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Sharing a Polluted River under Waste Flow Control

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Abstract

When the cleaning up of a polluted transboundary river requires the cooperation of several agents (countries, regions, firms or cities) located along it, a challenging issue is how should the pollutant-cleaning costs be shared among them. An important factor ignored by literature so far concerns the ability for wastewater treatment of the river itself depending on both sediment types and ecological units (hydrophyte filter beds, aerobic digesters,. . .) in order to control waste flow from upstream to downstream. First, we introduce and implement a new cost sharing method for polluted river problems under waste flow control, called the Downstream Compensation method, which combines the two well-known conflicting theories in international river disputes, namely the Absolute Territorial Sovereignty and the Unlimitted Territorial Integrity. When the river does not have any wastewater treatment ability, the Downstream Compensation method coincides with the Downstream Equal Sharing method. At the other extreme case of full wastewater treatment within the river, the Downstream Compensation method corresponds to the Local Responsibility Sharing method. Second, we show that the Downstream Compensation method is obtained as the Shapley value of appropriately defined cooperative games with transferable utility. Finally, we prove that these games satisfy the concavity property, meaning that the proposed cost allocation scheme belongs to the core.

Keywords: Polluted river; Wastewater treatment rate; Cost sharing; Shapley value; Core
JEL Classification System: C71; D61; D62

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1 Introduction

The study of allocation problems to resolve international disputes related to transboundary rivers has developed in two directions. On the one hand, a growing literature has investigated the problem of sharing clean water resources among several agents (countries, regions, firms or cities) located along a river, see e.g. Ambec and Sprumont (2002 [3]), Parrachino *et al.* (2006 [24]), van den Brink *et al.* (2007 [11]), Ambec and Ehlers (2008 [2]), Khmelnitskaya (2010 [17]), Wang (2011 [32]), Ansink and Weikard (2012 [4]), van den Brink *et al.* (2012 [10]), van den Brink *et al.* (2014 [7]) and Béal *et al.* (2015 [6]). On the other hand, several works, including ours, have developed game theoretical models for studying how to share the costs of cleaning up a polluted river among agents located along it.

Water pollution is a major environmental problem faced by modern societies that leads to unprecedented ecological imbalances, chasing countless species from their habitats, destroying biodiversity and depleting land. Around the world, more than 200 rivers pass through national borders (Ambec and Sprumont 2002 [3], Barrett 2010 [5]), and many others flow across borders between regions or cities. From a theoretical point of view, the so-called cost sharing problem on a river network, shortly called polluted river problem, was first introduced by Ni and Wang (2007 [23]) for single spring rivers, and generalized by Dong *et al.* (2012 [12]) for rivers with multiple springs. They proposed and characterized three cost sharing methods based on the two well-known conflicting theories in international river disputes, namely the Absolute Territorial Sovereignty (ATS) and the Unlimitted Territorial Integrity (UTI).[1] The Local Responsibility Sharing (LRS) method applies the local responsibility principle implied by the ATS theory, and requires that each agent should pay for the cleaning cost in its own territory. The Upstream Equal Sharing (UES) (Downstream Equal Sharing (DES), respectively) method corresponds to the downstream responsibility (upstream responsibility, respectively) principle implied by the UTI theory, and assigns each agent with its own cleaning cost plus the equal sharing of the downstream (upstream, respectively) costs.[2] The authors also showed that these methods were obtained as the Shapley value of appropriately defined cooperative games with transferable utility (henceforth TU-games), and as solutions belonging to the core of these games. Moreover, van den Brink *et al.* (2018 [9]) proved that the UES and DES methods correspond to the conjunctive permission value (van den Brink and Gilles, 1996 [8]) of an associated cooperative game with a permission structure. They also introduced a new cost sharing method, called the Upstream Limited Sharing method, by applying the disjunctive permission value (Gilles and Owen, 1994 [13]) to every polluted river problem.

[1]While the ATS theory stipulates that *a country has absolute sovereignty over the area of any river basin within its territory*, the UTI theory stipulates that *a country should not alter the natural conditions within its own territory to the disadvantage of a neighboring country.* We refer to Godana (1985 [15]) and Kilgour and Dinar (1996 [18]) for more discussions on the ATS and the UTI theories.

[2]Ni and Wang (2007 [23]) first proposed the LRS and UES methods. Dong *et al.* (2012 [12]) extended these two methods to rivers with multiple springs and introduced the DES method based on a new interpretation of the UTI theory.

Recently, Sun *et al.* (2019 [27]) proposed and characterized a cost allocation scheme based on a one-by-one formation of the grand coalition (Shapley, 1953 [25]) which turned out to be a convex combination of the LRS and UES methods.

An important factor ignored by literature on polluted river problems so far is that the river itself may have some ability for wastewater treatment in order to control waste flow from upstream to downstream. Generally speaking, water purification process depends on both natural conditions such as sediment types (see e.g. Wijesiri *et al.*, 2019 [33]) and ecological units such as hydrophyte filter beds and aerobic digesters[3] among others (see e.g. Vanderpoorten, 1999 [31], and Wu *et al.*, 2013 [34]). In this article, we generalize the polluted river model introduced by Ni and Wang (2007 [23]) by taking into account the ability for wastewater treatment of the river itself. Precisely, we assume that a river is divided into several segments, each one having an exogenous wastewater treatment rate of between 0 (null wastewater treatment) and 1 (full wastewater treatment). The waste transfer rate from an upstream segment to a downstream one is then equal to the product of the wastewater treatment rates of the segments between these two segments, including the upstream one. This enables us to define the transferred cleaning cost an agent has to pay for the cleaning up of the waste flow from its upstream river segments.

First, we propose a new cost sharing method for polluted river problems under waste flow control, called the Downstream Compensation (DC) method. Similarly to Shapley's idea of a one-by-one formation of the grand coalition (Shapley, 1953 [25]), we provide a procedural implementation of the DC method[4] which turns out to be a natural compromise between the ATS and the UTI theories. During the coalition formation process, each entrant bargains with the agents within the existing coalition in accordance with the ATS and the UTI theories. First, according to the local responsibility principle implied by the ATS theory, each entering agent undertakes its own cleaning cost. Second, once the entering agent cleans up the pollutants in its own territory, according to the upstream responsibility principle implied by the UTI theory,[5] it requires that each of its downstream agents compensates a part of the transferred cleaning costs of its upstream agents. Conversely, the upstream agents also require that the entering agent compensates a part of their own transferred cleaning costs. Proceeding in this way, once the grand coalition is formed, independently of the selected arrival order, the total pollutant-cleaning cost is fully distributed among the agents. We then show that this cost allocation scenario combining the ATS and the UTI theories implements the DC method. Interestingly, when the river segments do not have any wastewater treatment ability, the DC method coincides with the DES method (Dong et al, 2012 [12]) and the Airport Landing Fee solution (Littlechild and Owen, 1973 [19]).[6] At the other extreme case of full wastewater treatment of the river segments, the DC method corresponds to the LRS method (Ni and Wang, 2007 [23]).

[3] In what follows, we will assume that such ecological units can be installed at zero cost.

[4] Malawski (2013 [20]), Sun *et al.* (2017a [28]) and Sun *et al.* (2017b [29]) also focus on the dynamic formation process of the grand coalition in order to propose new classes of values for TU-games.

[5] According to the interpretation of Dong *et al.* (2012 [12]), the UTI theory requires monetary compensations from downstream agents to upstream ones.

[6] See also Tijs and Driessen (1986 [30]) and Hou *et al.* (2018 [16]) on that point.

Second, we prove that the DC method is obtained as the Shapley value (Shapley, 1953 [25]) of TU-games where the worth of any coalition is derived from the ATS and the UTI theories. Finally, we establish that these games satisfy the concavity property, meaning, in the context of cost games, that the cost allocation scheme induced by the DC method belongs to the core.

The paper is organized as follows. In Section 2, we define polluted river problems under waste flow control and introduce the DC method. In Section 3, we provide a procedural implementation of the DC method which combines the ATS and the UTI theories. In Section 4, we prove that the DC method is obtained as the Shapley value of appropriately defined TU-games. In Section 5, we explore the stability of the DC method by establishing the concavity of these games. In Section 6, we give some concluding remarks on another compensation method, called the Upstream Compensation (UC) method, based on the downstream responsibility principle implied by the UTI theory.

2 Preliminaries

2.1 Cooperative TU-games and solutions

A *cooperative game with transferable utility* (or simply a TU-game) on a finite set $N \subset \mathbb{N}$ of players is a mapping $v : 2^N \longrightarrow \mathbb{R}$ such that $v(\emptyset) = 0$. A non-empty subset $S \subseteq N$ is called a *coalition*, whose cardinality will be denoted by s. For any coalition $S \subseteq N$, $v(S)$ describes the *worth* that coalition S can achieve when all its members cooperate. The set of all TU-games is denoted by G.

A *payoff vector* for game $(N, v) \in G$ is an n-dimensional vector $x \in \mathbb{R}^N$ assigning a payoff $x_i \in \mathbb{R}$ to any player $i \in N$. A *solution* on G is a function φ which associates with each TU-game $(N, v) \in G$ a subset $\varphi(N, v) \subseteq R^N$ of payoff vectors. If φ assigns a unique payoff vector to each TU-game $(N, v) \in G$, then φ is called a *value*. One of the most well-known solutions for TU-games is the *Shapley value* (Shapley, 1953 [25]) given by

$$Sh_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} (v(S \cup \{i\}) - v(S)) \text{ for all } i \in N.$$

The Shapley value has appealing properties and has been successfully applied to a diversity of fields (Moretti and Patrone, 2008 [21]).

A TU-game can reflect costs or rewards. The following definition and property refer to costs and will be useful in the context of cost allocation of a polluted river. The *core* (Shapley, 1955 [26]) of a cost game $(N, v) \in G$ is given by

$$C(N, v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N), \sum_{i \in S} x_i \leq v(S) \text{ for all } S \subset N \right\}.$$

Given a payoff vector in the core, the grand coalition N forms and distributes the total cost to its members in such a way that no coalition can dispute this distribution of costs by breaking off from the grand coalition.

It is widely known that a convex game always has a non-empty core and, in particular the Shapley value belongs to the core (Moulin, 1988 [22]). For cost games, the convexity is instead replaced by the concavity of the game. A TU-game $(N, v) \in G$ is *concave* if for all $i \in N$, all $S, T \subseteq N \setminus \{i\}$, if $S \subseteq T$, then

$$v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S) \tag{1}$$

Thus, the game (N, c) is concave if the marginal contribution of a player to a coalition is monotone non-increasing with respect to set-theoretic inclusion.

2.2 Polluted river problems under waste flow control

A *polluted river problem under waste flow control* is given by a triple (N, c, γ), where $N = \{1, \ldots, n\}$ is a finite set of agents located along a river divided into n segments, $c \in \mathbb{R}^N_+$ is an n-dimensional cost vector, and $\gamma \in [0,1]^{N \setminus \{n\}}$ is an $(n-1)$-dimensional wastewater treatment rate vector. The river segments are indexed in a given order $i = 1, 2, \ldots, n$ from upstream to downstream, and each agent is located in one of segments according to this order. The cost vector $c \in \mathbb{R}^N_+$ is such that c_i corresponds to the cost of cleaning river segment i in order to satisfy environmental standards for water quality. The wastewater treatment rate vector $\gamma \in [0,1]^{N \setminus \{n\}}$ is such that γ_i represents the proportion of waste that is transferred from segment i to its unique downstream neighbor.[7]

A *cost allocation* for a polluted river problem under waste flow control (N, c, γ) is a vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^N_+$ such that $\sum_{i \in N} x_i = \sum_{i \in N} c_i$, where x_i is the cost to be paid by agent $i \in N$. A *cost sharing method* is a mapping g that assigns a cost alllocation $g(N, c, \gamma) \in \mathbb{R}^N_+$ to every (N, c, γ).

The following three cost sharing methods was respectively introduced by Ni and Wang (2007 [23]) and Dong *et al.* (2012 [12]). First, the *Local Responsibility Sharing (LRS) method* assigns to every agent its own cost, and is given by

$$g_i^{LRS}(N, c, \gamma) = c_i \text{ for all } i \in N.$$

The *Upstream Equal Sharing (UES) method* shares equally the cost of cleaning a certain river segment among all agents that are located upstream of that segment, and is given by

$$g_i^{UES}(N, c, \gamma) = \sum_{k=i}^{n} \frac{c_k}{k} \text{ for all } i \in N.$$

The *Downstream Equal Sharing (DES) method* shares equally the cost of cleaning a certain river segment among all agents that are located downstream of that segment, and is given by

[7] In our model, it is not necessary to specify the wastewater treatment rate of the last segment at the mouth of the river.

$$g_i^{DES}(N,c,\gamma) = \sum_{k=1}^{i} \frac{c_k}{n-k+1} \text{ for all } i \in N.$$

Obviously, the three above methods do not take into account the waste flows between river segments that depend on their wastewater treatment rates. Precisely, for any two river segments $i, j \in N$ such that $i < j$, the real number $\alpha_j^i = \prod_{k=i}^{j-1}(1-\gamma_k)$ is the *waste transfer rate* from i to j when every agent $k \in \{i, i+1, \dots, j-1\}$ does not clean up the waste in its own territory.[8] The product $\alpha_j^i c_i$ then represents the *transferred cleaning cost* agent j has to pay for the cleaning up of the waste flow from river segment i. Considering the waste flows between river segments from upstream to downstream, we propose a new cost sharing method, called the *Downstream Compensation (DC) method*, given by

$$g_i^{DC}(N,c,\gamma) = c_i + \sum_{k=1}^{i-1} \frac{\alpha_i^k c_k}{i-k+1} - \sum_{p=i+1}^{n} \sum_{k=1}^{i} \frac{\alpha_p^k c_k}{(p-k+1)(p-k)} \text{ for all } i \in N \qquad (2)$$

where, by convention, $\alpha_1^0 = \alpha_{n+1}^n = 0$. The DC method is a combination of the ATS and the UTI theories by which the cost agent $i \in N$ has to pay is composed of three parts. First, agent $i \in N$ covers his own cost c_i according to the local responsibility principle implied by the ATS theory. Second, agent $i \in N$ compensates every agent $k \in \{1, \dots, i-1\}$ according to the upstream responsibility principle implied by the UTI theory. This compensation is based on the transferred cleaning cost $\alpha_i^k c_k$. Noting that the number of river segments from k to i, $k \in \{1, \dots, i-1\}$, is $i-k+1$, it is common for i to compensate $\frac{\alpha_i^k c_k}{i-k+1}$. Third, following the upstream responsibility principle, every agent $p \in \{i+1, \dots, n\}$ compensates agent $i \in N$. This compensation corresponds to the transferred cleaning cost $\alpha_p^k c_k$ which is divided by the number $p-k+1$ of agents involved in the waste flow from k to p, and is then shared equally among the $p-k$ agents that are located upstream river segment p, i.e., $k+1, k+2, \dots, p$. Interestingly, it turns out that the LRS and the DES methods become special cases of the DC method.

Proposition 2.1. For every polluted river problem under waste flow control (N, c, γ), it holds that

(i) If $\alpha_j^i = 0$ for all $i, j \in N$ such that $i < j$, then the DC method coincides with the LRS method, i.e., $g_i^{DC}(N,c,\gamma) = g_i^{LRS}(N,c,\gamma)$ for all $i \in N$;

(ii) If $\alpha_j^i = 1$ for all $i, j \in N$ such that $i < j$, then the DC method coincides with the DES method, i.e., $g_i^{DC}(N,c,\gamma) = g_i^{DES}(N,c,\gamma)$ for all $i \in N$.

[8] If there exists an agent between i and j, including i itself, that absorbs all the pollution in its territory, i.e. $\gamma_k = 1$ for some $k \in \{i, i+1, \dots, j-1\}$, then $\alpha_j^i = 0$. On the contrary, if all the agents between i and j, including i itself, have no wastewater treament ability, i.e. $\gamma_k = 0$ for all $k \in \{i, i+1, \dots, j-1\}$, then $\alpha_j^i = 1$.

Proof. (i) The proof follows directly from (2) by applying $\alpha_j^i = 0$ for all $i, j \in N$ such that $i < j$.
(ii) Assume that $\alpha_j^i = 1$ for all $i, j \in N$ such that $i < j$. Then, it holds that

$$\begin{aligned}
g_i^{DC}(N,c,\gamma) &= c_i + \sum_{k=1}^{i-1} \frac{c_k}{i-k+1} - \sum_{l=i+1}^{n} \sum_{k=1}^{i} \frac{c_k}{(l-k+1)(l-k)} \\
&= c_i - \sum_{l=i+1}^{n} \frac{c_i}{(l-i+1)(l-i)} + \sum_{k=1}^{i-1} \frac{c_k}{i-k+1} - \sum_{l=i+1}^{n} \sum_{k=1}^{i-1} \frac{c_k}{(l-k+1)(l-k)} \\
&= c_i - \sum_{l=i+1}^{n} \left(\frac{c_i}{l-i} - \frac{c_i}{l-i+1} \right) + \sum_{k=1}^{i-1} \frac{c_k}{i-k+1} - \sum_{k=1}^{i-1} \sum_{l=i+1}^{n} \frac{c_k}{(l-k+1)(l-k)} \\
&= c_i - c_i \left(1 - \frac{1}{n-i+1} \right) + \sum_{k=1}^{i-1} \frac{c_k}{i-k+1} - \sum_{k=1}^{i-1} \sum_{l=i+1}^{n} \left(\frac{c_k}{l-k} - \frac{c_k}{l-k+1} \right) \\
&= \frac{c_i}{n-i+1} + \sum_{k=1}^{i-1} \frac{c_k}{i-k+1} - \sum_{k=1}^{i-1} \left(\frac{c_k}{i-k+1} - \frac{c_k}{n-k+1} \right) \\
&= \frac{c_i}{n-i+1} + \sum_{k=1}^{i-1} \frac{c_k}{n-k+1} \\
&= \sum_{k=1}^{i} \frac{c_k}{n-k+1} \\
&= g_i^{DES}(N,c,\gamma),
\end{aligned}$$

which concludes the proof. □

It is worth noting that when the river segments do not have any wastewater treatment ability as in (ii) of Proposition 2.1, the DC method also coincides with the Airport Landing Fee solution (Littlechild and Owen, 1973 [19]).

To conclude this preliminary section, we introduce a new set of TU-games which will be useful to establish the fairness and the stability of the DC method in Sections 4 and 5. The *Downstream-oriented game* $(N, v^D) \in G$ associated to the polluted river problem under waste flow control (N, c, γ) is defined for every coalition $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$, $i_1 < i_2 < \cdots < i_s$, by

$$v^D(S) = \sum_{i \in S} c_i + \sum_{k=1}^{s} \sum_{t=i_{(k-1)}+1}^{i_k - 1} \alpha_{i_k}^t c_t \tag{3}$$

where, by convention, $i_0 = 0$. The worth $v^D(S)$ characterizes the cleaning costs that coalition S should undertake when the upstream players of its members do not clean up the pollutants in their territories. Precisely, players in S are first responsible for their own total cost $\sum_{i \in S} c_i$ according to the ATS theory. Second, for every player $t \in \{i_{(k-1)}+1, \ldots, i_k-1\}$ where $k \in \{1, \ldots, s\}$, since t is not a member of S, the transferred waste from t heaps up in i_k. Hence, the transferred cleaning cost $\alpha^t_{i_k} c_t$ should also be covered by coalition S according to the UTI theory insofar as these players are located between segments $i_{(k-1)}$ and i_k. Note that if $i_{(k-1)}$ and i_k are located next to each other, player $i_{(k-1)}$ cleans up the waste in its own territory, and therefore the transferred cleaning cost $\alpha^{i_{(k-1)}}_{i_k} c_{i_{(k-1)}}$ does not appear in the expression of $v^D(S)$.

3 Procedural implementation of the DC method

It is generally assumed that the grand coalition has to be formed in order to get Pareto efficient solutions such as, for example, the Shapley value (Shapley, 1953 [25]) and the core (Shapley, 1955 [26]). The cost allocation scenario envisaged here to implement the DC method takes up Shapley's idea of a one-by-one formation of the grand coalition and consists of the following steps.

1. Choose any polluted river problem under waste flow control (N, c, γ) and any arrival order π of agents on N to gradually form the grand coalition.[9]

2. Each entering agent $i \in N$ has to pay for its own cost c_i.

3. Each entering agent $i \in N$ asks each agent $p \in N$ such that $p > i$ and $\pi(p) < \pi(i)$ to compensate it for the amount of $\sum_{k=1}^{i} \frac{\alpha^k_p c_k}{(p-k+1)(p-k)}$.

4. For each entering agent $i \in N$, each agent $q \in N$ such that $q < i$ and $\pi(q) < \pi(i)$ asks agent i to compensate it for the amount of $\sum_{k=1}^{q} \frac{\alpha^k_i c_k}{(i-k+1)(i-k)}$.

5. Steps 1-4 determine a cost allocation $x \in \mathbb{R}^N_+$ such that

$$x_i = c_i + \sum_{q=1}^{i-1} \sum_{k=1}^{q} \frac{\alpha^k_i c_k}{(i-k+1)(i-k)} - \sum_{p=i+1}^{n} \sum_{k=1}^{i} \frac{\alpha^k_p c_k}{(p-k+1)(p-k)} \text{ for all } i \in N \quad (4)$$

While Step 2 is based on the local responsibility principle implied by the ATS theory, Steps 3 and 4 are based on the downstream compensation principle implied by the UTI theory. In addition, an interesting observation is that cost allocation $x \in \mathbb{R}^N_+$ given by (4) at Step 5 is independent of the arrival order π selected at Step 1. This is a main difference with existing allocation scenarios usually proposed to implement values (see e.g. Malawski, 2013 [20]) that take the average of some contributions vectors over all the $n!$ possible arrival

[9] Formally, an arrival order π on N assigns a position $\pi(i)$ to each agent $i \in N$.

orders π on N.

The following result establishes the relationship between the cost allocation scenario described above and the DC method.

Theorem 3.1. For every polluted river problem under waste flow control (N, c, γ), the DC method coincides with the cost allocation given by (4).

Proof. It suffices to prove that the total cost compensation from agent $i \in N$ obtained by its upstream agents are equal in both expressions (2) and (4). Consequently, one has

$$\begin{aligned}\sum_{q=1}^{i-1}\sum_{k=1}^{q}\frac{\alpha_i^k c_k}{(i-k+1)(i-k)} &= \sum_{k=1}^{i-1}\sum_{k\leq q\leq i-1}\frac{\alpha_i^k c_k}{(i-k+1)(i-k)}\\ &= \sum_{k=1}^{i-1}(i-k)\frac{\alpha_i^k c_k}{(i-k+1)(i-k)}\\ &= \sum_{k=1}^{i-1}\frac{\alpha_i^k c_k}{(i-k+1)},\end{aligned}$$

completing the proof. □

The following three-agent example illustrates the cost allocation scenario.

Example 3.2. Consider a polluted river problem under waste flow control (N, c, γ) where $N = \{1, 2, 3\}$ and the arrival order π such that $\pi(1) = 2$, $\pi(2) = 3$ and $\pi(3) = 1$. Agent 3 first enters and undertakes its own cost c_3 with no costs for the other two agents. At this first step, the cost allocation is $(0, 0, c_3)$. Then, agent 1 joins the coalition. Due to the cost allocation scenario rules, agent 1 pays its own cost c_1, and asks agent 3 to compensate it by paying the amount of $\frac{\alpha_3^1 c_1}{3\times 2}$. At this second step, agent 2 still has no costs and the cost allocation is $(c_1 - \frac{\alpha_3^1 c_1}{6}, 0, \frac{\alpha_3^1 c_1}{6})$. Finally, agent 2 enters the coalition and undertakes its own cost c_2. It also asks agent 3 to compensate it by paying the amount of $\frac{\alpha_3^1 c_1}{3\times 2} + \frac{\alpha_3^2 c_2}{2}$. Furthermore, agent 2 compensates agent 1 by paying the amount of $\frac{\alpha_2^1 c_1}{2}$. At this third step, the cost allocation is $(-\frac{\alpha_2^1 c_1}{2}, c_2 + \frac{\alpha_2^1 c_1}{2} - (\frac{\alpha_3^1 c_1}{6} + \frac{\alpha_3^2 c_2}{2}), \frac{\alpha_3^1 c_1}{6} + \frac{\alpha_3^2 c_2}{2})$. Summating the costs covered at the three steps, we then obtain $(c_1 - \frac{\alpha_2^1 c_1}{2} - \frac{\alpha_3^1 c_1}{6}, c_2 + \frac{\alpha_2^1 c_1 - \alpha_3^2 c_2}{2} - \frac{\alpha_3^1 c_1}{6}, c_3 + \frac{\alpha_3^2 c_2}{2} + \frac{\alpha_3^1 c_1}{3})$.

4 The DC method and the Shapley value

In this section, we investigate the fairness of the DC method for polluted river problems under waste flow control. Precisely, we want to establish that the DC method is obtained as the Shapley value (Shapley, 1953 [25]) of the Downstream-oriented game given by (3). The following lemma brings forward the marginal contributions to the coalitions.

Lemma 4.1. Let $(N, v^D) \in G$ be a Downstream-oriented game. Then, for every coalition $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$ where $i_1 < i_2 < \cdots < i_s$, and all $i \notin S$, it holds that

(i) If $i < i_1$, then $v^D(S \cup \{i\}) - v^D(S) = c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_{i_1}^t c_t$;

(ii) If $i > i_s$, then $v^D(S \cup \{i\}) - v^D(S) = c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t$;

(iii) If there exists $p \in \{1, 2, \ldots, s-1\}$ such that $i_p, i_{(p+1)} \in S$ and $i_p < i < i_{(p+1)}$, then $v^D(S \cup \{i\}) - v^D(S) = c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_{i_{(p+1)}}^t c_t$.

Proof. Let $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$ be a coalition where $i_1 < i_2 < \cdots < i_s$.
(i) First, take any player $i \notin S$ such that $i < i_1$. It follows from (3) that

$$v^D(S \cup \{i\}) = \sum_{k \in S \cup \{i\}} c_k + \sum_{t=1}^{i-1} \alpha_i^t c_t + \sum_{t=i+1}^{i_1-1} \alpha_{i_1}^t c_t + \sum_{k=2}^{s} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t,$$

and

$$v^D(S) = \sum_{k \in S} c_k + \sum_{t=1}^{i_1-1} \alpha_{i_1}^t c_t + \sum_{k=2}^{s} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t.$$

Hence, it holds that

$$v^D(S \cup \{i\}) - v^D(S) = c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_{i_1}^t c_t.$$

(ii) Second, take any player $i \notin S$ such that $i > i_s$. It follows from (3) that

$$\begin{aligned} v^D(S \cup \{i\}) &= \sum_{k \in S \cup \{i\}} c_k + \sum_{k=1}^{s} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t \\ &= v^D(S) + c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t. \end{aligned}$$

(iii) Third, take any player $i \notin S$ such that $i_p < i < i_{(p+1)}$ for some $p \in \{1, 2, \ldots, s-1\}$. It follows from (3) that

$$v^D(S \cup \{i\}) = \sum_{k \in S \cup \{i\}} c_k + \sum_{k=1}^{p} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t + \sum_{t=i+1}^{i_{(p+1)}-1} \alpha_{i_{(p+1)}}^t c_t + \sum_{k=p+2}^{s} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t$$

and

$$v^D(S) = \sum_{k\in S} c_k + \sum_{k=1}^{p} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t + \sum_{t=i_p+1}^{i_{(p+1)}-1} \alpha_{i_{(p+1)}}^t c_t + \sum_{k=p+2}^{s} \sum_{t=i_{(k-1)}+1}^{i_k-1} \alpha_{i_k}^t c_t.$$

Hence, it holds that

$$\begin{aligned} v^D(S\cup\{i\}) - v^D(S) &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t + \sum_{t=i+1}^{i_{(p+1)}-1} \alpha_{i_{(p+1)}}^t c_t - \sum_{t=i_p+1}^{i_{(p+1)}-1} \alpha_{i_{(p+1)}}^t c_t \\ &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_{i_{(p+1)}}^t c_t, \end{aligned}$$

which concludes the proof. □

It follows with Lemma 4.1 about the marginal contributions that the DC method can be obtained by applying the Shapley value to the Downstream-oriented game.

Theorem 4.2. Let (N, c, γ) be a polluted river problem under waste flow control. Then $g^{DC}(N, c, \gamma) = Sh(N, v^D)$.

Proof. First, we deduce from the marginal contributions in Lemma 4.1 that the Shapley value of (N, v^D) for any player $i \in N$ consists of the following three items: c_i, $\alpha_i^k c_k$ with $1 \leq k \leq i-1$, and $\alpha_p^k c_k$ with $1 \leq k \leq i$ and $i+1 \leq p \leq n$. The coefficients of these items are respectively denoted by A, B and C so that

$$Sh_i(N, v^D) = Ac_i + \sum_{k=1}^{i-1} B\alpha_i^k c_k + \sum_{p=i+1}^{n} \sum_{k=1}^{i} C\alpha_p^k c_k.$$

Second, we want to study coefficients A, B and C of the above expression of the Shapley value. Regarding coefficient A, it obviously holds that

$$\begin{aligned} A &= \sum_{S\subseteq N\setminus\{i\}} \frac{s!(n-s-1)!}{n!} \\ &= 1. \end{aligned}$$

In order, then, to determine coefficient B, it is worth noting that $\alpha_i^k c_k$ where $1 \leq k \leq i-1$ appears in the three cases of Lemma 4.1. Furthermore, given a coalition $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$ where $i_1 < i_2 < \cdots < i_s$, and an agent $k \in N$ such that $k \leq i-1$, it holds that $k, k+1, \ldots, i-1, i \notin S$ if and only if one of the following three cases happens: (i) $i < i_1$ and $1 \leq k \leq i-1$; (ii) $i > i_s$ and $i_s + 1 \leq k \leq i-1$; and (iii) there exists

$p \in \{1, 2, \cdots, s-1\}$ such that $i_p < i < i_{(p+1)}$ and $i_p + 1 \leq k \leq i - 1$. Thus, coefficient B can be written as

$$
\begin{aligned}
B &= \sum_{\substack{k \in N: k \leq i-1, \\ k, k+1, \ldots, i-1, i \notin S}} \frac{s!(n-s-1)!}{n!} \\
&= \sum_{s=0}^{n-(i-k+1)} \binom{n-(i-k+1)}{s} \times \frac{s!(n-s-1)!}{n!} \\
&= \sum_{s=0}^{n-(i-k+1)} \frac{(n-(i-k+1))!}{(n-s-(i-k+1))!} \times \frac{(n-s-1)!}{n!} \\
&= \sum_{s=0}^{n-(t+1)} \frac{(n-(t+1))!}{(n-s-t-1)!} \times \frac{(n-s-1)!}{n!} \\
&= \frac{(n-t-1)!t!}{n!} \times \sum_{s=0}^{n-(t+1)} \binom{n-s-1}{t} \\
&= \frac{(n-t-1)!t!}{n!} \times \sum_{s=0}^{n-(t+1)} \left(\binom{n-s}{t+1} - \binom{n-s-1}{t+1} \right) \\
&= \frac{(n-t-1)!t!}{n!} \times \binom{n}{t+1} \\
&= \frac{1}{t+1} \\
&= \frac{1}{i-k+1},
\end{aligned}
$$

where $t = i - k$ and the sixth equality follows from Pascal's triangle with, by convention, $\binom{t}{t+1} = 0$.
In order finally to determine coefficient C, note that $\alpha_p^k c_k$ where $1 \leq k \leq i < p \leq n$ appears in cases (i) and (iii) of Lemma 4.1. Furthermore, given a coalition $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$ where $i_1 < i_2 < \cdots < i_s$, and two agents $k, p \in N$ such that $1 \leq k \leq i < p \leq n$, it holds that $k, k+1, \cdots, p-1 \notin S$ and $p \in S$ if and only if one of the following two cases happens: (i) $i < i_1 = p$; and (ii) there exists $l \in \{1, 2, \cdots, s-1\}$ such that $i_l < i < i_{(l+1)} = p$ and $i_l + 1 \leq k \leq i$. Thus, coefficient C is given by

$$
\begin{aligned}
C &= -\sum_{\substack{k,p\in N:1\leq k\leq i<p\leq n,\\ k,k+1,\cdots p-1\notin S, p\in S}} \frac{s!(n-s-1)!}{n!} \\
&= -\sum_{s=1}^{n-(p-k)} \binom{n-(p-k)-1}{s-1} \times \frac{s!(n-s-1)!}{n!} \\
&= -\sum_{s=1}^{n-t} \binom{n-t-1}{s-1} \times \frac{s!(n-s-1)!}{n!} \\
&= -\sum_{s=1}^{n-t} \frac{(n-t-1)!}{(n-t-s)!} \times \frac{s(n-s-1)!}{n!} \\
&= -\frac{(t-1)!(n-t-1)!}{n!} \times \sum_{s=1}^{n-t} s\binom{n-s-1}{t-1} \\
&= -\frac{(t-1)!(n-t-1)!}{n!} \times \binom{n}{t+1} \\
&= -\frac{1}{(t+1)t} \\
&= -\frac{1}{(p-k+1)(p-k)},
\end{aligned}
$$

where $t = p-k$ and the equality $\sum_{s=1}^{n-t} s\binom{n-s-1}{t-1} = \binom{n}{t+1}$ can be proved by induction on the number n of agents. □

The following three-agent example illustrates the result of Theorem 4.2.

Example 4.3. Consider the polluted river problem under waste flow control (N, c, γ) with $N = \{1, 2, 3\}$, cost vector $c = (1, 2, 1)$ and wastewater treatment rate vector $\gamma = (\frac{1}{2}, \frac{1}{2})$. The waste transfer rates are then given by $\alpha_2^1 = \frac{1}{2}$, $\alpha_3^1 = \frac{1}{4}$ and $\alpha_3^2 = \frac{1}{2}$. The cost allocation obtained by the DC method is given by $g^{DC}(N, c, \gamma) = (\frac{17}{24}, \frac{41}{24}, \frac{19}{12})$. Moreover, the Downstream-oriented game $(N, v^D) \in G$ defined by (3) is summarized in the following table.

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v^D(S)$	1	5/2	9/4	3	3	7/2	4

The Shapley value of this Downstream-oriented game is given by $Sh(N, v^D) = (\frac{17}{24}, \frac{41}{24}, \frac{19}{12})$ as predicted by Theorem 4.2.

5 The concavity of the Downstream-oriented game

In this section, we want to establish the concavity of the Downstream-oriented game. In the context of cost games, this property indicates that the cost allocation obtained by the DC method belongs to the core of (N, v^D) so that no agent or group of agents can contest this allocation.

Theorem 5.1. Let (N, c, γ) be a polluted river problem under waste flow control. Then $g^{DC}(N, c, \gamma) \in C(N, v^D)$.

Proof. Let $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$ be a coalition where $i_1 < i_2 < \cdots < i_s$. We want to show that for all $i, j \notin S$, we have $v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$ which is equivalent to the concavity property given by (1). To this end, we distinguish three cases.

Case 1 First, assume that $i < i_1$. We then distinguish three subcases.

Subcase 1.1 Assume that $j < i < i_1$. On the one hand, since $j, i_1 \in S \cup \{j\}$, it follows from point (iii) of Lemma 4.1 that

$$\begin{aligned} v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=j+1}^{i-1} \alpha_i^t c_t - \sum_{t=j+1}^{i} \alpha_{i_1}^t c_t \\ &= c_i - \alpha_{i_1}^i c_i + \sum_{t=j+1}^{i-1} (\alpha_i^t - \alpha_{i_1}^t) c_t. \end{aligned}$$

On the other hand, by point (i) of Lemma 4.1, one has

$$\begin{aligned} v^D(S \cup \{i\}) - v^D(S) &= c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_{i_1}^t c_t \\ &= c_i - \alpha_{i_1}^i c_i + \sum_{t=1}^{j} (\alpha_i^t - \alpha_{i_1}^t) c_t + \sum_{t=j+1}^{i-1} (\alpha_i^t - \alpha_{i_1}^t) c_t. \end{aligned}$$

Since $i < i_1$, we have $\alpha_i^t \geq \alpha_{i_1}^t$ for all $t \in \{1, \ldots, j\}$, and we conclude that $v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$.

Subcase 1.2 Assume that $i < j < i_1$. It follows from point (i) of Lemma 4.1 that

$$v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) = c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_j^t c_t,$$

and

$$v^P(S \cup \{i\}) - v^P(S) = c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_{i_1}^t c_t.$$

Since $j < i_1$, we have $\alpha_j^t \geq \alpha_{i_1}^t$ for all $t \in \{1, \ldots, i\}$. Thus, it holds that $-\sum_{t=1}^{i} \alpha_j^t c_t \leq \sum_{t=1}^{i} \alpha_{i_1}^t c_t$, and so $v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$.

Subcase 1.3 Assume that $i < i_1 < j$. By point (i) of Lemma 4.1 it holds that

$$\begin{aligned} v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=1}^{i-1} \alpha_i^t c_t - \sum_{t=1}^{i} \alpha_{i_1}^t c_t \\ &= v^D(S \cup \{i\}) - v^D(S). \end{aligned}$$

Case 2 Second, assume that $i > i_s$. We then distinguish three subcases.

Subcase 2.1 Assume that $i > i_s > j$. It follows from point (ii) of Lemma 4.1 that

$$\begin{aligned} v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t \\ &= v^D(S \cup \{i\}) - v^D(S). \end{aligned}$$

Subcase 2.2 Assume that $i > j > i_s$. By point (ii) of Lemma 4.1, one has

$$v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) = c_i + \sum_{t=j+1}^{i-1} \alpha_i^t c_t,$$

and

$$\begin{aligned} v^D(S \cup \{i\}) - v^D(S) &= c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t \\ &= c_i + \sum_{t=j+1}^{i-1} \alpha_i^t c_t + \sum_{t=i_s+1}^{j} \alpha_i^t c_t. \end{aligned}$$

Thus, we conclude that $v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$.

Subcase 2.3 Assume that $j > i > i_s$. On the one hand, since $j, i_s \in S \cup \{j\}$, it follows from point (iii) of Lemma 4.1 that

$$v^D(S \cup \{i, j\}) - v^D(S \cup \{j\}) = c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_s+1}^{i} \alpha_j^t c_t.$$

On the other hand, by point (ii) of Lemma 4.1, one has

$$v^D(S \cup \{i\}) - v^D(S) = c_i + \sum_{t=i_s+1}^{i-1} \alpha_i^t c_t.$$

Thus, we conclude that $v^D(S \cup \{i,j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$.

Case 3 Third, assume that $i_1 < i < i_s$. Since $i \notin S$, there exists $p \in \{1, 2, \cdots, s-1\}$ such that $i_p < i < i_{(p+1)}$. We distinguish three subcases.

Subcase 3.1 Assume that $j < i_p$ or $j > i_{(p+1)}$. It follows from point (iii) of Lemma 4.1 that

$$\begin{aligned} v^D(S \cup \{i,j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_{i_{(p+1)}}^t c_t \\ &= v^D(S \cup \{i\}) - v^D(S). \end{aligned}$$

Subcase 3.2 Assume that $i_p < j < i < i_{(p+1)}$. Since $j, i_{(p+1)} \in S \cup \{j\}$, it follows from point (iii) of Lemma 4.1 that

$$\begin{aligned} v^D(S \cup \{i,j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=j+1}^{i-1} \alpha_i^t c_t - \sum_{t=j+1}^{i} \alpha_{i_{(p+1)}}^t c_t \\ &= c_i - \alpha_{i_{(p+1)}}^i c_i + \sum_{t=j+1}^{i-1} (\alpha_i^t - \alpha_{i_{(p+1)}}^t) c_t, \end{aligned}$$

and

$$\begin{aligned} v^D(S \cup \{i\}) - v^D(S) &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_{i_{(p+1)}}^t c_t \\ &= c_i - \alpha_{i_{(p+1)}}^i c_i + \sum_{t=i_p+1}^{i-1} (\alpha_i^t - \alpha_{i_{(p+1)}}^t) c_t. \end{aligned}$$

Since $i_p < j$ and $\alpha_i^t \geq \alpha_{i_{(p+1)}}^t$ for all $t \in \{i_p+1, \ldots, i-1\}$, we conclude that $v^D(S \cup \{i,j\}) - v^D(S \cup \{j\}) \leq v^D(S \cup \{i\}) - v^D(S)$.

Subcase 3.3 Assume that $i_p < i < j < i_{(p+1)}$. On the one hand, since $i_p, j \in S \cup \{j\}$, it follows from point (iii) of Lemma 4.1 that

$$\begin{aligned} v^D(S \cup \{i,j\}) - v^D(S \cup \{j\}) &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_j^t c_t \\ &= c_i - \alpha_j^i c_i + \sum_{t=i_p+1}^{i-1} (\alpha_i^t - \alpha_j^t) c_t. \end{aligned}$$

On the other hand, by point (iii) of Lemma 4.1, one has

$$\begin{aligned} v^D(S\cup\{i\}) - v^D(S) &= c_i + \sum_{t=i_p+1}^{i-1} \alpha_i^t c_t - \sum_{t=i_p+1}^{i} \alpha_{i_{(p+1)}}^t c_t \\ &= c_i - \alpha_{i_{(p+1)}}^i c_i + \sum_{t=i_p+1}^{i-1} (\alpha_i^t - \alpha_{i_{(p+1)}}^t) c_t. \end{aligned}$$

Since $j < i_{(p+1)}$, we have $-\alpha_j^i < -\alpha_{i_{(p+1)}}^i$ and $\alpha_i^t - \alpha_j^t \leq \alpha_i^t - \alpha_{i_{(p+1)}}^t$ for all $t \in \{i_p+1, \dots, i-1\}$. Thus, we conclude that $v^D(S\cup\{i,j\}) - v^D(S\cup\{j\}) \leq v^D(S\cup\{i\}) - v^D(S)$. □

The following three-agent example illustrates the result of Theorem 5.1.

Example 5.2. Consider the polluted river problem under waste flow control (N, c, γ) with $N = \{1,2,3\}$, cost vector $c = (1,2,1)$ and wastewater treatment rate vector $\gamma = (\frac{2}{3}, \frac{1}{2})$. The waste transfer rates are then given by $\alpha_2^1 = \frac{1}{3}$, $\alpha_3^1 = \frac{1}{6}$ and $\alpha_3^2 = \frac{1}{2}$. The cost allocation obtained by the DC method is given by $g^{DC}(N,c,\gamma) = (\frac{29}{36}, \frac{59}{36}, \frac{14}{9})$. Moreover, the Downstram-oriented game $(N, v^D) \in G$ defined by (3) is summarized in the following table.

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v^D(S)$	1	7/3	13/6	3	3	10/3	4

The core $C(N, v^D)$ contains all payoff vectors $x \in \mathbb{R}_+^3$ such that $\sum_{i\in N} x_i = 4$, $\frac{2}{3} \leq x_1 \leq 1$, $1 \leq x_2 \leq \frac{7}{3}$, and $1 \leq x_3 \leq \frac{13}{6}$. Thus, we have $g^{DC}(N,c,\gamma) = (\frac{29}{36}, \frac{59}{36}, \frac{14}{9}) \in C(N, v^D)$ as predicted by Theorem 5.1.

6 Concluding remarks

In this paper we considered polluted river problems under waste flow control and introduced a new cost sharing method, called the DC method, for which the two standard LRS and DES methods are special cases, as well as the Airport Landing Fee solution (Littlechild and Owen, 1973 [19]). We showed that the DC method has a good interpretation in terms of International Water Law since it is implemented by a cost allocation scenario based on the ATS and the UTI theories (Theorem 3.1). We also showed that the DC method has appealing properties insofar as it coincides with the Shapley value of the Downstream-oriented game (Theorem 4.2), and that no agent or group of agents has any interest to contest this method since it is core stable (Theorem 5.1).

Although, similarly to Dong *et al.* (2012 [12]), we focused on the upstream responsibility principle in order to construct our cost sharing method, we mention that our work can be easily extended by considering the downstream responsibility principle as suggested by Ni and Wang (2007 [23]). In this way, we can propose an alternative cost sharing method, called the Upstream Compensation method, which is complementary to the DC method in the sense that any agent has to pay a part of the downstream transferred cleaning costs. Formally, the *Upstream Compensation (UC) method* is given by

$$g_i^{UC}(N,c,\gamma) = c_i + \sum_{k=i+1}^{n} \frac{\alpha_k^i c_i}{k-i+1} - \sum_{p=1}^{i-1}\sum_{k=i}^{n} \frac{\alpha_k^p c_p}{(k-p+1)(k-p)} \text{ for all } i \in N \qquad (5)$$

where $\alpha_1^0 = \alpha_{n+1}^n = 0$. The UC method is composed of three parts. First, agent $i \in N$ covers his own cost c_i. Second, agent $i \in N$ compensates every agent $k \in \{i+1,\ldots,n\}$ according to the downstream responsibility principle implied by the UTI theory. This compensation corresponds to the transferred cleaning cost $\alpha_k^i c_i$. Noting that the number of river segments from i to k, $k \in \{i+1,\ldots,n\}$, is $k-i+1$, it is common for i to compensate $\frac{\alpha_k^i c_i}{k-i+1}$. Third, following the downstream responsibility principle, every agent $p \in \{1,\ldots,i-1\}$ compensates agent $i \in N$. This compensation corresponds to the cost $\alpha_k^p c_p$ which is divided by the number $k-p+1$ of agents involved in the waste flow from p to k, and is then shared equally among the $k-p$ agents that are located downstream river segment p, including agent i itself. Consequently, the LRS and the UES methods become special cases of the UC method.

Furtermore, the cost allocation scenario that implements the UC method consists of the following steps.

1. Choose any polluted river problem under waste flow control (N,c,γ) and any arrival order π of agents on N to gradually form the grand coalition.

2. Each entering agent $i \in N$ has to pay for its own cost c_i.

3. Each entering agent $i \in N$ asks each agent $p \in N$ such that $p < i$ and $\pi(p) < \pi(i)$ to compensate it for the amount of $\sum_{k=i}^{n} \frac{\alpha_k^p c_p}{(k-p+1)(k-p)}$.

4. For each entering agent $i \in N$, each agent $q \in N$ such that $q > i$ and $\pi(q) < \pi(i)$ asks agent i to compensate it for the amount of $\sum_{k=q}^{n} \frac{\alpha_k^i c_i}{(k-i+1)(k-i)}$.

5. Steps 1-4 determine a cost allocation $x \in \mathbb{R}_+^N$ such that

$$x_i = c_i + \sum_{q=i+1}^{n}\sum_{k=q}^{n} \frac{\alpha_k^i c_i}{(k-i+1)(k-i)} - \sum_{p=1}^{i-1}\sum_{k=i}^{n} \frac{\alpha_k^p c_p}{(k-p+1)(k-p)} \text{ for all } i \in N \quad (6)$$

The relationship between the cost allocation scenario described above and the UC method is underlined by the following result.

Theorem 6.1. For every polluted river problem under waste flow control (N,c,γ), the UC method coincides the cost allocation given by (6).

The proof of this result is similar to the one of Theorem 3.1.

Moreover, the *Upstream-oriented game* $(N, v^U) \in G$ associated to the polluted river problem under waste flow control (N, c, γ) is defined for every coalition $S = \{i_1, i_2, \ldots, i_s\} \subseteq N$, $i_1 < i_2 < \cdots < i_s$, by

$$v^U(S) = \sum_{i \in S} c_i + \sum_{k=1}^{s} \sum_{t=i_k+1}^{i_{(k+1)}-1} \alpha_t^{i_k} c_{i_k}.$$

where, by convention, $i_{(s+1)} = n+1$. Thus, players in S are first responsible for their own total cost $\sum_{i \in S} c_i$. Second, for every player $t \in \{i_k+1, \ldots, i_{(k+1)}-1\}$ where $k \in \{1, \ldots, s\}$, the transferred cleaning cost $\alpha_t^{i_k} c_{i_k}$ is also covered by coalition S since these players are located dowstream of segment i_k and upstream of segment $i_{(k+1)}$. Note that if i_k and $i_{(k+1)}$ are located next to each other, player i_k cleans up the waste in its own territory, and therefore the transferred cleaning cost $\alpha_{i_{(k+1)}}^{i_k} c_{i_k}$ does not appear in the expression of $v^U(S)$.

Analogously to Theorems 4.2 and 5.1, the UC method can then be obtained by applying the Shapley value to the Upstream-oriented game, and also belongs to the core of this game.

Theorem 6.2. Let (N, c, γ) be a polluted river problem under waste flow control. Then $g^{UC}(N, c, \gamma) = Sh(N, v^U)$ and $g^{UC}(N, c, \gamma) \in C(N, v^U)$.

To conclude, in real life situations, the rivers often have a tree structure such as, for example, the Baiyangdian Lake Catchment in Northern China depicted in Figure 3 of Dong *et al.* (2012 [12]). Although such a river structure is not formalized in our model, we mention that the DC and the UC methods can also be applied to this river type by decomposing them into several tributaries with a line structure. Thus, our compensation methods are also appropriate solutions for sharing the pollutant-cleaning costs of any river for which the waste flows can be partially controlled.

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