URBANIZATION AND DEVELOPMENT: A SPATIAL FRAMEWORK OF RURAL-TO-URBAN MIGRATION

Documents de travail GREDEG
GREDEG Working Papers Series

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GREDEG WP No. 2019-12
https://ideas.repec.org/s/gre/wpaper.html

Urbanization and development: A spatial framework of rural-to-urban migration[†]

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GREDEG Working Paper No. 2019–12

Abstract

This paper relies upon some of the assumptions of the classical Alonso-Muth-Mills model in order to construct a spatial framework of rural-to-urban migration; specifically, it develops a spatial framework of migration where rural workers are uniformly distributed throughout a rural area, which it develops around a monocentric nuclear urban area. The spatial interactions between the rural and the urban areas are modeled via the two spatial variables of the rural rents and productivity spillover, whose effects of propagation from the urban to the rural area depend on the distance of the rural area from the urban area. From the model, it emerges how the rural rents affect the final levels of congestion in both the two areas, so that the urbanization level of the market solution can be inferior or superior with respect to the urbanization level set by the city planner. On the other hand, the inclusion of spatial variables does not seem to produce scenarios for urban growth which significantly differ from the ones detected in previous studies. Ultimately, these findings suggest the need for a city planner to design policies affecting the level of rural rents in order to modify the desired level of rural-to-urban migration, and hence the desired trade-off between urban growth and congestion.

JEL-codes: R12; R23; O43.

Keywords: Alonso-Muth-Mills model, Spatial analysis, Migration.

[†]This work has been supported by the French government, through the UCAJEDI Investments in the Future project managed by the National Research Agency (ANR) with the reference number ANR-15-IDEX-01.

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1 Introduction

The effect of urbanization on urban growth in developing countries represents a topic which has been widely analyzed in the literature. It is now a widely accepted fact, among economists, that in less developed economies, urbanization constitutes one of the main factors boosting the process of urban development, beside other factors such as capital accumulation and quality of the institutions (Turok and McGranahan, 2013[20]; Brülhart and Sbergami, 2009[6]; Henderson and Wang, 2009[14]; Henderson, 2003[11]).

As stated by Anderson (2005)[10], in the early stages of growth, economic development is characterized by urbanization, which can be defined as a spatial transformation of the economy, where the population moves through migration from an agricultural, rural-based existence, to one where production occurs in cities of endogenous numbers and size. In this process, public and local authorities usually control land markets and regulate migration patterns, and all this clearly affects the city formation progression, as well as the city size. As well, the role of national government policies regarding trade, labor market and investments in facilities and infrastructures seem to exert a relevant role in determining the outline of the progressing of the urban setting.

Some authors (Jedwab et al., 2014[15]; Rahman et al., 2006 [1]) further suggested the relevance of the speed of the process of urbanization over time. As a matter of fact, urbanization stimulates growth, but the more gradual and controlled will be the process of urbanization, the higher will be the likelihood of having higher standards of living among the population during the transition phase; in fact, it has been the case that countries which urbanized too fast and without enough control, eventually experienced higher negative externalities (Jedwab et al., 2014[15]).

Since the seminal contribution of Lewis (1954)[16], subsequent general-equilibrium models developed in the literature have tried to frame the long-run relationship between migration, human capital accumulation and urban growth within a rural-urban framework (see, e.g., Fan and Stark, 2008[8]; Bertinelli and Black, 2004[3]; Handerson and Wang, 2004[13])[1]; nonetheless, most of these studies did not consider the spatial properties of the urban or rural areas, treating the latter as merely abstract locations. On the other hand, one of the emerging challenges posited in the urban economic literature has been the formulation of a rigorous economic explanation for a variety of observed regularities in the spatial structures of real-world cities (Brueckner, 1987[5]). Intuitive examples are represented by the average decrease in house size and average increases in the price for land when approaching the city center. These two simple examples entail the presence of substitution effects, which, to the best of our knowledge, have virtually never been considered in theoretical models of rural-to-urban migration. Per contrary, in the urban economic literature, since the seminal contributions of Alonso, 1964[2], Muth, 1969[18] and Mills, 1967[17], modeling the size and the structure of the urban environment has become a powerful tool to take into account substitution effects, thus providing a more truthful depiction of reality. Specifically, in the simplest framework of the Alonso-Muth-Mills model, the city is spatially represented by a monocentric Central Business District (CBD) around which the rest of the urban area is developed. Urban dwellers travel to the CBD to work, and in order to reduce commuting time, they would choose, *ceteris paribus*, to live closer to the city center. This inevitably increases the price of the houses, which becomes compensated by a minor housing surface (and this help explaining, for instance, why skyscrapers are generally located closer to city centers). In the end, the overall city size and structure will be simultaneously affected by the population size, commuting costs and the cost of the land.

In light of these considerations, we aim to contribute to the literature by developing a spatial framework of rural-to-urban migration drawing upon some key assumptions of the Alonso-Muth-Mills model; this has the main advantage of allowing to go beyond the classic assumption which considers the urban and rural areas as pure abstract locations.

[1] For a detailed literature review on rural-to-urban migration models, see Henderson, 2004[12].

Specifically, we model the size and the structure of the rural-urban framework by introducing a monocentric urban area surrounded by a rural area. We rely upon the (non-spatial) models developed by Handerson and Wang (2004)[13]; Bertinelli and Black (2004)[3] and Black and Henderson (1999)[4] to study the relationship between substitution effects, migration, human capital accumulation and urban growth. Specifically, we draw upon Handerson and Wang (2004)[13] by introducing the substitution effect of land rents in the rural area. The latter, in the Alonso-Muth-Mills model, have been proven to exert a significant impact on urban dwellers' decision to relocate to the CBD. Adopting the same logic, rural rents are likewise expected to affect the decision of rural dwellers to migrate to the urban area. Then, following Black and Henderson (1999)[4], we allow for the presence of a productivity spillover originating in the urban area, and whose effects spatially propagate throughout the rural area. The main reason why we account for a productivity spillover is to consider the so called radiation effect (QI Jin-li, 2003[19]), which basically consists of urban improvements in science, education, health care, etc., that tend to produce positive externalities in the surrounding rural areas, thus spreading knowledge and technological diffusion. Finally, we follow Bertinelli and Black (2004)[3] to derive the optimal share of the urbanization level, as well as to study the evolution of the urban growth process over time.

The rest of the paper is organized as follows: Section 2 outlines the model, where we discuss the spatial properties of the urban and the rural areas. Subsequently, we analyze the optimal levels of urbanization conditioned upon the substitution effects exerted by the rural rents and the productivity spillover. Then, we study the long-run process of urban growth conditioned on human capital accumulation and level of urbanization. Finally, section 3 concludes.

2 Model

2.1 Spatial properties of the urban and rural areas

As specified above, we draw upon the Alonso-Muth-Mills model in order to construct a simple spatial framework of rural-to-urban migration. To this aim, consider a settlement composed by a monocentric urban area (the Central Business District, or CBD) surrounded by a rural area. The total population of the settlement is expressed by $\bar{\Psi}$ (> 0), with δ_t $(0 \leq \delta_t \leq 1)$ denoting the fraction of the population living in the urban area at time t, and $(1 - \delta_t)$ the fraction of the population living in the rural area. The population is composed by individual workers living in the time period t, who decide their amount of investment in human capital at the beginning of the period[2]. In the rural area, we additionally assume absence of human capital, due to the presence of low skilled tasks requiring an inferior amount of knowledge. Always in the rural area, the gross income of workers depends on κ^r (> 0), expressing the exogenous productivity factor of the rural area[3] and π_t (> 0), the overall initial level of technology for the whole economy at time t. In addition, the gross income of rural workers is influenced by a productivity spillover effect deriving from the CBD, which propagates throughout the rural area, and whose effect decreases with the distance d from the urban area[4]. Besides the distance, the final effect of the productivity spillover on the rural income will also depend on the absolute level of ω (≥ 0), measuring the decay of the spillover itself; that is, when ω increases, the spillover benefit

[2] Following the literature, for the sake of simplicity, no physical capital is included in the model.

[3] The exogenous productivity factor includes, for instance, the level of the rural infrastructures, the efficiency and quality of the public utilities, etc.

[4] Namely, the parts of the rural area closer to the CBD will benefit more from the spillover effect compared to the more distant ones. It is possible to think about the productivity spillover effect as an indirect economic impact exerted by the industrial urban activities towards the rural area; such an impact could be expressed, for instance, by the relocation of urban industrial facilities, job creations, improvements in the rural infrastructures, and so on.

in the rural area will go down, thus leading to a worsening in the income condition of rural workers; on the contrary, when the decay of ω is flatter, rural workers will benefit a higher beneficial effect from the productivity spillover, thus enjoying higher rural salaries[5]. Finally, rural workers face a rent for their house given by $r(d)$ (≥ 0), representing the land rent for each unit of occupied living surface they have to pay[6].

The equation for the rural income hence writes as follows:

$$y_t^r = \kappa^r \pi_t - \omega(d) - r(d) \tag{1}$$

Rural workers living in the rural area will then maximize their income with respect to the distance:

$$\max_{d(d)} y_t^r = \max_{d(d)} [\kappa^r \pi_t - \omega(d) - r(d)L]$$

which leads to:

$$r'(d) = -\omega'(d)$$

The above equation entails that the rent is decreasing with respect to $\omega(d)$, in order to compensate a loss in the rural income from a lower productivity spillover as we approach the farthest regions of the rural area. Alternatively, more sensible the decay of the spillover effect is with respect to the distance from the CBD, the higher will be the rents in getting closer to the CBD itself.

Subsequently, to investigate the geometric properties of the settlement, assume for the sake of simplicity that the productivity spillover is linear with respect to the distance (i.e., $\omega(d) = \omega d$), so that also the gradient of the rent will be linear. In addition, further assume an homogeneous linear distribution of the rural population around the monocentric urban area, along two segments of equal length. As depicted in Figure 1, the urban area can therefore be represented by a nuclear CBD, while the two segments of equal length, [$\bar{d}$, CBD] and [CBD, $\bar{d}$] represent the linear distribution of the rural area around the CBD. The two $\bar{d}$ denote, respectively, the left and the right extremes of the rural area, i.e., the two farthest rural regions from the CBD.

Figure 1: Linear distribution of the rural area.

The distribution of rural workers across the rural area will hence be given by the following equation:

$$\bar{d} = ((1 - \delta_t)\bar{\Psi})/2$$

Given then the linearity of the productivity spillover effect and, accordingly, of the gradient of the rents, it is possible to derive the following expression:

[5]Here, the terms *income*, *salary* and *earning* are considered interchangeable.

[6]Even if rural workers own the house, because of living inside of it, they always face an opportunity cost given by a potential rent they loose from not renting the house to other workers.

$$r(d) = r(0) - \omega d$$

where $r(0)$ represents the rent in the closest rural region to the CBD, and $\bar{r}$ the rent at $\bar{d}$, so that we have:

$$r(\bar{d}) = \bar{r} = r(0) - \omega(\bar{d})$$

from which:

$$r(0) = \bar{r} + \omega((1 - \delta_t)\bar{\Psi}/2)$$

and therefore:

$$r(d) = \bar{r} + \omega((1 - \delta_t)\bar{\Psi}/2) - \omega d$$

After some algebric manipulations, the equation for the rural income can be rewritten as follows:

$$y_t^r = \kappa^r \pi_t - \bar{r} - \omega(1 - \delta_t)\bar{\Psi}/2 \tag{2}$$

From (2), some preliminary facts emerge. Firstly, the rural income is increasing in the gross earning but decreasing in $\bar{r}$; the increase in the opportunity cost for the land brings in fact to an increment of $r(d)$, whatever is the distance from CBD. As for ω, the latter, as it was easy to expect, decreases the rural income in increasing the distance from the CBD (moreover, as already stressed, an increase in the sensibility of $\omega(d)$ to the distance increases the gradient of the rent). Finally, the rural income results to be increasing in the number of workers living in the urban area, and decreasing in the number of workers living in the rural area; indeed, as more workers remain in the rural area, the higher the negative effect exerted by the rents in the rural area results to be, due to a higher competition for the land among rural workers.

When considering the urban area, assume conversely that here, workers have access to more sophisticated production technologies; hence, they can gain investments in human capital through investments in education (which however require a certain fixed amount of effort). Rural workers are free to relocate from the rural to the urban area (we assume no restrictions on rural-to-urban migration) and invest as well in human capital. At the same time, an excessive displacement of rural workers in the urban area imposes the problem of congestion externalities due to an excessive increase in the number of the urban population[7]. This inevitably generates negative spillovers (for instance, a worsening in the urban level of health conditions, public services and so on) which decrease the general well-being of urban citizens.

In light of these considerations, the equation for the urban income writes as follows:

$$y_t^u = \kappa^u \pi_t [s_t]^\gamma - [\phi s_t + q\delta_t \bar{\Psi}] \tag{3}$$

Where the first term on the right hand side of (3) represents the gross earning received by a urban worker (being already a urban citizen or a rural migrant who decided to relocate to the urban area), with κ^u expressing the exogenous

[7]The issue of congestion mainly affects urban areas (as the latter generally face strict space constraints), whereas rural areas are usually not (or remarkably less) affected by (severe) issues of congestion provoked by overpopulation. This is why congestion externalities were only modeled in the urban area and not in the rural area.

productivity factor of the urban area. s_t is the amount of human capital owned by the urban worker at time t, with the parameter γ ($0 < \gamma < 1$) denoting decreasing returns to education. ϕ (> 0) is the effort paid by the worker to obtain the amount of human capital he/she owns. Finally, the last term takes into account the congestion effect of over urbanization, with q being a positive constant[8]. In the urban area, workers maximize their income given their optimal level of human capital investment; deriving the FOC from (3) with respect to s_t, yields the optimal level of human capital for a urban worker:

$$\tilde{s_t} = (\gamma\kappa^u\phi^{-1}\pi_t)^{1/(1-\gamma)} \tag{4}$$

Following Harris and Todaro, 1970[9], we assume that rural-to-urban migration is driven by the gap in the net income between urban and rural workers. Supposing an initial gap between a superior urban income and a lower rural income, migration of rural workers towards the urban area will continue up to the point when an equilibrium is achieved. In the end, at this equilibrium level, the rural income will be equal to the urban income. Hence, we will have the following expression:

$$\kappa^r\pi_t - \bar{r} - \frac{\omega}{2}\bar{\Psi} + \frac{\omega}{2}\delta_t\bar{\Psi} = \kappa^u\pi_t\tilde{s_t}^\gamma - \phi\tilde{s_t} - q\delta_t\bar{\Psi}$$

Once this equilibrium condition is achieved, there will be no migration anymore.

2.2 Optimal levels of urban congestion

From the equation capturing the equilibrium condition among the urban and the rural incomes, it is possible to derive the fraction of the population $\hat{\delta_t}$ which eventually locates to the urban area at time t according to the market solution:

$$\hat{\delta_t} = \frac{2(\kappa^u\pi_t\tilde{s_t}^\gamma - \phi\tilde{s_t} - \kappa^r\pi_t + \bar{r}) + \bar{\Psi}\omega}{\bar{\Psi}(2q+\omega)} \tag{5}$$

An investigation on the conditions of existence for $\hat{\delta_t}$ is necessary. Analyzing the urban and rural incomes, we notice how their absolute value is influenced by the share of people living in their respective areas. Indeed, it was previously demonstrated how the urban income is negatively affected by the congestion costs that increase in the number of urban workers[9], while the rural income is positively influenced by the decrease in the number of workers living in the rural area, since this lowers the competition for the land (affecting in turn the rural rents). Therefore, considering expression (3) for the urban income, we see that the latter reaches its maximum when $\hat{\delta_t} = 0$, i.e., when there is no urban population, whereas the rural income, from expression (2), reaches its highest when $1 - \hat{\delta_t} = 0$ (or $\hat{\delta_t} = 1$), i.e., when there is no rural population. The necessary condition for the existence of the equilibrium at $\hat{\delta_t}$ is thus provided by the following expression:

$$\kappa^r\pi_t - \bar{r} - \frac{\omega}{2}\bar{\Psi} < \kappa^u\pi_t\tilde{s_t}^\gamma - \phi\tilde{s_t} < \kappa^r\pi_t - \bar{r} + q\bar{\Psi}$$

The graph in Figure 2 better helps in understanding this condition. For very low levels of urbanization (i.e., for levels of $\delta_t < \hat{\delta_t}$, whose extreme point at $\delta_t = 0$ represents the lowest value for the rural income, $\kappa^r\pi_t - \bar{r} - \frac{\omega}{2}N$, and the highest value for the urban income, $\kappa^u\pi_t\tilde{s_t}^\gamma - \phi\tilde{s_t}$), urban workers enjoy remarkably high incomes, due to a

[8]We further assume a response one for one of the amount of urban population with respect to changes in congestion costs.
[9]We assume that rural workers relocating to the urban area do not consider congestion costs.

minimal presence of congestion costs; on the other hand, the incomes in the rural areas are very low, given a high number of rural workers competing for the land. At the opposite, for very high levels of urbanization (i.e., for levels of $\delta_t > \hat{\delta}_t$, whose extreme point at $\delta_t = 1$ denotes the highest value for the rural income, $\kappa^r \pi_t - \bar{r}$, and the lowest value for the urban income, $\kappa^u \pi_t \tilde{s_t}^\gamma - \phi \tilde{s_t} - q\bar{\Psi}$), congestion costs are remarkably high (and this lowers the urban income), while the rural income results to be high (because of reduced land competition). Two different scenarios can hence take place. In the first scenario, when the urban income exceeds the rural income (i.e., when $0 < \delta_t < \hat{\delta}_t$), rural workers will migrate to the urban area, in this way increasing the urbanization level and lowering the urban income. Per contrary, in the scenario where the rural income exceeds the urban income (i.e., when $\hat{\delta}_t < \delta_t < 1$) we will assist to a migration of urban workers towards the rural area, and this will decrease the urbanization level. In both the two scenarios, the differential between the two incomes will adjust the urbanization level accordingly, until the equilibrium point at $\hat{\delta}_t$ is achieved. Here, no migration will occur anymore.

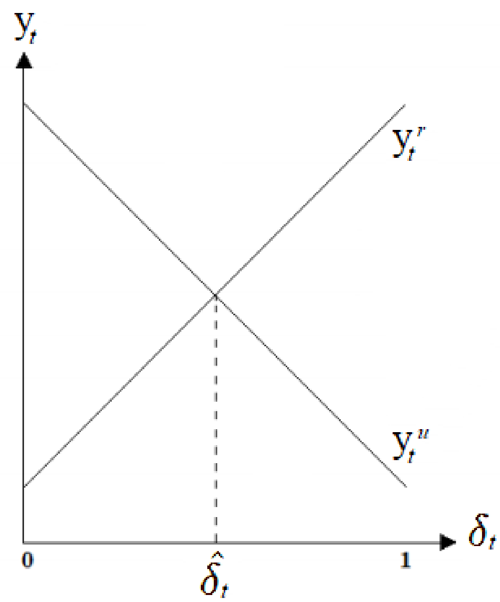

Figure 2: Equilibrium conditions.

Following Wheaton (1974)[21], we subsequently perform comparative static analysis on the equilibrium level of urban density in order to better evaluate the effect of some parameters of interest. Differentiating from (5) yields:

$$\frac{\partial \hat{\delta}_t}{\partial \bar{r}} > 0, \frac{\partial \hat{\delta}_t}{\partial \kappa^u} > 0, \frac{\partial \hat{\delta}_t}{\partial \kappa^r} < 0, \frac{\partial \hat{\delta}_t}{\partial \omega} > 0$$

These results are straightforward. Higher productivity factors in the urban and rural area will lead, respectively, to higher and lower rates of migration from the rural area to the CBD. In addition, increases in the level of land rents and in the decay of the productivity spillover will worsen rural workers' income, pushing them to migrate to the urban area.

Contrarily to rural migrants, the city planner in charge of designing the urban policies will consider the increase in congestion costs provoked by an excessive urban population. Subsequently, the city planner will accordingly set, at time t, the best feasible number of urban workers maximizing the overall per capita net output of the economy, $\bar{y}_t$:

$$\max_{\delta_t} \bar{y}_t = (1-\delta_t)[\kappa^r \pi_t - \bar{r} - \omega(1-\delta_t)\bar{\Psi}/2] + \delta_t[\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s}_t - q(\delta_t \bar{\Psi})]$$

Deriving the FOC from the above new expression of equilibrium, we obtain the proportion of workers $\bar{\bar{\delta}}_t^u$ that the urban planner will hence find as an optimal solution:

$$\bar{\bar{\delta}}_t^u = \frac{\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t} - \kappa^r \pi_t + \bar{r} + \bar{\Psi}\omega}{\bar{\Psi}(2q+\omega)} \tag{6}$$

When comparing (5) with (6), the market solution for the optimal share of urban population will exceed in magnitude the one of the city planner only under certain conditions. Specifically, $\bar{\bar{\delta}}_t^u < \hat{\delta}_t$ verifies iff $\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t} - \kappa^r \pi_t + \bar{r} > 0$; conversely, $\bar{\bar{\delta}}_t^u > \hat{\delta}_t$ occurs iff $\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t} - \kappa^r \pi_t + \bar{r} < 0$[10]. We will now discuss more accurately these two different cases.

First case ($\bar{\bar{\delta}}_t^u < \hat{\delta}_t$)

This first case arises when the gross urban income net of the cost to acquire human capital exceeds the gross rural income net of the negative effect exerted by the rents in the rural area, which entails: $\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s}_t > \kappa^r \pi_t - \bar{r}$; re-expressing this condition in terms of ratio between the shares of the net rural and urban income, we obtain: $\frac{\kappa^r \pi_t - \bar{r}}{\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t}} < 1$. This condition will lead to $\bar{\bar{\delta}}_t^u < \hat{\delta}_t$, since the final economic well-being, at the eyes of workers, will result to be higher in the urban area than in the rural area, provoking necessarily a higher urban agglomeration, since more rural workers would have eventually relocated to the urban area.

Second case ($\bar{\bar{\delta}}_t^u > \hat{\delta}_t$)

The second condition conversely arises whenever $\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t} < \kappa^r \pi_t - \bar{r}$, or, in terms of ratio between shares, when $\frac{\kappa^r \pi_t - \bar{r}}{\kappa^u \pi_t \tilde{s_t}^{\gamma} - \phi \tilde{s_t}} > 1$. This second case is likely to occur when the economic conditions of the rural area are particularly prosper and therefore rural workers enjoy considerably high rural salaries. Because of this, they will end up neglecting the negative effect exerted by the rents (since the latter will likely result to be minimal), so that the final result will be a higher agglomeration level in the rural area. The city planner, who considers instead congestion costs and the level of the rents, will be able to set a solution by which $\bar{\bar{\delta}}_t^u > \hat{\delta}_t$ to counterbalance the myopic attitude of rural workers. Intuitively, the necessary condition for the emergence of the second case could also derive from a situation of lower rural incomes, as far as, however, the net urban income results to be remarkably low (because of, for instance, a situation of urban economic downturn), always coupled by a negligence by rural workers in considering rural rents.

Therefore, to summarize, even in the presence of congestion costs associated to overpopulation in the urban area, not necessarily the urbanization level of the market solution will exceed the urbanization level set by the city planner; conversely, the opposite may also occur. This will particularly be the case in the presence of remarkably high rural salaries[11] compared to urban salaries, or when the urban economic conditions are not thriving (e.g., because of an

[10] In order for $\bar{\bar{\delta}}_t^u$ to be positive, the condition for which $\bar{\Psi}\omega > 2(\kappa^u \pi_t \tilde{s}_t^{\gamma} - \phi \tilde{s_t} - \kappa^r \pi_t + \bar{r})$ must always hold.

[11] This condition likely verifies when the exogenous productivity factor in the rural area increases, for instance because of an improvement in the quality of the infrastructures; the latter has in fact proven to exert a crucial role in increasing the productivity level in rural areas (see, e.g., Duranton and Turner, 2012 [7]).

economic recession), or given a high cost of human capital accumulation (for instance, because of the presence of bureaucratic barriers) and/or low returns from education (due to, for instance, a poor education system). In all these cases, the role exerted by the rural rents results to be crucial, given the impact exerted on the gross rural income, which eventually will affect the final choice of location of rural workers. Indeed, when the level of the rents is considerably low and the gross rural income is higher than the urban income, rural workers will not consider the negative effect of the rents, and this will consequently lead to more agglomeration in the rural area. On the other hand, the city planner, who as stressed, takes into account both the effect of urban congestion costs and rents, does not see the issue of a potential mass relocation of rural workers from the rural to the urban area, and therefore can increase the desirable share of workers in the CBD. Nonetheless, the gap between a higher rural income against a lower rural income could also be due to other reasons, for instance, in case of urban economic downturns, because of an increased difficulty in human capital acquisition, remarkably low returns to scale in education, etc. In all these cases, increasing the number of urban workers could potentially help to augment the chances of fostering the urban growth.

Conversely, when the net urban salaries greatly exceed the gross rural salaries net of the values of the rents, the degree of agglomeration set by the urban planner will result to be higher than the one of the market solution; for this scenario, there are two feasible explanations; first, it can be the case that the urban area has moderate congestion costs and registers a consistent economic growth, boosted by a good system of urban infrastructures, high returns from education and low costs of acquisition of human capital, thus attracting workers from the rural area. On the other hand, it may also be the case that in reality the urban net salaries are much lower, due to the presence of high congestion costs; the problem arises because the latter are not perceived by rural workers, thus providing them the false impression that in the urban area, workers truly enjoy higher wages. In all these cases, the city planner will hence narrow the share of desirable urban population, in order to decrease the congestion costs within the urban area.

2.3 Scenarios of urban growth

After having analyzed the drivers influencing the dynamics of rural-to-urban migration, we then study how the latter affects the process of long-run urban growth, with particular consideration of the effect exerted by the spatial variables of the rural rents and productivity spillover. Following the literature, we postulate that urban growth is measured by increases in the level of technology, π_t, in the time period $t+1$. Specifically, the equation of motion linking the overall initial level of technology π_t to the level of technology in the next time period, π_{t+1}, the degree of urbanization, the average level of human capital[12], and the parameter ρ providing for δ_t and s_t decreasing returns (i.e., $0 < \rho < 1$), writes as follows:

$$\pi_{t+1} = \max f(\pi_t, [\delta_t s_t]^\rho) \tag{7}$$

This expression entails that technological progress is induced by human capital accumulation and urbanization. What is crucial for the economy at time t are hence the investments in human capital at time $t-1$. The degree of urbanization[13], the other variable entering (7), affects the growth process since the higher the number of urban workers,

[12] The average level of human capital in the whole economy solely depends on the average level of urban human capital, since in the rural area we have postulated no human capital. Indeed, denoting as $\bar{s}_t$ the average level of human capital in the whole economy, we have that: $\bar{s}_t = (1-\delta_t)0 + \delta_t s_t$. Thus, $\bar{s}_t = \delta_t s_t$. Moreover, for the sake of simplicity, no depreciation is assumed.

[13] Specifically, the degree of urbanization affects the growth process in two ways, directly and indirectly. Directly as it appears in equation (7), and indirectly because, affecting the level of technology, it automatically influences the optimal level of human capital chosen by urban workers (as the latter depends on π_t), which will eventually end up affecting the final equation of motion. It is possible to see

the higher the investments in human capital boosting urban growth. In addition, the resulting higher productivity thus generated will attract more rural workers to the CBD, who will invest in turn in human capital, creating in this way a positive circle. Moreover, these investments will rise according to the increase in the technological level. In the end, the urban economy will experience different growth paths depending on the values of the parameters in (7), *in primis* depending on the level of π_t. First, if the initial level of technology is too low, the urban area will remain stuck in a development trap without economic growth, and will be likely to remain with its current technological level forever, without experiencing urbanization. Nonetheless, if the initial level of technology is sufficiently high, the urban growth process will begin; this process will lead to a final steady state of partial or full urbanization, through the respective functions governing the technological path of the economy, π^p_{t+1} and π^f_{t+1}.

To sum up, according to the different dynamic paths that the level of technology $\{\pi_t\}_{t=0}^{\infty}$ will take, we eventually obtain these three following scenarios: no-urbanization, partial urbanization, and full urbanization.

$$\pi_{t+1} = \begin{cases} \pi^0 & if \quad \delta_t = 0 \\ \pi^p_{t+1} & if \quad 0 < \delta_t < 1 \ , \ \pi^p_{t+1} > 0 \ and \ (\delta_t s_t)^\rho > \pi_t \\ \pi^f_{t+1} & if \quad \delta_t = 1 \ , \ \pi^f_{t+1} > 0 \ and \ (\delta_t s_t)^\rho > \pi_t \end{cases}$$

where the equations of motion π^p_{t+1} and π^f_{t+1} were obtained by plugging expressions (4) and (5) into $(\delta_t s_t)^\rho$. In the end, the path of the economy will eventually depend on the magnitude and shape of the same functions of motion. We will now proceed in the analysis of each one of the three scenarios.

First scenario: No urbanization

As hinted above, the start of the urbanization process requires an initial threshold level of technology. For lower initial levels of A_t, the urbanization process will never begin, and the economy will remain stuck in a development trap, with $\delta_t = 0$ and $y^r_t > y^u_t$. Plugging (4) into (3), this will occur when the initial level of technology is[14]:

$$\pi_0 \leq \left[\frac{\kappa^r}{\kappa^{u\,\frac{1}{1-\gamma}} \phi^{\frac{\gamma}{\gamma-1}} \left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}} \right)} \right]^{(1-\gamma)/\gamma}$$

In this case, the per capita gross earning for both urban and rural workers will end up to be the same: $\kappa^r \pi_t = \kappa^r \pi_0, \forall t$. Graphically, this scenario is represented in Figure 3. In the scenario of no urbanization, the shapes of the π^p_{t+1} and π^f_{t+1} functions are irrelevant, since the final equation of motion will always lie below the 45° line[15]. In this scenario, the initial level of technology π_0 is too low and the incentives to invest in human capital are not sufficient enough; therefore, the dynamic path of the technological level will always remain below the 45° line and the urban development process will never take place.

this by plugging $(\delta_t s_t)^\rho$ into expression (4), to obtain: $s_t = \left[\frac{\gamma \kappa^u (\delta_{t-1} s_{t-1})^\rho}{\phi} \right]^{1/(1-\gamma)}$, which in turn will impact on the final equation of motion in (7). Of course, an essential condition for generating growth is that the levels of human capital and degree of urbanization at time $t-1$ must be higher than the current level of technology at time t (i.e., $(\delta_{t-1} s_{t-1})^\rho > \pi_t$).

[14]Since $\delta_t = 0$, the values for the rents and the spillover effect (which depend on the distance from the CBD) in the rural income will be zero.

[15]π^p_{t+1} will result to be concave iff $(1-\gamma)/\gamma < 1$; at the opposite, for certain low levels of γ, it can also be the case that $(1-\gamma)/\gamma > 1$, and therefore the same function of motion will end up being convex. In latter case, however, nothing would change as long as the function π^f_{t+1} is concave and crosses the function π^p_{t+1} below the 45° line.

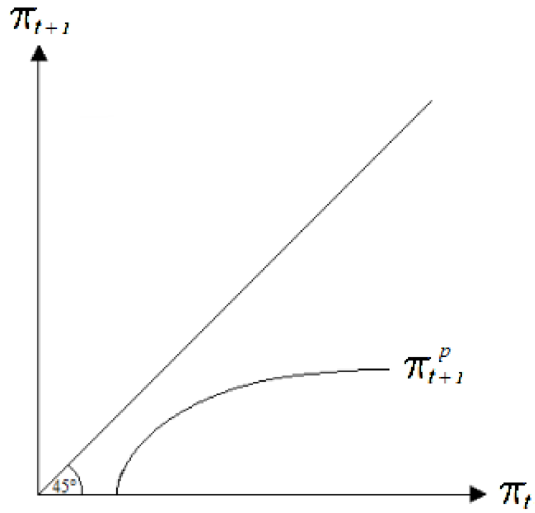

Figure 3: No Urbanization.

Second scenario: Partial urbanization

The necessary condition for the emergence of urban growth is represented by an initial level of technology above the threshold level. In addition, the urban income must be higher than the rural income, in order to boost the urbanization process (which requires $0 < \delta_t < 1$); in other words, this entails that the urban income net of the cost to invest in human capital results to be superior to the rural income net of the effect exerted by rural rents and the productivity spillover. In this situation, the equation of the dynamic path will be obtained by plugging expressions (4) and (5) into (7), to obtain:

$$\pi_{t+1}^{p} = \left[\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^{u}\pi_t\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) + \Theta^{1/(1-\gamma)}\frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^{r}\pi_t}{\bar{\Psi}(2q+\omega)}\right]^{\rho}$$

where $\Theta = \gamma\kappa^{u}\phi^{-1}\pi_t$.

As graphically shown in Figure 4, π_{t+1}^{p} will be concave, with two possible steady state levels for the technological path (see Appendix 1).

The first steady state level, $\tilde{\pi_1}$, is unstable. For all the values of π_t ranging from 0 up this point, the initial level of technology is not sufficiently high to start the urban growth process. As well, the workers' income and their incentive to invest in human capital result to be too low. Because of this, the economy will remain stuck in a development trap, and the level of growth of the urban area (represented by the dotted arrows) will converge back to zero. However, if the initial level of technology is equal or superior to $\tilde{\pi_1}$, the urban economy will start growing (boosted by the increasing rates of investments in human capital and urbanization) until it reaches the steady state level $\tilde{\pi_2}$. At this point, the growth rate of investments in human capital and urbanization will stop and the growth level of the urban economy will enter a condition of stable equilibrium[16].

[16]The initial level of technology could also start above $\tilde{\pi_2}$; in this case, however, there will be a downturn in investments in human capital (as well as in the urbanization rate), which will eventually bring back the equilibrium to the steady state level of $\tilde{\pi_2}$.

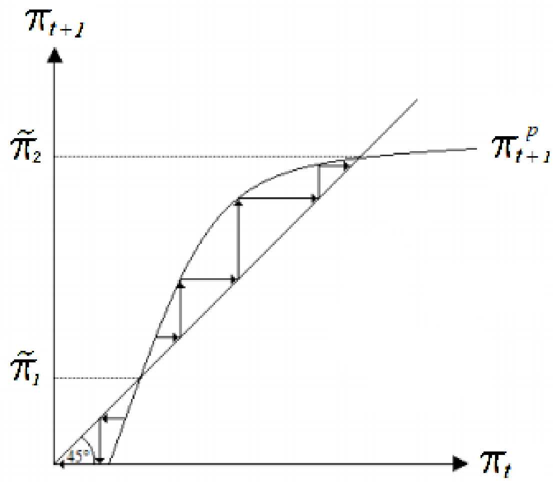

Figure 4: Partial urbanization.

Third scenario: Full urbanization

Finally, in this last scenario, the urban area experiences full urbanization, with two possible long paths of growth which are depicted in Figures 5 and 6, respectively. Specifically, the kink of the dynamic function at the point $\tilde{\pi}_2$ of Figures 5 (first sub-scenario) and 6 (second sub-scenario) represents the point where, given a highly enough technological level, the entire population will decide to invest in human capital and will locate to the urban area.

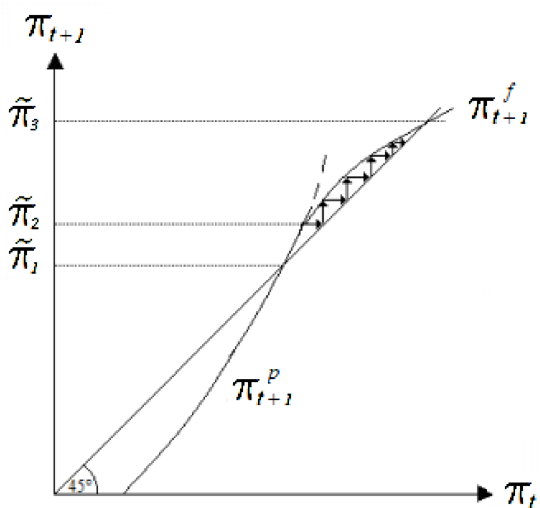

Figure 5: Full urbanization with steady state.

The final equation of motion will be equal to the one of the partial urbanization case, but being now $\delta_t = 1$ we will have:

$$\pi_{t+1}^f = \Theta^{\rho/(1-\gamma)}$$

In this third scenario, π_{t+1}^p will be convex, whereas π_{t+1}^f will either be concave (Figure 5) or convex (Figure 6) (see Appendix 1). In addition, the slope of π_{t+1}^p will always exceed the slope of π_{t+1}^f, since the condition $\partial\pi_{t+1}^p/\partial\pi_t > \partial\pi_{t+1}^f/\partial\pi_t$ is always verified (Proof in Appendix 2).

In the first sub-scenario (Figure 5), where π_{t+1}^f is concave, we assist to a faster urban growth in the first transition phase of partial equilibrium (given that $\partial\pi_{t+1}^p/\partial\pi_t > \partial\pi_{t+1}^f/\partial\pi_t$). Once the economy reaches point $\tilde{\pi_2}$, the speed of growth decelerates, since the second share of rural workers who move to the CBD is smaller in comparison to the first share; as a result, there will be a lower number of new investments in human capital boosting urban growth. Eventually, the economy reaches full urbanization at the new (higher) steady state level $\tilde{\pi_3}$, with per capita earnings equal to $[\tilde{\pi_3}\kappa^u]^{\frac{1}{1-\gamma}}\phi^{\frac{\gamma}{\gamma-1}}(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}) - q\bar{\Psi}$ (being $\delta_t = 1$).

Finally, the second sub-scenario (Figure 6) entailing a convex π_{t+1}^f function, requires the condition $\gamma + \rho \geq 1$ (see Appendix 1). This means that the returns to investments in human capital are remarkably high and/or the effect of human capital on technology is considerable. In this second sub-scenario, the urban economy and the level of technology will grow boundless; in particular, the urban economy will firstly reach the stage of full urbanization (at point $\tilde{\pi_2}$), to then continue its path, with per capita earnings (and growth rates) increasing forever.

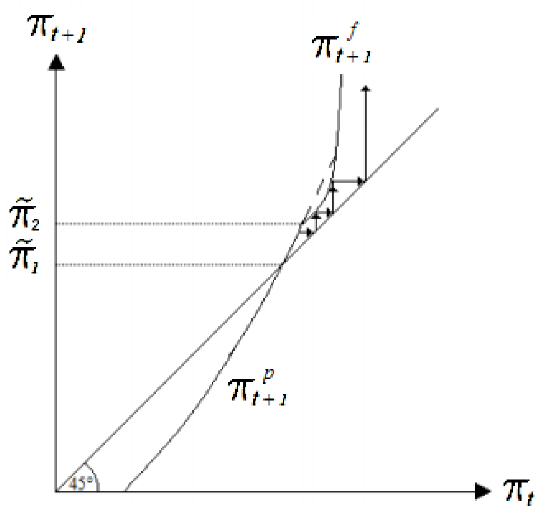

Figure 6: Full urbanization with unbounded growth.

3 Conclusion

This paper aimed at contributing to the literature by extending the classical models of rural-to-urban migration so far developed in previous studies by introducing a spatial framework for the urban and rural areas. Specifically, we developed a spatial framework of rural-to-urban migration, relying upon some of the key assumptions of the Alonso-Muth-Mills model. Hence, we defined a settlement composed by a monocentric nuclear urban area surrounded by a linearly distributed rural area. Following the literature, we conditioned urban growth as a function of human capital (the latter being accumulated only in the urban area), with rural-to-urban migration triggered by the income differential between urban and rural workers. The novelty introduced in this paper was to model the spatial interaction between the urban and the rural areas by introducing the spacial variables of the rural rents and the productivity spillover, whose effect on rural workers' income depend on the distance from the urban area.

With respect to the findings of previous studies, in this model, not necessarily the urbanization level associated to the market solution exceeds the urbanization level set by the city planner; in this regard, the spatial variable of rural rents exerts a main impact in affecting the two optimal levels of urbanization. Specifically, when rural workers enjoy considerably high salaries, they inevitably end up neglecting the effect exerted by the rural rents and accordingly, the market solution for the urbanization level will be lower, in absolute value, than the city planner solution, thus leading to a higher level of agglomeration in the rural area. On the other hand, in presence of higher urban salaries and/or reduced levels of rural rents, rural workers will be attracted to move to the urban area, so that, contrarily to the previous case, the market solution will now be higher, in absolute value, than the city planner solution. Subsequently, we studied the dynamic evolution of urban growth over time, following the assumption posited in the literature for which urban growth is measured by increases in the level of technology, with the variables of the rural rents and the productivity spillover entering the equations of motion. According to the initial level of technology, different growth scenarios emerged. For too low levels of technology, a development trap could arise, since investments in human capital are not sufficiently high. This scenario always entail lower rural incomes compared to urban incomes. Nonetheless, when the initial level of technology is sufficiently high, the urban process begins, leading to an increase in the technology level and output over time. A necessary condition for the start of the urbanization process is that the urban income must be higher than the rural income; this requires necessarily that the urban income net of the cost to invest in human capital results to be superior to the rural income net of the effect exerted by rural rents and the productivity spillover. When the growth process begins, the final level of steady state will eventually depend on the level of returns to human capital and impact of human capital on technology; for remarkably high values of returns to education, the urban area might also experience unbounded growth. However, these growth benefits deriving from over- (full) urbanization also entail the emergence of congestion costs, which will ultimately lower the quality of life of urban workers. Therefore, in the end, when the urbanization level of the market solution exceeds the urbanization level set by the city planner, the latter will face a trade-off between growth and negative urban congestion externalities; within this context, if the aim of the city planner is to reduce the congestion level of the urban area, it is likely that policies designed to decrease, for instance, the value of rural rents, could contribute to improve the standard of living of rural workers, thus resulting in lower rates or rural-to-urban migration. On the other hand, urban policies aimed at positively influencing the urban productivity factor (e.g., policies designed to improve the quality of urban infrastructures) may help increasing the optimal level of urbanization, thus allowing the relocation in the urban area of a larger share of rural workers.

Appendix 1

Dynamics of π^p_{t+1} and π^f_{t+1} functions.

As specified in the main text, the dynamic evolution of the technological level of the economy will depend on the shape of the function π_{t+1}. Specifically:

$$\pi_{t+1} = \begin{cases} \pi^0 & if \quad \delta_t = 0 \\ \pi^p_{t+1} & if \quad 0 < \delta_t < 1 \ , \ \pi^p_{t+1} > 0 \ and \ (\delta_t s_t)^\rho > \pi_t \\ \pi^f_{t+1} & if \quad \delta_t = 1 \ , \ \pi^f_{t+1} > 0 \ and \ (\delta_t s_t)^\rho > \pi_t \end{cases}$$

where:

$$\pi^p_{t+1} = \left[\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u\pi_t\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) + \Theta^{1/(1-\gamma)}\frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^r\pi_t}{\bar{\Psi}(2q+\omega)}\right]^\rho$$

and:

$$\pi^f_{t+1} = \Theta^{\rho/(1-\gamma)}$$

Now; $\pi^p_{t+1} < 0$ when the following conditions are met: $\pi_t < \frac{\phi}{\kappa^u\Theta^{(\gamma-1)/(1-\gamma)}} \quad \wedge \quad \pi_t < \frac{2\bar{r}+\bar{\Psi}\omega}{2\kappa^r}$, or $\pi_t < \frac{\phi}{\kappa^u\Theta^{(\gamma-1)/(1-\gamma)}} \quad \wedge \quad \pi_t > \frac{2\bar{r}+\bar{\Psi}\omega}{2\kappa^r}$, with $\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u\pi_t\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) > \Theta^{1/(1-\gamma)}\frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^r\pi_t}{\bar{\Psi}(2q+\omega)}$. The same conditions are always satisfied when $\pi_t > \frac{\phi}{\kappa^u\Theta^{(\gamma-1)/(1-\gamma)}} \quad \wedge \quad \pi_t < \frac{2\bar{r}+\bar{\Psi}\omega}{2\kappa^r}$, with $\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u\pi_t\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) < \Theta^{1/(1-\gamma)}\frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^r\pi_t}{\bar{\Psi}(2q+\omega)}$. Otherwise, the function π^p_{t+1} is strictly positive.

In differentiating π^p_{t+1} with respect to π_t, we obtain:

$$\frac{\partial\pi^p_{t+1}}{\partial\pi_t} = \rho\left[\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u\pi_t\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) + \Theta^{1/(1-\gamma)}\frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^r\pi_t}{\bar{\Psi}(2q+\omega)}\right]^{\rho-1} \times \left[\frac{2\Theta^{2/(1-\gamma)}\kappa^u\Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} - \frac{2\Theta^{1/(1-\gamma)}\kappa^r}{\bar{\Psi}(2q+\omega)}\right]$$

In the relevant range where π^p_{t+1} and δ_t are both positive, the first expression in brackets of $\frac{\partial\pi^p_{t+1}}{\partial\pi_t}$ is positive. The second expression in brackets of $\frac{\partial\pi^p_{t+1}}{\partial\pi_t}$ is positive as long as $B > (\frac{\kappa^r}{\kappa^u})^{\frac{1-\gamma}{\gamma}}$, which can be rewritten as $\pi_t > \frac{\phi}{\gamma}\kappa^{r\,\frac{1-\gamma}{\gamma}}(\frac{1}{\kappa^u})^{\frac{1}{\gamma}}$, which is always the case in the relevant range in question. The function reaches its minimum at $\pi_t = \frac{\phi}{\gamma}\kappa^{r\,\frac{1-\gamma}{\gamma}}(\frac{1}{\kappa^u})^{\frac{1}{\gamma}}$, and is monotonically increasing thereafter.

The evolution of the economy will eventually depend on the curvature of the π_{t+1} function. In particular, the number of times that this function intersects the 45° line (where $\pi^p_{t+1} = \pi_t$) will be a crucial factor for the dynamic path of the economy. Subsequently, in taking the second derivative of π^p_{t+1}, we get:

$$\frac{\partial^2 \pi^p_{t+1}}{\partial {\pi_t}^2} = \rho(\rho-1)\Bigg[\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u \pi_t \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) + \Theta^{1/(1-\gamma)} \times \frac{2\bar{r} + \bar{\Psi}\omega - 2\kappa^r \pi_t}{\bar{\Psi}(2q+\omega)}\Bigg]^{\rho-2}\left[\frac{2\Theta^{2/(1-\gamma)}\kappa^u \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} - \frac{2\Theta^{1/(1-\gamma)}\kappa^r}{\bar{\Psi}(2q+\omega)}\right]^2$$

To simplify the calculations, label $\Theta^{2/(1-\gamma)}\left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u \pi_t \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)}\right) = \Phi$ and $\Theta^{1/(1-\gamma)} \times \frac{2\bar{r}+\bar{\Psi}\omega-2\kappa^r\pi_t}{\bar{\Psi}(2q+\omega)} = \Omega$.

The function π^p_{t+1} will be convex when $\frac{\partial^2 \pi^p_{t+1}}{\partial {\pi_t}^2}$ is positive and this happens when both $\Phi < 0$, $\Omega < 0$. Or when $\Phi > 0$ and $\Omega < 0$, with $|\Phi| < |\Omega|$. Or $\Phi < 0$ and $\Omega > 0$, with $|\Phi| > |\Omega|$. In these three case, the slope of the monotonically increasing function of motion will be increasing for $\forall \pi_t$ and therefore there will always be a single intersection point where $\pi^p_{t+1} = \pi_t$.

Conversely, π^p_{t+1} will be concave when $\frac{\partial^2 \pi^p_{t+1}}{\partial {\pi_t}^2}$ is negative and this happens when both $\Phi > 0$, $\Omega > 0$. Or when $\Phi > 0$ and $\Omega < 0$, with $|\Phi| > |\Omega|$. Or $\Phi < 0$ and $\Omega > 0$, with $|\Phi| < |\Omega|$. In these cases, the slope of the monotonically increasing function of motion will be decreasing and therefore there could be zero, one or two values of π_t to allow tangency points (i.e., when $\pi^p_{t+1} = \pi_t$); depending on the values of the parameters it can hence be the case that for $\forall \pi_t$, $\pi^p_{t+1} < \pi_t$ and in this case there will not be any tangency point. It could then be the case that the condition $\pi^p_{t+1} = \pi_t$ is verified and therefore we have one single crossing point. Or, if $\pi^p_{t+1} > \pi_t$ there will be two tangency points.

Finally, for the study of the π^f_{t+1} function, the calculations are relatively easier. In differentiating π^f_{t+1}, we get:

$$\frac{\partial \pi^f_{t+1}}{\partial \pi_t} = \frac{\rho}{1-\gamma}\Theta^{\frac{\rho}{1-\gamma}-1}$$

The function is hence monotonically increasing. Taking the second derivative, we obtain:

$$\frac{\partial^2 \pi^f_{t+1}}{\partial {\pi_t}^2} = \frac{\rho}{1-\gamma}\left(\frac{\rho-1+\gamma}{1-\gamma}\right)\Theta^{\frac{\rho}{1-\gamma}-2}$$

Therefore π^f_{t+1} will be monotonically increasing and globally concave if $\gamma + \rho < 1$, and monotonically increasing but globally convex if $\gamma + \rho > 1$.

Appendix 2

$\partial \pi^p_{t+1}/\partial \pi_t > \partial \pi^f_{t+1}/\partial \pi_t$.

Proof. Consider the point where $\delta_t = 1$, and $\pi^p_{t+1} = \pi^f_{t+1}$. Recalling that:

$$\frac{\partial \pi^p_{t+1}}{\partial \pi_t} = \rho \left[\Theta^{2/(1-\gamma)} \left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u \pi_t \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} \right) + \Theta^{1/(1-\gamma)} \frac{2\bar{r} + \bar{\Psi}\omega - 2\kappa^r \pi_t}{\bar{\Psi}(2q+\omega)} \right]^{\rho-1} \times \left[\frac{2\Theta^{2/(1-\gamma)} \kappa^u \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} - \frac{2\Theta^{1/(1-\gamma)} \kappa^r}{\bar{\Psi}(2q+\omega)} \right]$$

and

$$\frac{\partial \pi^f_{t+1}}{\partial \pi_t} = \frac{\rho}{1-\gamma} \Theta^{\frac{\rho}{1-\gamma}-1}$$

at the intersection of the two functions we have that:

$$\frac{\partial \pi^f_{t+1}}{\partial \pi_t} = \frac{\rho}{1-\gamma} \pi^f_{t+1} {\pi_t}^{-1} = \frac{\rho}{1-\gamma} \pi^p_{t+1} {\pi_t}^{-1}$$

Assume now that $\frac{\partial \pi^p_{t+1}}{\partial \pi_t} > \frac{\partial \pi^f_{t+1}}{\partial \pi_t}$. This entails that:

$$\rho \left[\Theta^{2/(1-\gamma)} \left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u \pi_t \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} \right) + \Theta^{1/(1-\gamma)} \frac{2\bar{r} + \bar{\Psi}\omega - 2\kappa^r \pi_t}{\bar{\Psi}(2q+\omega)} \right]^{\rho-1} \times \left[\frac{2\Theta^{2/(1-\gamma)} \kappa^u \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+\omega)} - \frac{2\Theta^{1/(1-\gamma)} \kappa^r}{\bar{\Psi}(2q+\omega)} \right] > \frac{\rho}{1-\gamma} {\pi_t}^{-1} \left[\Theta^{2/(1-\gamma)} \left(\frac{-2\phi}{\bar{\Psi}(2q+\omega)} + \frac{2\kappa^u \pi_t \Theta^{(\gamma-1)/(1-\gamma)}}{\bar{\Psi}(2q+t)} \right) + \Theta^{1/(1-\gamma)} \frac{2\bar{r} + \bar{\Psi}\omega - 2\kappa^r \pi_t}{\bar{\Psi}(2q+\omega)} \right]^{\rho}$$

Simplifying, we obtain:

$$\left(\frac{1}{1-\gamma} - 1 \right) \frac{2\Theta^{(\gamma+1)/(1-\gamma)} \kappa^u}{\bar{\Psi}(2q+\omega)} + \left(1 - \frac{1}{1-\gamma} \right) \frac{2\Theta^{1/(1-\gamma)} \kappa^r}{\bar{\Psi}(2q+\omega)} + \frac{1}{1-\gamma} \left[\frac{-2\Theta^{2/(1-\gamma)} \phi}{\pi_t \bar{\Psi}(2q+\omega)} + \Theta^{1/(1-\gamma)} \frac{2\bar{r} + \bar{\Psi}\omega}{\pi_t \bar{\Psi}(2q+\omega)} \right] < 0$$

Which leads to:

$$(\pi_t \kappa^u)^{1/(1-\gamma)} (\gamma \phi^{-1})^{\gamma/(1-\gamma)} - \gamma^{\gamma/(1-\gamma)} (\pi_t \kappa^u)^{1/(1-\gamma)} \phi^{\gamma/(\gamma-1)} + \gamma^{-1} \left[\bar{r} + \frac{\bar{\Psi}\omega}{2} \right] < \kappa^r \pi_t$$

or:

$$\frac{\bar{r} + \frac{\bar{\Psi}\omega}{2}}{\gamma} < \kappa^r \pi_t$$

Since $y^r_t \geq 0$ and $0 < \delta_t \leq 1$, given (2), the above inequality is always true. ■

References

[1] Ahmad Abdel-Rahman, Mohammad Safarzadeh, and Michael Bottomley. "Economic growth and urbanization: A cross-section and time-series analysis of thirty-five developing countries". In: *International Review of Economics* 53.3 (2006), pp. 334–348. DOI: https://doi.org/10.1007/BF03029785.

[2] William Alonso. *Location and land use. Toward a general theory of land rent*. Harvard University Press, 1964.

[3] Luisito Bertinelli and Duncan Black. "Urbanization and growth". In: *Journal of Urban Economics* 56.1 (2004), pp. 80–96.

[4] Duncan Black and Vernon Henderson. "A theory of urban growth". In: *Journal of Political Economy* 107.2 (1999), pp. 252 –284. DOI: http://dx.doi.org/10.1086/250060. eprint: https://doi.org/10.1086/250060.

[5] Jan K. Brueckner. "Chapter 20: The structure of urban equilibria: A unified treatment of the muth-mills model". In: *Urban Economics*. Vol. 2. Handbook of Regional and Urban Economics. Elsevier, 1987, pp. 821 –845. DOI: https://doi.org/10.1016/S1574-0080(87)80006-8. URL: http://www.sciencedirect.com/science/article/pii/S1574008087800068.

[6] Marius Brulhart and Federica Sbergami. "Agglomeration and growth: Cross-country evidence". In: *Journal of Urban Economics* 65.1 (2009), pp. 48 –63. ISSN: 0094-1190. DOI: https://doi.org/10.1016/j.jue.2008.08.003. URL: http://www.sciencedirect.com/science/article/pii/S0094119008000715.

[7] Gilles Duranton and Matthew A Turner. *Urban Growth and Transportation*. CEPR Discussion Papers 6633. C.E.P.R. Discussion Papers, Jan. 2008.

[8] Simon C. Fan and Oded Stark. "Rural-to-urban migration, human capital, and agglomeration". In: *Journal of Economic Behavior & Organization* 68.1 (2008), pp. 234 –247. ISSN: 0167-2681. DOI: https://doi.org/10.1016/j.jebo.2008.04.003. URL: http://www.sciencedirect.com/science/article/pii/S0167268108000966.

[9] John R. Harris and Michael P. Todaro. "Migration, unemployment and development: A two-sector analysis". In: *American Economic Review* 60.1 (1970), pp. 126 –142. URL: https://www.jstor.org/stable/1807860?seq=1#page_scan_tab_contents.

[10] Vernon Henderson. *Handbook of economic growth*. Vol. 1. Elsevier, 2005.

[11] Vernon Henderson. "The urbanization process and economic growth: The so-what question". In: *Journal of Economic Growth* 8.1 (2003), pp. 47 –71. DOI: https://doi.org/10.1023/A:102286080.

[12] Vernon Henderson. "Urbanization and growth". 2004. URL: http://www.brown.edu/Departments/Economics/Faculty/henderson/papers/Urbangrowhandbook261004.pdf.

[13] Vernon Henderson and Hyoung Gun Wang. "Urbanization and city growth". 2004. URL: http://www.econ.brown.edu/Faculty/henderson/papers/UrbanizationandCityGrowth0804.pdf.

[14] Vernon Henderson and Hyoung Gun Wang. "Urbanization and city growth: The role of institutions". In: *Regional Science and Urban Economics* 37.3 (2007), pp. 283 –313. ISSN: 0166-0462. DOI: https://doi.org/10.1016/j.regsciurbeco.2006.11.008. URL: http://www.sciencedirect.com/science/article/pii/S0166046206001049.

[15] Remi Jedwab, Luc Christiaensen, and Marina Gindelsky. "Rural push, urban pull and... urban push? New historical evidence from developing countries". In: *Institute for International Economic Policy Working Paper Series - IIEP-WP-2014-4* (2014). URL: https://www2.gwu.edu/~iiep/assets/docs/papers/Jedwab_IIEPWP_2014-4.pdf.

[16] Arthur W. Lewis. “Economic development with unlimited supplies of labour”. In: *The Manchester School* 22.2 (1954), pp. 139 –191. DOI: 10.1111/j.1467-9957.1954.tb00021.x. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-9957.1954.tb00021.x. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9957.1954.tb00021.x.

[17] Edwin S. Mills. “An aggregative model of resource allocation in metropolitan areas”. In: *American Economic Review* 57.2 (1967), pp. 197–210.

[18] Richard F. Muth. *Cities and housing : the spatial pattern of urban residential land use*. University of Chicago Press, 1969.

[19] Jin-li Qi. “Agglomeration and radiation effect of the pull of urbanization”. In: *Chinese Geographical Science* 13.3 (2003), pp. 224–227. ISSN: 1002-0063.

[20] Ivan Turok and Gordon McGranahan. “Urbanization and economic growth: the arguments and evidence for Africa and Asia”. In: *Environment and Urbanization* 25.2 (2013), pp. 465 –482. DOI: https://doi.org/10.1177/0956247813490908.

[21] William C. Wheaton. “A comparative static analysis of urban spatial structure”. In: *Journal of Economic Theory* 9.2 (1974), pp. 223 –237. ISSN: 0022-0531. DOI: https://doi.org/10.1016/0022-0531(74)90068-4. URL: http://www.sciencedirect.com/science/article/pii/0022053174900684.

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