the monetary interest rate depends on different factors.

- $\frac{\partial C}{\partial \delta}$: which can be defined as the expected price effect which stems from changes in the risk-free interest rate, *ceteris paribus*. Any change in the risk-free interest rate induces the social cost to moves in the same direction, i.e. $\frac{\partial C}{\partial \delta} > 0$. The higher the risk-free interest rate the more costly the crisis resolution, all things remaining unchanged;

- $\frac{d\alpha}{d\delta}$: which is defined as the risk-taking effect. Its sign determines the impact of the change in the risk-free rate on the bank's risk appetite. As shown in the preceding section, the effect of monetary policy on bank risk depends on the risk sensitivity of the capital requirement (see Proposition 2 in Section 4). When the capital requirement is less risk sensitive than the bank's intermediation margin, a monetary easing leads to more risk-taking, i.e. $\frac{d\alpha}{d\delta} < 0$. Inversely, when the capital requirement is more risk sensitive than the bank's intermediation margin, a monetary easing induces a reduction in the bank's risk, i.e. $\frac{d\alpha}{d\delta} > 0$;

- $\frac{\partial C}{\partial \alpha}$: which captures the impact of a change in the bank equilibrium risk level on the cost of bank failure. This impact stems from two effects since changes in α impact the social cost through a change in both the probability of failure and the capital requirement ratio (the bank's leverage), and:

$$\frac{\partial C}{\partial \alpha} = \frac{\partial(1 - P(\alpha))}{\partial \alpha}.[\frac{1}{(k(\alpha))} - 1] + \frac{\partial([\frac{1}{(k(\alpha))} - 1])}{\partial \alpha}.(1 - P(\alpha))$$

  Let's denote $Frag = \frac{\partial(1-P(\alpha))}{\partial \alpha}.[\frac{1}{(k(\alpha))} - 1]$ and $Lev = \frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial \alpha}.(1 - P(\alpha))$.

  In this sense, the two components of effect of a change in the bank's risk on the cost of bank failure are:

  - $Frag = \frac{\partial(1-P(\alpha))}{\partial \alpha}.[\frac{1}{(k(\alpha))} - 1]$ that can be interpreted as a fragilization effect which is positively correlated to the bank's risk level. In fact since the the probability of default increases with the risk level ($\frac{\partial(1-P(\alpha))}{\partial \alpha} > 0$), and that $k(\alpha) < 1$ which induces that $[\frac{1}{(k(\alpha))} - 1] > 0$, we have $\frac{\partial(1-P(\alpha))}{\partial \alpha}.[\frac{1}{(k(\alpha))} - 1] > 0$. A rise in the bank's risk level triggers a hike in the probability of default (frequency of crises), then the expected social cost increases, *ceteris paribus*;

  - $Lev = \frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial \alpha}.(1 - P(\alpha))$ that can be interpreted as a leverage effect and captures the effect of the bank's leverage on the expect cost of a crisis resolution. All things remaining unchanged, the higher the bank's leverage the more expensive is the cost of a crisis resolution. However, this effect is negatively correlated to the bank's risk level. A rise in the bank's risk level tightens the capital requirement ratio which decreases the bank's leverage. On the one hand, the higher the risk level the more capital the bank should provision and then the lower the deposit proportion, i.e. $\frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial \alpha} < 0$. On the other hand, for any non null risk level, the probability of default is positive, i.e. $(1 - P(\alpha)) > 0$. As a result, the leverage effect is negatively correlated to the bank's risk, i.e. $\frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial \alpha}.(1 - P(\alpha)) < 0$. Hence, all things being equal, the higher the risk level the higher the capital requirement, the lower the bank's leverage and the lower the social cost.

Finally, equation 4 can be written as follows:

$$dC = \left[\frac{\partial C}{\partial \delta} + (Frag + Lev).\frac{d\alpha}{d\delta}\right].d\delta \tag{4'}$$

This equation describes the various effects that determine the change in the social cost following a change in the risk-free rate. The overall effect then depends on the combination of four effects: the price effect, the risk-taking effect, the fragilization effect and the leverage effect. As a result, the direction in which the social cost moves following a change in monetary policy depends on the relative intensity of these four different effects.

To have a precise understanding of the underlying intuition, let's analyse the changes in the cost of bank failure following, on the one hand, a monetary easing ($d\delta < 0$), and on the other hand, a monetary contraction ($d\delta > 0$). Note that any change in the monetary policy and the bank's risk can be negatively ($\frac{d\alpha}{d\delta} < 0$) or positively ($\frac{d\alpha}{d\delta} > 0$) related. In the following development, we will successively examine each case.

5.2 Impact of monetary easing on the expected cost of bank failure

The decrease in the risk-free interest rate ($d\delta < 0$) entails a direct decrease in the expected social cost (the price effect), since $\frac{\partial C}{\partial \delta} > 0$. However, this decrease of interest rate has an indirect effect on the social cost given that there is an increase in the bank's risk or not.

Case 1: The bank's risk increases with decreased interest rate, $\frac{d\alpha}{d\delta} < 0$.

We can summarize the overall mechanism as follows. Recall that the total differential in the social cost is given by:

$$dC = \left[\overset{(+)}{\frac{\partial C}{\partial \delta}} + (\overset{(+)}{Frag} + \overset{(-)}{Lev}).\overset{(-)}{\frac{d\alpha}{d\delta}}\right].\overset{(-)}{d\delta}$$

With $\frac{\partial C}{\partial \delta}$ the price effect, $\frac{d\alpha}{d\delta}$ the risk-taking effect, $Frag$ the fragilization effect and Lev the leverage effect.

Monetary easing directly decreases the cost of a bank failure (the price effect). However, the bank's risk increases following monetary easing (the risk-taking effect), the probability of default also increases (the fragilization effect) since $\frac{\partial(1-P(\alpha))}{\partial\alpha}.[\frac{1}{(k(\alpha))}-1] > 0$. Then, bank failures become more frequent. At the same time, the loan volume decreases (the leverage effect) in response to the tightening of the capital requirement, $\frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial\alpha}.(1-P(\alpha)) < 0$, which results in a decrease in the cost of crisis. Finally, the effective cost of crisis resolution (driven by the price effect and the leverage effect) decreases but the resolution mechanism is frequently activated (due to the fragilization effect). As a result:

- The expected social cost decreases, $dC < 0$, if the decrease in the magnitude of crises (driven by the price effect and the leverage effect) is large enough to compensate for the increase in the frequency of crises (the fragilization effect);
- The expected social cost increases, $dC > 0$, if the decrease in the magnitude of crises (driven by the price effect and the leverage effect) does not compensate for the increase in the frequency of crises (the fragilization effect).

However, it appears that the risk-taking effect is an important element in the variation in the cost of bank failure. This is because the risk-taking effect determines the frequency (fragilization effect) and also the magnitude (through the leverage effect) of crisis.

Since $d\delta < 0$, we can derive that:

$$dC > 0 \Leftrightarrow \quad \frac{\partial C}{\partial \delta} + (Frag + Lev).\frac{d\alpha}{d\delta} < 0$$
$$(Frag + Lev).\frac{d\alpha}{d\delta} < -\frac{\partial C}{\partial \delta}$$

Assuming that there is a risk-taking following a decrease in interest rate, $\frac{d\alpha}{d\delta} < 0$, then $dC > 0$ if $\boldsymbol{\frac{d\alpha}{d\delta} < -\frac{\partial C}{\partial \delta}/[Frag + Lev] \equiv S_1 < 0}$.

That is, there is a threshold (S_1) in the response of the bank's risk to monetary easing below which a monetary easing whets bank's risk and entails an increase in the expected cost of bank failure. Below this threshold, the increase in the expected cost is mainly driven by the fragilization effect. In other words, although the price and leverage effects decrease the cost of resolution in the event of crisis, the higher frequency of crises makes it costly to solve crisis. However, when the response of the bank's risk to monetary easing is above this threshold (S_1) a monetary easing whets bank's risk but there is a decrease in the expected cost of bank failure. In this latter case, the decrease in the interest rate and in the bank's leverage makes it cheaper to solve crises even though they are more frequent.

Now, let's examine how change in interest rate may impact the expected cost of bank failure when there is no risk-taking following a monetary easing.

Case 2: The bank's risk decreases with decreased interest rate, $\frac{d\alpha}{d\delta} > 0$.

The total differential in the social cost is still given by:

$$dC = \begin{bmatrix} \overset{(+)}{\frac{\partial C}{\partial \delta}} + (\overset{(+)}{Frag} + \overset{(-)}{Lev}). \overset{(+)}{\frac{d\alpha}{d\delta}} \end{bmatrix} . \overset{(-)}{d\delta}$$

The decrease in the risk-free interest rate entails a direct decrease in the expected social cost (the price effect), since $\frac{\partial C}{\partial \delta} > 0$. However, this decrease of interest rate has an indirect effect on the social cost. As the bank's risk decreases following a monetary easing, the probability of default also decreases (the fragilization effect plays positively) since $\frac{\partial(1-P(\alpha))}{\partial \alpha}.[\frac{1}{(k(\alpha))} - 1] > 0$, and the crises are less frequent. The loan volume increases simultaneously (the leverage effect) in response to the loosening of the capital requirement, $\frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial \alpha}.(1 - P(\alpha)) < 0$, and increases the cost of crisis resolution. As a result:

- The social cost decreases, $dC < 0$, if the net change in the magnitude of crises (resulting from the power relationship between the price effect and the leverage effect) is lower than the decrease in the frequency of crises (the fragilization effect);
- The social cost increases, $dC > 0$, if the net change in the magnitude of crises (resulting from the power relationship between the price effect and the leverage effect) is higher than the decrease in the frequency of crises (the fragilization effect).

In line with the preceding development (since $d\delta < 0$), there is a threshold in the risk-taking effect which may determine an increase or a decrease in the expected cost of bank failure. We can derive that:

$$dC > 0 \Leftrightarrow \quad \frac{\partial C}{\partial \delta} + (Frag + Lev).\frac{d\alpha}{d\delta} < 0$$
$$(Frag + Lev).\frac{d\alpha}{d\delta} < -\frac{\partial C}{\partial \delta}$$

Since $\frac{d\alpha}{d\delta} > 0$, then $dC > 0$ if $\boldsymbol{0 < \frac{d\alpha}{d\delta} < -\frac{\partial C}{\partial \delta}/[Frag + Lev] \equiv S_2}$.

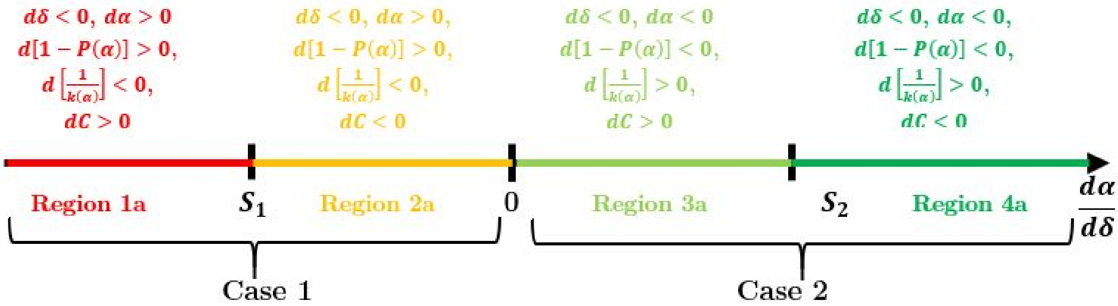
That is, when monetary policy does not whet bank's risk appetite, there is a threshold (S_2) in the response of the bank's risk to monetary easing below which there is an increase in the expected cost of bank failure. Below this threshold (S_2), the higher bank's leverage makes it more expensive to solve crises albeit crises become rare (less frequent) and that deposits are cheaper (lower interest rate). However, when the response of the bank's risk to monetary easing is above this threshold (S_2) a monetary easing lowers the bank's risk level and is followed by a decrease in the expected cost of bank failure. There are two possible reasons. First, the lower expected cost of crises resolution may be due to the fact that the occurrence of crises becomes almost improbable (near-zero frequency of crisis). Second, while the frequency of crises is lower, the combination of the increase in the bank's leverage with the decrease in deposits cost (the price effect) is such that the effective cost of a crisis become smaller.

Given the conditions mentioned above, there are four possible scenarios that can be depicted using Figure 4.

As depicted in Figure 4, we can observe that monetary easing have different effects on bank's risk and the resulting effects on the expected social cost are not unique. Let's describe each region of the graph.

- Region 1a: in this region, a monetary easing leads to an increase of the bank's risk

Figure 4: Decomposition of the effects of monetary easing on the expected social cost of bank failure.



level. This increase in the risk entails an increase in the probability of default (higher frequency of crises). However, the increase in the bank's risk level also induces a decrease in the bank's leverage (magnitude of a crisis) due to a tightening of the capital requirement. However, the increase in the probability of default is sufficiently high to exceed the decrease both in the price and in the volume of deposits. In other words, the effective cost of crisis is smaller, but since crises become more frequent, the overall expected cost of crises resolution increases.

$d\delta < 0 \implies Price < 0;$

$$d\alpha > 0 \implies \begin{cases} Frag & > 0 \\ Lev & < 0 \end{cases} \implies dC > 0 \quad \text{Since} \quad \overset{(+)}{Frag} > (\overset{(-)}{Lev} + \overset{(-)}{Price})$$

- Region 2a: a monetary easing leads to an increase of the bank's risk level. This increase in the risk triggers an increase in the probability of default (higher frequency of crises). However, the increase in the bank's risk level also induces a decrease in the bank's leverage (magnitude of a crisis) due to a tightening of the capital requirement. Finally, the decrease in the magnitude of a crisis, driven by the decrease both in the risk-free interest rate (the price effect) and in the volume of deposits (the leverage effect), is sufficient to overcome the increase in the frequency of crises driven by the increase in the probability of default. As a result, there is a decrease in the expected cost of bank failures since crises are frequent but are cheaper.

  $d\delta < 0 \implies Price < 0;$

  $$d\alpha > 0 \implies \begin{cases} Frag & > 0 \\ Lev & < 0 \end{cases} \implies dC < 0 \quad \text{Since} \quad \overset{(+)}{Frag} < (\overset{(-)}{Lev} + \overset{(-)}{Price})$$

- Region 3a: there is a decrease in the bank's risk level as a response to a monetary easing. This decrease in the risk results in a decrease in the probability of default (lower frequency of crises). However, the decrease in the bank's risk level induces an increase in the bank's leverage (magnitude of a crisis) due to a loosening of the capital requirement. Therefore, the situation is such that there is an increase in the severity of the crisis because the increase in the magnitude of a crisis driven by the hike in the volume of deposits (the bank's leverage) exceeds the decrease in its magnitude driven by a fall in the deposits cost (due to the decrease in the interest rate). Finally, the increase in the magnitude of a crisis increases the expected cost of crises resolution even if crises become less frequent (decrease in the probability of default). That is, there is an increase in the expected social cost of a bank failure.

$d\delta < 0 \implies Price < 0;$

$$d\alpha < 0 \implies \begin{cases} Frag & < 0 \\ Lev & > 0 \end{cases} \implies dC > 0 \quad \text{Since} \quad \overset{(+)}{Lev} > (\overset{(-)}{Frag} + \overset{(-)}{Price})$$

- Region 4a: like in region 3a, in response to a monetary easing, there is a decrease in the bank's risk level and in the probability of default (lower frequency of crises). There is also a higer bank's leverage (magnitude of a crisis) due to a loosening of the capital requirement. But, there is a decrease in the severity of crises because the decrease in their magnitude driven by a fall in the deposits cost (due to the decrease in the interest rate) exceeds the increase in their magnitude of a crisis driven by the hike in the volume of deposits (the bank's leverage). And since the crises also become less frequent (decrease in the probability of default), consequently, there is a decrease in the expected social cost of a bank failure.

  $d\delta < 0 \implies Price < 0;$

$$d\alpha < 0 \implies \begin{cases} Frag & < 0 \\ Lev & > 0 \end{cases} \implies dC < 0 \quad \text{Since} \quad \overset{(+)}{Lev} < (\overset{(-)}{Frag} + \overset{(-)}{Price})$$

In sum, it appears that monetary easing has different effects in terms of risk-taking and in terms of expected cost of bank failure. The main point is that even if monetary easing could induces more risk-taking, it does not necessarily translate into higher expected cost of bank failure. After having examined how the expected social cost of bank failure is impacted by a monetary easing, let's turn to the impact of a monetary contraction.

5.3 Impact of monetary contraction on the expected cost of bank failure

The increase in the risk-free interest rate entails a direct increase in the expected social cost (the price effect), since $\frac{\partial C}{\partial \delta} > 0$. Besides this direct effect, the increase in interest rate indirectly impacts the social cost given that there is an increase in the bank's risk or not.

Case 1': The bank's risk decreases with increased interest rate, $\frac{d\alpha}{d\delta} < 0$.

Similarly to what we stated in the preceding subsection (i.e. 4.2), recall that the total differential in the social cost is given by:

$$dC = \left[\overset{(+)}{\frac{\partial C}{\partial \delta}} + (\overset{(+)}{Frag} + \overset{(-)}{Lev}). \overset{(-)}{\frac{d\alpha}{d\delta}} \right] \overset{(+)}{.\, d\delta}$$

With $\frac{\partial C}{\partial \delta}$ the price effect, $\frac{d\alpha}{d\delta}$ the risk-taking effect, $Frag$ the fragilization effect and Lev the leverage effect.

A monetary tightening makes a crisis resolution more expensive. So, it induces a direct increase in the expected cost of bank failure (the price effect). As the bank's risk decreases following monetary contraction (the risk-taking effect), the probability of default also decreases (the fragilization effect) since $\frac{\partial(1-P(\alpha))}{\partial\alpha}.[\frac{1}{(k(\alpha))}-1]>0$. In other words, the frequency of crises decreases. At the same time, the loan volume increases (the leverage effect) in response to looser capital requirement, $\frac{\partial([\frac{1}{(k(\alpha))}-1])}{\partial\alpha}.(1-P(\alpha))<0$, and exacerbates the magnitude of a crisis. As a result:

- The expected social cost increases, dC>0, if the increase in the magnitude of the crisis (driven by the price effect and the leverage effect) is larger than the decrease in the frequency of crises (driven by the fragilization effect);
- The expected social cost decreases, $dC<0$, if the increase in the magnitude of the crisis (driven by the price effect and the leverage effect) is smaller than the decrease in the frequency of crises (driven by the fragilization effect).

Again, we can show that there is a threshold value in the risk-taking effect that determines whether the cost of bank failure may increase or decrease following a monetary contraction.

Given that $d\delta>0$, we can derive that:

$$\begin{aligned} dC>0 \Leftrightarrow \quad & \frac{\partial C}{\partial\delta}+(Frag+Lev).\frac{d\alpha}{d\delta}>0 \\ & (Frag+Lev).\frac{d\alpha}{d\delta}>-\frac{\partial C}{\partial\delta} \end{aligned}$$

Since $\frac{d\alpha}{d\delta}<0$, then $dC>0$ if $\quad \boldsymbol{0>\frac{d\alpha}{d\delta}>-\frac{\partial C}{\partial\delta}/[Frag+Lev]\equiv S_3}$.

So, there is a threshold (S_3) in the response of the bank's risk to monetary contraction above which, although a monetary contraction reduces bank's risk, there is an increase in the expected cost of bank failure. However, when the response of the bank's risk to monetary contraction is below this threshold (S_3) the expected cost of bank failure decreases. Below this threshold, following a monetary contraction, the frequency of crises decreases enough to compensate for the increase in their magnitude.

Case 2': The bank's risk decreases with increased interest rate, $\frac{d\alpha}{d\delta}>0$.

Once more, the total differential in the social cost is given by:

$$dC=\begin{bmatrix} \overset{(+)}{\frac{\partial C}{\partial\delta}}+\overset{(+)\qquad(-)}{(Frag+Lev)}.\overset{(+)}{\frac{d\alpha}{d\delta}} \end{bmatrix}.\overset{(+)}{d\delta}$$

As stated in case 1', the increase in the risk-free interest rate directly increases the expected social cost (the price effect), since $\frac{\partial C}{\partial\delta}>0$. However, this increased interest rate increases the bank's risk. As a result, the probability of default also increases (the fragilization effect), and so do the frequency of crises. At the same time, the loan volume decreases (the leverage effect) in response to a tighter capital requirement, decreasing the severity of a crisis. To sum up:

- The social cost increases, $dC > 0$, if the increase in the frequency of crises (due to the fragilization effect) exceeds the change in their magnitude (resulting from the price effect and the leverage effect power relationship);
- The expected social cost decreases, $dC < 0$, if the net decrease in the magnitude of crises (resulting from the price effect and the leverage effect power relationship) is larger than the increase in their frequency (due to the fragilization effect).

As in case 1', we can evidence a threshold value in the risk-taking effect that imply either an increase or a decrease in the cost of bank failure following a monetary contraction according to the position from this threshold.

We can derive that:

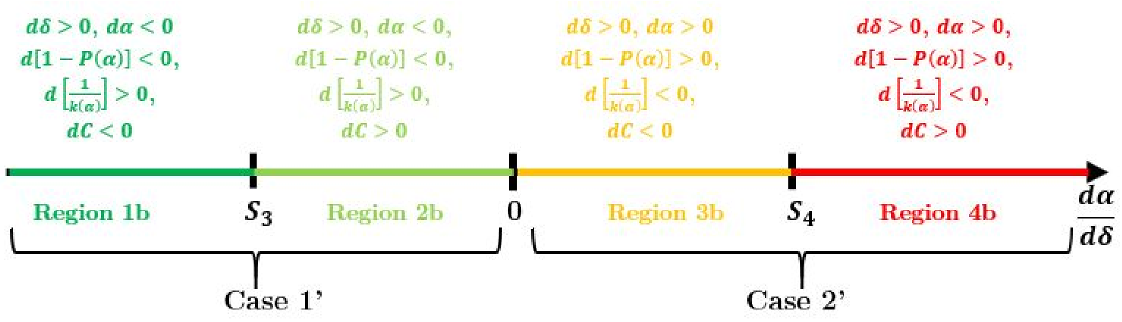
$$dC > 0 \Leftrightarrow \frac{\partial C}{\partial \delta} + (Frag + Lev).\frac{d\alpha}{d\delta} > 0$$
$$(Frag + Lev).\frac{d\alpha}{d\delta} > -\frac{\partial C}{\partial \delta}$$

Since $\frac{d\alpha}{d\delta} > 0$, then $dC > 0$ if $\boldsymbol{\frac{d\alpha}{d\delta} > -\frac{\partial C}{\partial \delta}/[Frag + Lev] \equiv S_4 > 0}$.

When monetary contraction does whet bank's risk appetite, there is also a threshold (S_4) in the response of the bank's risk to monetary contraction above which there is an increase in the expected cost of bank failure. However, when the response of the bank's risk to monetary contraction is below this threshold (S_4) a monetary contraction increases the bank's risk level but there is a decrease in the expected cost of bank failure.

Given the above conditions, there are four possible scenarios that can be depicted using Figure 5.

Figure 5: Decomposition of the effects of monetary contraction and bank's risk on the expected social cost of bank failure.



As depicted in Figure 5, we can observe that monetary contraction also has different effects on bank's risk and that the resulting effects on the expected social cost of bank failure are also ambiguous. Let's describe each region of the graph.

- Region 1b: in this region, a monetary contraction leads to a decrease in the bank's risk level. The decrease in the risk entails a decrease in the probability of default, i.e. there is a decrease in the frequency of crises. However, the bank's leverage increases (leverage effect) due to a loosening of the capital requirement. Nevertheless, the decrease in the frequency of crises is sufficient to compensate for the increase in their magnitude (stemming from the increase both in the price and in the bank's leverage). The resulting effect is a decrease in the expected social cost of a bank failure.

  $d\delta > 0 \implies Price > 0;$

  $$d\alpha < 0 \implies \begin{cases} Frag & < 0 \\ Lev & > 0 \end{cases} \implies dC < 0 \quad \text{Since} \quad \overset{(-)}{Frag} > (\overset{(+)}{Lev} + \overset{(+)}{Price})$$

- Region 2b: a monetary contraction leads to a decrease in the bank's risk level. The decrease in the risk translates into a more stable financial system, making crises less frequent. However, the bank's leverage increases (leverage effect) due to a loosening of the capital requirement. Finally, the increase in the magnitude of crises (stemming from the increase both in the price and in the bank's leverage) is sufficient to compensate for the decrease in their frequency. This leads to an increase in the expected social cost of a bank failure.

  $d\delta > 0 \implies Price > 0;$

  $$d\alpha < 0 \implies \begin{cases} Frag & < 0 \\ Lev & > 0 \end{cases} \implies dC > 0 \quad \text{Since} \quad \overset{(-)}{Frag} < (\overset{(+)}{Lev} + \overset{(+)}{Price})$$

- Region 3b: there is an increase in the bank's risk level following a monetary contraction. The increase in the risk translates into a more unstable financial system, making crises more frequent. However, the bank's leverage decreases (leverage effect) due to a tightening of the capital requirement. Finally, the net decrease in the magnitude of crises (stemming from the power relationship between the price and the leverage effects) is sufficient to compensate for the increase in their frequency. Then, there is a decrease in the expected social cost of a bank failure.

  $d\delta > 0 \implies Price > 0;$

  $$d\alpha > 0 \implies \begin{cases} Frag & > 0 \\ Lev & < 0 \end{cases} \implies dC < 0 \quad \text{Since} \quad \overset{(+)}{Frag} < (\overset{(-)}{Lev} + \overset{(+)}{Price})$$

- Region 4b: in this region, there is also an increase in the bank's risk level as a response to a monetary contraction. The increase in the risk makes the system more unstable, inducing more frequent crises. However, the bank's leverage decreases (leverage effect) due to a tightening of the capital requirement. But, the net change in the magnitude of crises (stemming from the power relationship between the price and the leverage effects) is insufficient to compensate for the increase in their frequency. So, there is an increase in the expected social cost of a bank failure.

  $d\delta > 0 \implies Price > 0;$

  $$d\alpha > 0 \implies \begin{cases} Frag & > 0 \\ Lev & < 0 \end{cases} \implies dC > 0 \quad \text{Since} \quad \overset{(+)}{Frag} > (\overset{(-)}{Lev} + \overset{(+)}{Price})$$

As we can see in Sections 4 and 5, the impact of monetary policy in terms of financial stability and expected social cost of bank failure highly depends on the risk sensitivity of the microprudential tool (here the capital requirement ratio). On the one hand, the strength of the capital requirement ratio determines how the bank responds to change in the risk-free interest rate. On the other hand, this strength of the capital requirement ratio also determines the force with which the leverage effect counter the fragilization effect in defining the response of the social cost to change in monetary policy. Then, it appears interesting to examine the macroprudential policy to be implemented according to monetary stance and the different situations (Region 1a to Region 4b).

6 Combination of monetary policy with prudential policy

The previous sections show that a change in monetary policy has different consequences in terms of bank soundness and social cost. We have situations where a monetary easing induces a increase in the bank's risk and is followed by an decrease in the expected social cost, and vice versa.

We can summarize the preceding analysis (from Section 5) in Table 1

Table 1: Summary of regions of changes in risk and cost of bank failure

Effect of Monetary policy on the bank's risk	**Monetary Policy**		**Change in the cost of bank failure**
	$d\delta < 0$	$d\delta > 0$	
$\frac{d\alpha}{d\delta} < 0$	Region 2a	Region 1b	$dC < 0$
	Region 1a	Region 2b	$dC > 0$
$\frac{d\alpha}{d\delta} > 0$	Region 4a	Region 3b	$dC < 0$
	Region 3a	Region 4b	$dC > 0$

Let's first define each combination of monetary policy with prudential policy using an efficiency criteria. We consider as:

- efficient (or totally efficient) any combination of monetary policy with prudential policy leading to a decrease in both the bank's risk and the expected social cost of bank failure;
- partially efficient any combination of monetary policy with prudential policy leading to a decrease in only one of the two objective variables. In this case, we can consider two types of partial efficient combination:

- a partial efficient (combination) of type 1: any combination of monetary policy with prudential policy leading to an increase in the bank's risk but to a decrease in the expected social cost of bank failure;
- a partial efficient (combination) of type 2: any combination of monetary policy with prudential policy leading to a decrease in the bank's risk but to an increase in the expected social cost of bank failure.

- inefficient (or totally inefficient) any combination of monetary policy with prudential policy leading to an increase in both the bank's risk and the expected social cost of bank failure.

Let's now examine each combination successively.

Regions 4a and 1b are regions where the combinations of monetary policy with microprudential tool are considered as (totally) efficient since they lead to a positive outcome in terms of more stable financial system and lower cost of crises resolution. In fact, these combinations lead to a decrease in both the bank's risk and the expected social cost of bank failure. In other words, in each of these cases, the microprudential tool is set such that a monetary easing, in one case, or a monetary contraction, in the other case, leads the bank to reduce its risk level, fostering the financial stability. In addition, the lower risk level entails a relaxation of the regulatory constraint, leading to an increase in the bank's leverage, and in the amount of financed projects (the credit volume granted to the economy). At the same time, the combination of monetary policy with the microprudential tool is such that the expected cost of bank failure is also reduced.

Conversely, regions 1a and 4b are regions where the combinations of monetary policy and microprudential tool are considered as (totally) inefficient since the outcome for the economy is negative. In fact, these combinations are such that there is an increase in both the bank's risk and the expected social cost of bank failure. In short, the microprudential tool is set such that a monetary easing, in one hand, or a monetary contraction, in the other hand, triggers more risk-taking, jeopardizing the financial stability. In addition, the higher risk level implies a tightening of the regulatory constraint, leading to a decrease in the credit volume provided to the economy. Simultaneously, the combination of monetary policy with the microprudential tool is such that the expected cost of bank failure grows up. As we have already shown, in Regions 1a and 4b, the increase in the expected social cost is driven by a high increase in the probability of default. That is, a need for a more risk sensitive microprudential tool that limits the increase in the bank's risk level may appear effective in limiting the increase in the expected cost of bank failure. In other words, a more stringent microprudential policy may be effective both in limiting the increase in the probability of default and in offsetting the increase in the social cost as well.

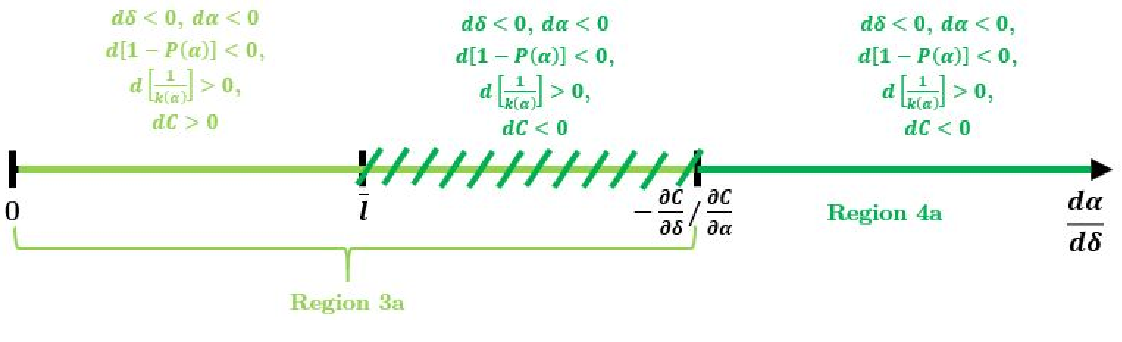
Finally, the remaining cells in Table 1 refer to combination of monetary policy with microprudential policy that are considered as partially efficient.

For instance, Regions 2a and 3b are regions where the combination of monetary policy and microprudential tool is partially efficient of type 1 since the outcome for the economy is negative in terms of financial stability and positive in terms of expected cost of bank failure. These combinations are such that there is an increase in the bank's risk but a decrease in the expected social cost of bank failure. In other words, the microprudential tool is set such that a monetary easing, in one hand, or a monetary contraction, in the other hand, leads the bank to take more risk, thus threatening the financial stability. The higher risk level implies a tightening of the regulatory constraint, leading to a decrease in the bank's leverage (decrease in the amount of financed projects by the bank). However, the combination of monetary policy with the microprudential tool is such that the expected cost of bank failure goes down.

Regions 3a and 2b are regions where the combination of microprudential tool and monetary policy is partially efficient of type 2 since the outcome for the economy is positive in terms of financial stability and negative in terms of expected cost of bank failure. In fact, these combinations are such that there is a decrease in the bank's risk but an increase in the expected social cost of bank failure. In other words, the microprudential tool is set such that a monetary easing, in one hand, or a monetary contraction, in the other hand, lowers the bank's risk, then fostering the financial stability. Besides, the lower risk level implies a relaxation of the regulatory constraint, leading to an increase in the bank's leverage (increase in the amount of financed projects). Nevertheless, the combination of monetary policy with the microprudential tool is such that the cost of bank failure grows up. Albeit, the monetary easing leads the bank to reduce its risk level, this induces an increase in the bank's leverage which itself exacerbates the severity of crises, and increases the expected social cost of bankruptcy. In this sense, although the microprudential tool improves the soundness of individual bank, it fails in mitigating the increases in the magnitude of crises. A macroprudential tool such as a cap on the leverage ratio may be suitable to complement the microprudential policy in such a case. In fact, imposing a leverage ratio may obliges the bank to not grant more loans even if it chooses a lower risk level. Hence, introducing a leverage ratio may limit the bank's leverage and may limit the increase in the expected social cost at the expense of the funding of the economy (amount of financed projects).

Let's consider Figure 4 and focus on Region 3a to illustrate the fact that introducing a leverage ratio may help in containing the expected social cost in this specific case. The result is given on Figure 6

Figure 6: Changes induced by a leverage ratio on the expected social cost of bank failure in region 3a.



Given that in Region 3a monetary easing is followed by a decrease in the bank's risk and an increase in the expected social cost of a bank failure, there is a minimum leverage ratio $\bar{l}$ that allows the social cost to decrease. In this case, imposing a minimum leverage ratio $\bar{l}$ limits the increase in the bank's leverage following a decrease in the bank's risk. The resulting effect is an enlargement of Region 4 at the expense of Region 3. The more binding the leverage ratio the smaller the region 3. In other words, the more constraining is the leverage ratio the more likely is the fact that the decrease in the cost driven by the decrease both in the price and in the probability of default exceeds the increase in cost driven by the increase in the volume of deposits. Put it differently, it may be possible for the regulator to attenuate the magnitude of the crisis by mitigating the bank's leverage. However, the more constraining the leverage ratio the lower the bank's leverage and the lower the amount of financed projects. That is, the macroprudential regulator faces a trade-off between either reducing the cost of bank failure followed by lower funding of the economy or ensuring a higher funding of the economy while facing the risk of a more severe crisis.

In a nutshell, the combination of monetary and prudential policies produces different outcomes for the economy in terms of financial stability (the bank's risk level) and in terms of the expected cost of bank failure. The combination can be totally efficient and no additional action is needed. It can be inefficient and a better calibration of the microprudential tool is suitable. When the combination of monetary and microprudential policies is such that there is lower risk but higher expected cost in the event of a crisis, then a leverage ratio appears to be effective to counteract the increase in this cost since the latter is driven by a leverage effect.

Conclusion

We built a partial equilibrium model of bank regulation and interaction with monetary policy, with perfect information. The two policies are assumed to be exogenous. The monetary policy set interest rate piloting inflation and GDP growth. Prudential regulator cares about the financial stability and is endowed with a capital requirement ratio that is risk sensitive (and increases with bank's risk).

We show that the bank's optimal risk level is determined according the risk-free interest rate and the intensity of the microprudential policy. Assuming the level of capital is exogenous fixed, the bank's leverage is then endogenously determined. Our model shows that monetary policy effectively impacts financial stability, thus supporting the existing literature on the bank risk channel of monetary policy. However, we show that the impact of monetary interest rate on the bank's risk is not independent from the sensitivity of the capital requirement to risk. A monetary easing or tightening could lead to more risk-taking according to risk sensitivity of the microprudential tool. For instance, when the risk sensitivity of the capital requirement is lower than the risk sensitivity of the bank's intermediation margin, the well-known risk-taking channel of monetary policy operates. In this case, decreased interest rates push up the bank's risk. Inversely, when the risk sensitivity of the capital requirement is higher than the risk sensitivity of the bank's intermediation margin, the bank's risk drops following monetary easing.

However, any change in monetary policy do also have important implication in terms of expected social cost of a bank failure. There is a direct effect that stems from change in the deposit cost following changes in the monetary policy. There is also an indirect effect (the risk-taking effect) driven by changes in the bank's risk. This indirect effect leads also two possible effects on the expected social cost: the fragilization effect and the leverage effect. The final change in the social cost of a bank failure depends on the relative intensity of these effects. In fact, a monetary easing can lead to more risk-taking but be followed by a decrease in the expected cost of bank failure. This is the case when the magnitude of the price effect and the leverage effect is larger than the magnitude of the fragilization effect. Furthermore, the analysis highlights cases where a more stringent microprudential tools is effective in mitigating both financial fragility and the expected social cost of a bank failure. There are some cases where a macroprudential tool, such as a leverage ratio, may complement the microprudential tool in limiting the cost of bank failure. This is particularly the case when there is a decrease in the bank's risk but an increase in the expected cost of bank failure induced by a leverage effect.

The main policy implication of our findings is that monetary and prudential policies must be coordinated. In fact, since the effect of monetary policy on bank's risk is not unique, it appears necessary for economic authorities to best manage the coordination of these two policies. If the coordination is mismanaged, the policy-mix (monetary and microprudential policy) may generate more instability and induce higher expected cost of bank failure. In the same vein, a mismanagement of the combination of microprudential policy with macroprudential policy may generate undesired outcomes in terms of providing funds to the economy (reduction of the bank's leverage and so the amount of financed projects) without much improvement in terms of financial stability. Even if prudential and monetary policies target two separate goals, financial stability for the former, and GDP growth and stable inflation for the second, their interaction should be well understood by policymakers for their combined effects to be fine tuned. Note that our study focus on a precise microprudential tool, the capital requirement ratio. It does not account for liquidity issues as introduced by Basel III.

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Appendices

Appendix 1: Proof of lemma 1

The expected return of a project is given by:

$$R^e(\alpha,\delta) = R(\alpha,\delta)P(\alpha)$$

Let's show that there is a unique $\overline{\alpha}$, with $0 < \overline{\alpha} < 1$, such that $R^e(\alpha,\delta)$ is maximum for $\overline{\alpha}$.

$$\frac{\partial R^e(\alpha,\delta)}{d\alpha} = R'_\alpha(\alpha,\delta)P(\alpha) + R(\alpha,\delta)P'_\alpha(\alpha) = R^{e'}_\alpha(\alpha,\delta)$$

According to our assumptions, we have:

1. $\lim_{\alpha\to 0} P'_\alpha(\alpha) \to 0$, $\lim_{\alpha\to 0} R'_\alpha(\alpha,\delta) \to +\infty$ and $\lim_{\alpha\to 0} R^{e'}_\alpha(\alpha,\delta) \to +\infty$
2. $\lim_{\alpha\to 1} P'_\alpha(\alpha) \to -\infty$, $\lim_{\alpha\to 1} P(\alpha) \to 0$ and $\lim_{\alpha\to 1} R^{e'}_\alpha(\alpha,\delta) \to -\infty$
3. $\frac{\partial^2 R^e(\alpha,\delta)}{\partial\alpha^2} = R^{e''}_\alpha(\alpha,\delta) = R''_\alpha(\alpha,\delta)P(\alpha) + R(\alpha,\delta)P''_\alpha(\alpha) + 2R'_\alpha(\alpha,\delta)P'_\alpha(\alpha) < 0$

As $R^{e'}_\alpha(\alpha,\delta)$ is continuous on $[0;1]$, there is a unique $0 < \overline{\alpha} < 1$, such that $R^e(\alpha,\delta)$ is maximum for $\overline{\alpha}$.

Appendix 2: Proof of proposition 1

i) We prove that there is a unique risk level that maximizes the bank's profit

Let's denote $A(\alpha,\delta) = \frac{d\pi(\alpha,\delta)}{d\alpha}$ with $\pi(\alpha,\delta) = L[R(\alpha,\delta)P(\alpha) - (1-k(\alpha))\delta P(\alpha) - k(\alpha)\rho]$

We have:

$$A(\alpha,\delta) = P(\alpha)[R'_\alpha(\alpha,\delta) + k'_\alpha(\alpha)\delta] + P'_\alpha(\alpha)[R(\alpha,\delta) - [1-k(\alpha)]\delta] - k'_\alpha(\alpha)\rho$$

Let's show that there is a unique interior solution $\alpha^* \in]0;1[$ such that $A(\alpha^*,\delta) = 0$.

$A(\alpha,\delta)$ is a continuous function for $\alpha \in [0;1]$.

For $\alpha = 0$, we have:

$$A(0,\delta) = P(0)[R'_\alpha(0,\delta) + k'_\alpha(0)\delta] + P'_\alpha(0)[R(0,\delta) - [1-k(0)]\delta] - k'_\alpha(0) * \rho$$

Since $P(0) = 1$, $R(0, \delta) = \delta$, and $k(0) = \underline{k}$, we can write

$$A(0, \delta) = R'_\alpha(0, \delta) + P'_\alpha(0)\underline{k}\delta + k'_\alpha(0)(\delta - \rho)$$

According to our assumptions, we have:

- $\lim_{\alpha \to 0} P'_\alpha(\alpha) \to 0$
- $\lim_{\alpha \to 0} R'_\alpha(\alpha, \delta) \to +\infty$

Therefore, $A(0, \delta) \to +\infty$

For $\alpha = 1$, we have:

$$A(1, \delta) = P(1)[R'_\alpha(1, \delta) + k'_\alpha(1)\delta] + P'_\alpha(1)[R(1, \delta) - [1 - k(1)]\delta] - k'_\alpha(1)\rho$$

Since $P(1) = 0$, $R(1, \delta) = \overline{R}$, and $k(1) = \overline{k}$, we can write

$$A(1, \delta) = P'_\alpha(1)[\overline{R} - (1 - \overline{k})\delta] - k'_\alpha(1)\rho$$

According to our assumptions, $\lim_{\alpha \to 1} P'_\alpha(1) \to -\infty$ and $\overline{R} > \delta$, $\overline{R} - (1 - \overline{k})\delta > 0$

Moreover, since $k'_\alpha(1)\rho > 0$, we can conclude that $A(1, \delta) \to -\infty$.

Finally, $A'_\alpha(\alpha, \delta) = \frac{\partial A(\alpha, \delta)}{\partial \alpha}$ is equal to

$$\begin{aligned} A'(\alpha, \delta) = &P'_\alpha(\alpha)[R'_\alpha(\alpha, \delta) + k'_\alpha(\alpha)\delta] + P(\alpha)[R''_\alpha(\alpha, \delta) + k''_\alpha(\alpha)\delta] \\ &P''_\alpha(\alpha)[R(\alpha, \delta) - (1 - k(\alpha))\delta] + P'_\alpha(\alpha)[R'_\alpha(\alpha, \delta) + k'_\alpha(\alpha)\delta] \\ &- k''_\alpha(\alpha)\rho \end{aligned}$$

$$\begin{aligned} A'(\alpha, \delta) = &2P'_\alpha(\alpha)[R'_\alpha(\alpha, \delta) + k'_\alpha(\alpha)\delta] + P(\alpha)[R''_\alpha(\alpha, \delta) + k''_\alpha(\alpha)\delta] \\ &+ P''_\alpha(\alpha)[R(\alpha, \delta) - (1 - k(\alpha))\delta] - k''_\alpha(\alpha)\rho \end{aligned}$$

Let's remind that:

- $R'_\alpha(\alpha, \delta) > 0$ and $R''_\alpha(\alpha, \delta) < 0$
- $P'_\alpha(\alpha) < 0$ and $P''_\alpha(\alpha) < 0$
- $k'_\alpha(\alpha) > 0$ and $k''_\alpha(\alpha) < 0$
- And $R(\alpha, \delta) - (1 - k(\alpha))\delta > 0$

As a result, we have:

- $2P'_\alpha(\alpha)[R'_\alpha(\alpha,\delta) + k'_\alpha(\alpha)\delta] < 0$
- $P(\alpha)[R''_\alpha(\alpha,\delta) + k''_\alpha(\alpha)\delta] < 0$
- $P''_\alpha(\alpha)[R(\alpha,\delta) - (1 - k(\alpha))\delta] < 0$
- And $k''_\alpha(\alpha)\rho < 0$

We can rewrite $A'_\alpha(\alpha,\delta)$ as follows:

$A'_\alpha(\alpha,\delta) = k''_\alpha(\alpha)[P(\alpha)\delta - \rho] + 2P'_\alpha(\alpha)[R'_\alpha(\alpha,\delta) + k'_\alpha(\alpha)\delta] + P(\alpha)R''_\alpha(\alpha,\delta)) + P''_\alpha(\alpha)[R(\alpha,\delta) - (1 - k(\alpha))\delta]$

Let's find the condition on $k''_\alpha(\alpha)$ such that $A'_\alpha(\alpha,\delta) < 0$.

$$\boldsymbol{A'_\alpha(\alpha,\delta) < 0 \Leftrightarrow k''_\alpha(\alpha) > -\frac{X}{Y}}$$

With:
$X = 2P'_\alpha(\alpha)[R'_\alpha(\alpha,\delta) + k'_\alpha(\alpha)\delta] + P(\alpha)R''_\alpha(\alpha,\delta)) + P''_\alpha(\alpha)[R(\alpha,\delta) - (1 - k(\alpha))\delta]$
and $Y = P(\alpha)\delta - \rho$

This condition supposes that the capital requirement function should be concave, but not concave enough.

So under the condition that $0 > k''_\alpha(\alpha) > -\frac{X}{Y}$, we can conclude that $A'_\alpha(\alpha,\delta) < 0$. Consequently, $A(\alpha,\delta)$ is a decreasing function on]0;1[. Moreover, since $A(\alpha,\delta)$ moves from positive to negative value ($A(0,\delta) \to +\infty$ and $A(1,\delta) \to -\infty$), and is strictly decreasing, there is a unique $\alpha^* \in]0;1[$ such that $A(\alpha^*,\delta) = 0$

Furthermore, as $A'_\alpha(\alpha,\delta) < 0$, we can deduce that the bank's profit curve is inverted U-shaped and $\pi(\alpha^*,\delta)$ is a maximum.

ii) Since the bank chooses its optimal risk α^*, the bank will finance the amount of projects $L^* = \frac{\overline{K}}{k(\alpha^*)}$. In other words, the bank leverage is endogenously determined since it depends on the bank's risk choice.

Appendix 3: Proof of proposition 2

We seek to determine the impact of the monetary policy on the bank's optimal risk level, α^*. In other words, we aim at determining the sign of $\frac{d\alpha^*}{d\delta}$. To reach our objective, we use the total differential in the $A(\alpha,\delta)$.

Let's remind that $A(\alpha,\delta) = P(\alpha)[R'_\alpha(\alpha,\delta) + k'_\alpha(\alpha)\delta] + P'_\alpha(\alpha)[R(\alpha,\delta) - [1-k(\alpha)]\delta] - k'_\alpha(\alpha) * \rho$

The total derivative of $A(\alpha,\delta)$ (according both to α and δ) is given by:

$$dA(\alpha,\delta) = A'_\alpha(\alpha,\delta).d\alpha + A'_\delta(\alpha,\delta).d\delta$$

Where $A'_\alpha(\alpha,\delta)$ and $A'_\delta(\alpha,\delta)$ are the partial derivatives of $A(\alpha)$ according respectively to α and δ.

Consequently:

$$\begin{aligned} dA(\alpha^*,\delta) = 0 \Leftrightarrow & A'_\alpha(\alpha^*,\delta).d\alpha^* + A'_\delta(\alpha^*,\delta).d\delta = 0 \\ & A'_\alpha(\alpha^*,\delta).d\alpha^* = -A'_\delta(\alpha^*,\delta).d\delta \\ \text{and} \quad & \frac{d\alpha^*}{d\delta} = -\frac{A'_\delta(\alpha^*,\delta)}{A'_\alpha(\alpha^*,\delta)} \end{aligned}$$

In the proof of proposition 1, we have shown that $A'_\alpha(\alpha^*,\delta) < 0$. This means that the sign of $\frac{d\alpha^*}{d\delta}$ mainly depends on the sign of $A'_\delta(\alpha^*,\delta)$,

$$A'_\delta(\alpha^*,\delta) = P'_\alpha(\alpha^*)[R'_\delta(\alpha^*,\delta) - (1-k(\alpha^*))] + k'_\alpha(\alpha^*)P(\alpha^*)$$

Since $P'_\alpha(\alpha^*) < 0$, $k'_\alpha(\alpha^*) > 0$, $P(\alpha^*) > 0$ and under the assumption that $R'_\delta(\alpha^*,\delta) - [1-k(\alpha^*)] > 1$, we have:

- $A'_\delta(\alpha^*,\delta) < 0$ if $|P'_\alpha(\alpha^*)|[R'_\delta(\alpha^*,\delta) - (1-k(\alpha^*))] > k'_\alpha(\alpha^*)P(\alpha^*)$, and $\frac{d\alpha^*}{d\delta} < 0$. The bank's risk decreases with monetary rates if the net expected marginal gain of taking additional risk does not overcome the marginal funding cost induced by the strengthening of the capital requirement ratio.

- $A'_\delta(\alpha^*,\delta) > 0$ if $|P'_\alpha(\alpha^*)|[R'_\delta(\alpha^*,\delta) - (1-k(\alpha^*))] < k'_\alpha(\alpha^*)P(\alpha^*)$, and $\frac{d\alpha^*}{d\delta} > 0$. The bank's risk increases with monetary rates if the net expected marginal gain of taking additional risk overcomes the marginal funding cost induced by the strengthening of the capital requirement ratio.

- $A'_\delta(\alpha^*,\delta) = 0$ if $|P'_\alpha(\alpha^*)|[R'_\delta(\alpha^*,\delta) - (1-k(\alpha^*))] = k'_\alpha(\alpha^*)P(\alpha^*)$, and $\frac{d\alpha^*}{d\delta} = 0$.The bank's optimal risk level remains unchanged whatever the change in the monetary rates. In that case, the bank has no interest in changing its level of risk, otherwise it will have to suffer loss of profit.

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