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WEAKENING DESCRIPTION INVARIANCE TO DEAL WITH FRAMING EFFECTS: AN AXIOMATIC APPROACH

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Weakening description invariance to deal with framing effects: an axiomatic approach*

Dino Borie† Dorian Jullien‡

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Abstract

The goal of this paper is to provide an axiomatic framework that can account for framing effects violating the so-called axiom of description invariance. Most existing discussions of the latter in economics are made with respect to Kahneman and Tversky's 1980s work. However, other psychologists have, over the last twenty-five years or so, progressively refined the conditions under which framing effects violating description invariance hold. Our axiomatic framework is motivated by these developments. We argue that description invariance is an implicit axiom of the standard model primarily on the formal structure of its object of choice and only derivatively on the formal structure of preferences. The conditions under which it is violated in psychologists' experiments are a useful guide to make it formally explicit. Furthermore, they also provide normative justifications for weakening it. We propose a way to do so in a mathematically tractable fashion that can account for all the variations in framing effects for which prospect theory cannot (at least straightforwardly) account for.

JEL Classification: D80, D81, D01, D03.

Keyword: Description invariance, Framing effect, Extensionality, Consequentialism, Description-dependence.

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1 Introduction

The goal of this paper is to provide an axiomatic framework that can account for framing effects violating the so-called axiom of description invariance. The paradigmatic example is Tversky and Kahneman's Asian Disease problem (1981, p.543):

> Imagine that the U.S. is preparing for the outbreak of an unusual Asian Disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:
>
> If Program A is adopted, 200 people will be saved.
>
> If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.
>
> If Program C is adopted, 400 people will die.
>
> If Program D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Decision makers are only presented with either the choice set $\{A, B\}$ or $\{C, D\}$. Both choice sets have identical consequences, they only differ in the descriptions of these consequences. A framing effect is observed in both within- and between-subject designs because the modal preferences are $A \succ B$ and $D \succ C$, i.e., a preference reversal that violate description invariance.

Existing discussions of description invariance often highlight five points[1]. (1) Description invariance is an implicit axiom of the standard model, the violations of which are (2) descriptively pervasive, (3) normatively unjustifiable, (4) mathematically intractable and (5) explained by prospect theory.

These points are usually made by discussing Kahneman and Tversky's work on framing in the 1980s. This is a narrow focus because the conditions under which this framing effect holds have been refined by other psychologists over the last twenty-five years. Our axiomatic framework is empirically motivated by three of these developments. The first development concerns the effects of further redescriptions of the same consequence, e.g.,"200 people will be saved and 400 people will not be saved", "400 people will not be saved", etc. The second is about the direct impact of the description of an object of choice on the utility derived from the occurrence of its associated consequence, e.g., being better off when the rescue of 200 people and the death of 400 occurs from having chosen A than from having chosen C. The last development studies how the one who describes the decision problem makes choices from sets of different descriptions of the same object of choice and how these very choices (of one description over other possible ones) may influence the decision maker, e.g., the former may signal that he would prefer that the sure program be chosen by describing it as "200 people will be saved" instead of "400 people will die" and the decision maker may infer that signal.

Especially regarding this latter point, we want to make a distinction throughout this paper between the perspective that the decision maker (henceforth denoted by d_{ma}) takes on a given decision problem and the perspective that what we will call the decision modeler (henceforth denoted by d_{mo}) takes on the same decision problem. This distinction is especially useful in applications of our framework where the d_{mo} can be identified with the one who poses the decision problem (an experimenter, a policy maker, a firm, one particular self of a multiple-selves d_{ma} etc.) or the one who just observes the decision situation (an economist, a psychologist, a politician etc.).

[1] See Tversky and Kahneman (1981; 1986, p.S253; 1992, p.298), Arrow (1982, pp.6-8), Smith (1985), Machina (1987, pp.144-146), Kreps (1988, p.197), Sugden (1991, p.759), Quiggin (1993, §14.5; 2014, p.713), Camerer (1995, p.652; 1999, p.10577; 2004, p.386), Rabin (1996, pp.46-47; 1998, pp.37-38; 2002, p.662), Starmer (2000, pp.338-9, p.352), Luce (2000, p.8, pp.34-35, pp.216-217), Camerer and Loewenstein (2004, p.12, p.14), Samuelson (2005, pp.93-95), Varian (2006, pp.549-550), Fudenberg (2006, p.699, p.700, p.708), Fudenberg and Levine (2006, p.1472), Bernheim and Rangel (2007, p.66), Thaler and Sunstein (2008, p.37), Köszegi and Rabin (2008, p.1824), Loewenstein and Haisley (2008, pp.219), Wakker (2010, p.234, pp.241-2, p.250, p.265, p.350, pp.377-378), Bardsley et al. (2010, pp.130-1), Kahneman (2011, chap.34), Angner and Loewenstein (2012, p.663, p.668).

It captures the main condition of possibility for the existence of framing effects, namely that there can be *different* perspectives on the *same* decision situation.

We propose an axiomatic framework that essentially refines the formal structure of the objects of choice within the standard model. In a nutshell, we construct a choice set with descriptions of objects of choice instead of objects of choice, an equivalence relation determining which descriptions are about the same object of choice from the d_{mo}'s perspective and a concatenation operation joining different equivalent descriptions to form a new description (e.g., "200 people will be saved" can be concatenated with "400 people will not be saved" to form "200 people will be saved and thus 400 people will not be saved"). The d_{ma}'s preferences over descriptions of objects of choice will be a standard weak order. Our results determine how the d_{mo}'s equivalence relation is articulated with the d_{ma}'s indifference relation. This allows to characterize the key implicit axiom of the standard model that makes it blind to framing effects, which we call "Independence of Common Description" (ICD). We weaken the latter to allow such effects to be formally represented. Further axioms characterizing finer degrees of dependence to descriptions are distinguished, along with informational measures derivable from our framework.

Our intended contribution can be summarized with respect to the five points about description invariance noted above. (1) Description invariance is an implicit axiom of the standard model that we make explicit by working primarily on the formal structure of its object of choice and only derivatively on the d_{ma}'s preferences. We are driven by the conditions under which it is violated in psychologists' experiments on the Asian Disease (2) to make it formally explicit, (3) to provide normative justifications for weakening it, and (4) to do so in a mathematically tractable way. (5) We account for variations in framing effects for which prospect theory cannot (at least straightforwardly) account for.

One difference between our framework and the few other formal contributions on framing effects is that our framework is presented for decision under *certainty*. We show in the applications how it extends straightforwardly to decisions under risk and uncertainty so that violations of description invariance can be represented in expected utility models, rank-dependent utility models and cumulative prospect theory models. We do so not only because some psychologists have demonstrated violations of description invariance under certainty, but also because some others have shown that the framing effects in the Asian Disease is clearly unrelated to its probabilistic dimension. Hence one implication of our difference with the other contributions is that, while we focus on the d_{ma}'s preferences and on the consequences in the objects of choice, they all focus on the d_{ma}'s beliefs and/or on the formal structure of the events constitutive of the state space. Though we rely on more empirical evidence from psychology than these contributions, we are more minimalist in making less assumptions about unobservable psychological processes (our axioms have independent testable implications through choice data). We emphasize that, despite the apparent complexity of framing effects often mentioned in the literature, the contribution of our framework lies in its theoretical and formal simplicity (none of the results are very technical). This notably allows it to be compatible with other contributions, in which it can be integrated to explain further framing phenomena. Finally, by virtue of the distinction between the d_{mo}'s and d_{ma}'s perspectives, our framework makes more precise how framing effects can be seen as rational and irrational, without being tied to a universal position whereby they are all necessarily either rational or irrational.

The body of evidence motivating the axiomatic framework is presented in section 2. The framework and the results are presented in section 3. Applications and extensions under risk and uncertainty are discussed along with the relation to previous formal accounts of framing in economics in section 4.

2 Motivation

As emphasized in one of the most cited review and meta-analysis of framing effects in psychology, "the likelihood of obtaining choice reversals [is] directly related to the similarity between features of a given study and features of Tversky and Kahneman's (1981) original "Asian disease problem."" (Levin et al., 1998, p.157). The effect is robust to changes in scenario and type of consequences,

e.g., money instead of lives[2]. Hence, it makes sense to focus on the Asian Disease's empirical structure to guide theoretical developments on framing effects violating description invariance.

The first set of results that will guide our theoretical development is the d_{mo}'s equivalent redescriptions of consequences that trigger a variety of d_{ma}s' modal preferences. Here are three examples (the modal preference is bolded). Kühberger (1995, experiment 2) redescribed the sure consequence as follows, and the modal preference did not exhibit framing effects anymore but a slight tendency for the risky programs:

> If Program A' is adopted, 200 people will be saved and 400 people will not be saved.
>
> If **Program** B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.
>
> If Program C' is adopted, 400 people will die and 200 people will not die.
>
> If **Program** D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Kühberger (1995, experiment 1) redescribed again the sure consequence by only retrieving the original description from the previous experiment, and the modal preference exhibited a reverse framing effects:

> If Program A'' is adopted, 400 people will not be saved.
>
> If **Program** B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.
>
> If **Program** C'' is adopted, 200 people will not die.
>
> If Program D is adopted, there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

Mellers and Locke (2007, p.362) redescribed the sure consequence by mixing its two original descriptions, and the modal preference went for it:

> If **Program** A''' is adopted, 200 people will be saved and 400 people will die
>
> If Program B''' is adopted, there is 1/3 probability that 600 people will be saved and 2/3 probability that 600 people will die.

At least the two former examples are robust in both between- and within-subject designs[3]. The last one have not been systematically investigated to best of our knowledge. Prospect theory's account of the original Asian Disease relies on the argument that the semantics of "saved" and "die" induce changes in the reference point, from the expectation that everybody will die in the "save frame" to the expectation that everybody can be saved in the "die frame", thereby implying risk aversion and risk seekingness respectively. That the semantics does not change in the first two examples makes prospect theory unable to explain straightforwardly the patterns of preferences presented here[4].

The second set of results that motivate our theoretical development is best understood by first introducing a classical result from psychology on so-called attribute framing effects. Levin and Gaeth (1988) described the same ground beef to their d_{ma}s in two different ways: either as "75% lean ground beef" in one frame or as "25% fat ground beef" in another frame. The d_{ma} have to rate the ground beef on some numerical scales (bad/good-tasting, greasless/greasy, high/low quality, lean/fat) either after or before they actually consume the ground beef. There is a framing effect whereby the ground beef in the "75% lean ground beef" frame gets systematically higher ratings than in the "25% fat ground beef" frame, *even in the condition where ratings were made*

[2]Other review and meta-analysis include Kühberger (1997; 1998), Kühberger et al. (1999), Piñon and Gambara (2005), Maule and Villejoubert (2007), Keren (2011b), Mandel and Vartanian (2012), Takemura (2014, Part. V). The results of Kühberger et al. (2002) is of particular interest for economics, as they show in a problem with monetary consequences that the that framing effect disappears with small stakes but is amplified with large stakes.

[3]See Kühberger and Tanner (2010, Table 2), Schulte-Mecklenbeck and Kühberger (2014), Tombu and Mandel (2015), and the references therein about further variations of equivalences through redescriptions triggering different modal preferences.

[4]For more detailed theoretical discussions, see Kühberger (1997) and Takemura (2014, chap.10).

after the tasting (though the magnitude of the framing effect was indeed *slighlty* reduced). Hence, different descriptions of the same ground beef influence the utility derived from its experienced taste[5].

Similar results exist on the Asian Disease. Kühberger and Gradle (2013, experiment 3) presented the four original programs to their d_{ma}s in a joint evaluation mode, i.e., $\{A, B, C, D\}$ simultaneously. They had to rate all four programs on three scales: (1) likeliness to choose, (2) believed efficacy and (3) emotional impact. The modal preferences from (1) are $A \succ B \sim D \succ C$, which highly correlate with the efficacy ratings from (2), both of which weakly correlate with the emotional ratings from (3) in the overall, though the correlation is much stronger when only the emotional ratings for the sure programs are considered. More akin to economists' choice experiment, the modal preferences $A \succ B \sim D \succ C$ are also found in their last experiments where subjects had to rank directly (i.e., no ratings anymore) the four programs[6]. In short, there is a *direct* dependence of preferences on the descriptions of the consequences.

The third and last set of results motivating our axiomatic framework comes from the growing trend in behavioral decision research that demonstrates how d_{mo}s' notions of equivalence between decision problems are not necessarily shared by the d_{ma}s, because the former's *choice* of one description over another may "leak" information about the decision situation that the latter may infer. For instance, Sher and McKenzie (2008, pp.89-90) first put their subjects into the shoes of the d_{ma} by eliciting their choices and strength of preferences (on a scale) between the two programs in a mixed frame (i.e., $\{A''', B'''\}$). Then they put them into the shoes of the d_{mo} using their "embedded creativity" experimental methodology (Sher and McKenzie, 2006) where subjects construct descriptions of the consequences in order to present the problem to a d_{ma}. The main result is that among those who, as d_{ma}, prefer the sure program there is, as d_{mo}, a modal framing of it as "200 saved", and the greater the strength of preference the more likely this occurs. This suggests that information about the d_{mo}'s preferences "leak" in the descriptions of the consequences. van Buiten and Keren (2009) also put their subjects in the shoes of the d_{mo} in the Asian Disease, by investigating their choices of frame, i.e., $\{A, B\}$ *versus* $\{C, D\}$, while asked to promote either the sure program *or* the risky program. They found that d_{mo}s choose $\{A, B\}$ if they have to promote the sure program and $\{C, D\}$ if they have to promote the risky one *only when* they are put in the shoes of the d_{ma}s by considering both frames in a separate evaluation mode (i.e., sequentially $\{A, B\}$ then $\{C, D\}$) *but not when* they consider them in a joint evaluation mode (i.e., simultaneously $\{\{A, B\}, \{C, D\}\}$). Hence, in the former case, the d_{mo} leaks an implicit information about the decision situation, namely an implicit recommendation imposed on him.

Besides preferences and implicit recommendations, other studies have documented leakage of information about reference point, trustworthiness and other choice-relevant characteristics of decision problems[7]. The general argument takes an information theoretic form. d_{mo}s' *choice* of one presentation over another may send signals to d_{ma}s in the form of tacit information about the decision situation that are not contained in the literal messages constituting the decision problems. What exactly is this leaked information about is highly contingent upon subtle details of the linguistic interaction between the d_{mo} and the d_{ma}. If "psychologically salient properties recruit congruent linguistic terms", then information leakage is likely to be systematic through the choices of descriptions, hence informational leaking cannot be dismissed as random noise (Sher

[5]Intuitively, it can be argued that this is not a very surprising result. Think of fancy restaurants and customers spending so much efforts and money to respectively make and consume well presented food, with sometimes very weird and long descriptions on the menus. For further nonconventional evidence supporting this intuitive argument, see the story that made the news in October 2014 about McDonald's food being untruthfully presented as "organic food" to food experts for tests, most of whom declared that it tasted way better than McDonald's, e.g., http://www.dailymail.co.uk/news/article-2804875/Dutch-pranksters-fool-food-experts-organic-convention-believing-MCDONALD-S-actually-new-healthy-dish.html, last consulted 23/01/2016.

[6]The distinction between joint and separate evaluation modes is discussed by Hsee (1996). On the central role played by the "saved" framing of the sure program in the Asian Disease see also van Buiten and Keren (2009, experiment 3). De Martino et al. (2006) found the same framing effects in a monetary version of the Asian Disease where only the description of sure consequence varies across frame; see Mandel and Vartanian (2012) for further evidence using DeMartino et al.'s designs in line with the next set of results.

[7]See McKenzie and Nelson (2003), Sher and McKenzie (2006; 2008; 2011), McKenzie et al. (2006), Keren (2007; 2011), van Buiten and Keren (2009), Mandel (2012; 2015), Tombu and Mandel (2015), Kühberger and Tanner (2010), Kühberger and Gradl (2013); and Bowles and Polanía-Reyes (2012) for a link with social preferences. See also Frisch (1993) for an early demonstration of the non-equivalence of frames in framing experiments from the d_{ma}'s perspective.

and McKenzie 2006, p.488). If the d_{ma} has a "sensitivity to subtle linguistic cues" (ibid, p.488), then he absorbs this information. Exactly how remains unclear. It does not seem to be a classic inductive inference because creativity and imagination[8] are involved in the process of going beyond literal information. But at the same time it seems to be an unconscious process, "[o]therwise, the non-equivalence of attribute frames would have been self-evident prior to our analysis, and no disturbing conclusions about human rationality would have been drawn from attribute framing effects" (p.489). Following Sher and McKenzie (2006, pp.491), it can be argued that "framing effects are best understood, not as paradoxes of rationality [when the d_{ma} correctly infers a choice relevant information], but as paradoxes of measurement": "framing effects without information equivalence raise the question of whether the analysis of preferences is being undertaken at sufficiently high resolution" (p.492).

At the axiomatic level, it seems reasonable only to increase the resolution of the analysis a little bit, just to take account of the fact that different descriptions of the same object of choice can have different informational contents. We propose an axiomatic framework to account for this in the next section.

3 The framework

Our framework has four primitives. The first one is X the nonempty set of objects of choice. It elements are descriptions of objects of choice instead of objects of choice. For the construction of the framework, our objects of choice are going to be "described consequences", in contrast to the standard model's "consequences". Furthermore, we will work on static decision problems under certainty because violations of description invariance can occur at this basic level, e.g., the attribute framing discussed above and other framing effects discussed later. Abstracting from risk, the Asian Disease provides a handy illustrative example throughout, which we will use with the following notation. Consequences are denoted by letters within brackets, e.g., $[x]$, described consequences are denoted by letters with subscripts differentiating different descriptions of a same consequence, e.g., x_1, x_2, and the symbol $\equiv$ reads 'can be empirically instantiated as'; see Table 1.

Consequences	Described consequences
$[x]$	$x_1 \equiv$ "200 people will be saved"
	$x_2 \equiv$ "400 people will die"
	$x_3 \equiv$ "400 people will not be saved"
	$x_4 \equiv$ "200 people will not die"
$[y]$	$y_1 \equiv$ "600 people will be saved"
	$y_2 \equiv$ "nobody will die"
	$y_3 \equiv$ "600 people will not be saved"
	$y_4 \equiv$ "600 people will not die"
$[z]$	$z_1 \equiv$ "no people will be saved"
	$z_2 \equiv$ "600 people will die"
	$z_3 \equiv$ "600 will not be saved"
	$z_4 \equiv$ "600 people will not die"

All 1s and 2s are fom Kahneman and Tversky's (1981) original problem, x_3 and x_4 are from Kühberger (1995), the others 3s and 4s have not yet been implemented to the best of our knowledge.

Table 1: Consequences and Described consequences in Asian Disease problems

The second primitive is a non-trivial binary relation $\approx$ on X. Two described consequences related by this binary relation are such that, from the "decision modeler" (d_{mo}) perspective, they are two descriptions of the same consequence (and conversely). Observe that $\approx$, by definition, is an equivalence relation; if $x \approx y$, we say that x and y are *consequentially equivalent*. In other words, the d_{mo}'s perspective defines equivalence classes of described consequences. By contrast with the standard model where it is implicitly assumed that d_{ma} and d_{mo} share the same perspective on

[8] Possibly through narrativity, see Igou (2011) and Akerlof and Snower (2016).

what counts as a consequence, we want the d_{ma}'s perspective to be analytically free from the d_{mo}'s, at least within the primitives of our model.

The third primitive is a partial binary operation $\circ$ on X. If $x \approx y$ we may wish to consider them, taken together, as forming a single described consequence (i.e., a single object of choice under certainty) denoted $x \circ y$ and called the *concatenation* of x and y. The concatenation $x \circ y$ is the joint description of the described consequences x and y, and is consequentially equivalent to either one of them. To illustrate with the Asian Disease example, if the d_{mo} considers that "200 people will be saved" (x_1) and "400 people will not be saved" (x_3) are two different descriptions of the same consequences, i.e., are consequentially equivalent ($x_1 \approx x_3$), then they can be concatenated and the concatenation is itself consequentially equivalent to either one of the two described consequences. In other words, we have three different descriptions of the same consequence: x_1, x_3 and $x_1 \circ x_3 \equiv$ "200 people will be saved and 400 people will not be saved".

Definition 1. Let X be a nonempty set, $\approx$ a nontrivial binary relation on X, and $\circ$ a partial binary operation on X. The triple $\langle X, \approx, \circ \rangle$ is a *descriptive structure* iff $\approx$ is an equivalence relation and for all $x, y \in X$, $x \approx y$ iff $x \circ y$ is defined; and when both conditions hold $x \circ y \approx x$.

Three related points need to be emphasized. First, the concatenation of described consequences is different from the concatenation of consequences. The former is akin to the combination of information about one consequence. The latter is akin to the combination of at least two consequences. This contrast can be illustrated by a comparison with the operation of joint receipt (Luce and Fishburn, 1991). While the joint description of one glass of water "half-full" with one glass of water "half-empty" is the description of a glass of water "half-full half-empty", the joint receipt of one glass of water half-full with one glass of water half-empty is a full glass of water. [9]

Second, the operation of concatenation is not to be confused with the operations of addition or multiplication (it can only define these operations if more structure is added). Thus, concatenation of descriptions does not necessarily imply the addition of information, it just explicitly puts next to each other what the d_{mo} takes to be consequentially equivalent. In other words, the concatenation of information does not necessarily imply more information.

Third, the operation of concatenation is not an algorithm for the translation between (1) the mathematical representations of described consequences in the formal language of our setup and (2) the empirical description of consequences in the ordinary language of, say, an experimental setup testing framing effects. This is not the case because, by contrast with formal languages, the concatenation of two described consequences in ordinary language is more vague (there are multiple representations of a concatenation) but also more efficient (recursivity within the representations avoids redundant repetitions which decreases the amount of symbols). For instance, take $x \equiv$ "a glass of water half-full" and $y \equiv$ "a glass of water half-empty". In ordinary language, it would be natural that $x \circ y \equiv$ "a glass of water half-full and half-empty" or "a glass of water half-full or half-empty" or "an half-full half-empty glass of water " or yet other formulations. But it is very unlikely that such formulations include "a glass of water half-full a glass of water half-empty" or "a glass of water half-full, a glass of water half-empty". These are confusing, and implied by the operation of concatenation taken as a translation algorithm. Therefore, our framework is agnostic about the *exact* empirical counterparts of the concatenation operation $\circ$. This is in part why we said that $\equiv$ reads as '*can* be empirically instantiated as'(and not 'is necessarily instantiated as'), which allows $\circ$ to be empirically described by "and", "or", ",", " " (just a space), etc. We just want to formally capture the possibility condition of framing effects: the simple fact that one consequence can be presented under different descriptions, and that some of these descriptions are combinations of other descriptions.

Nevertheless, common sense derived from our everyday uses of ordinary language, and, most importantly, the large experimental literature on framing effects, provide fairly clear guidance for the empirical application of our framework. As an illustration, consider the structure of the sure thing in Asian Disease experiments. A described consequence involves expressions of four dimensions of the consequence, i.e., (1) of a quantity ("200", "400"), (2) of what is being quantified ("people"), (3) of an attribute of what is being quantified ("saved", "die") and (4) of the implicit

[9] Assuming fungibility of water, which seems less problematic than fungibility of money, and that consequences are only about water, not its physical container.

assertion of the attribute ($\emptyset$) or the explicit negation of it ("will $\emptyset$ be saved"/"will $\emptyset$ die", "will not be saved"/"will not die"). Empirically speaking, what could be called the 'minimal descriptive structure' of a consequence requires a combination of at least one expression of each dimension; and concatenations of described consequences often involve the repetition, and always requires a combination, of at least two different expressions of (at least) one dimension: "200 people will be saved and 400 people will not be saved", (or possibly) "400 people will die and 200 will not". Where we are formally agnostic is on the empirical manifestation of those dimensions, the repetition of those dimensions, and the combination order of those dimensions.

To be clear on the interpretations of our primitives, let us contrast three possibilities. Firstly, if X would be the set of all typographical symbols, the concatenation operation could be interpreted as forming words and sentences, e.g., $w \circ i \circ l \circ l \circ \circ d \circ i \circ e \equiv$ "will die". Secondly, if X would be the set of all expressions of the four dimensions of the minimal descriptive structure of a consequence, the concatenation operation could be interpreted as forming described consequences, e.g., with $x \equiv$ "200", $y \equiv$ "people" $z \equiv$ "will be saved", $x \circ y \circ z \equiv$ "200 people will be saved". Clearly, both these levels of analysis are not fitted for economics[10]. Hence, the third interpretation is the one we take in this paper, where X is the set of described consequences, empirically corresponding the expressions of *at least* one instance of each of the four dimensions of the minimal descriptive structure of a consequence. What we want to formalize is that described consequences can *generate* further described consequences, by concatenation. Formally, let $\langle X, \approx, \circ \rangle$ be a descriptive structure, $x \in X$ be a described consequence and $[x]_\approx := \{y \in X \mid y \approx x\}$ be the equivalence class of x by $\approx$.

Definition 2. $[x]_\approx$ is said to be generated by $D \subset [x]_\approx$ iff every element of $[x]_\approx$ can be expressed as the concatenation of finitely many elements of D; and when both conditions hold, $[x]_\approx = \langle D \rangle$ and the elements in D are called *generators*.

Identifying potential generators in the experimental study of framing effects is useful to get a picture of the conditions under which the effects hold and vary across different descriptions of the same consequences. For example, in the literature on the Asian Disease problem, D contains at least four generators of the sure program, namely x_1, x_2, x_3 and x_4 in Table 1. Their concatenations generate further descriptions of the sure program, as shown in Table 2.

	Kahn. and Tver. (1981)	Kühb. (1995, exp.1)	Kühb. (1995, exp.2)	Mell. and Lo. (2007, p.362)
As	x_1	x_3	$x_1 \circ x_3$	$x_1 \circ x_2$
Cs	x_2	x_4	$x_2 \circ x_4$	

Table 2: Generators and their concatenations in Asian Disease problems

The last primitive is the d_{ma}'s preference relation $\succsim$ on X. $x \succsim y$ means that x is at least as desirable as y. As usual, the strict preference $\succ$ and indifference $\sim$ relations are respectively defined as the asymmetric and symmetric parts of $\succsim$. We emphasize that $\succsim$ is defined over described consequences. This contrasts with the standard model, where d_{ma}'s preferences are defined over consequences. That is, in the standard model, the d_{ma}'s preferences are implicitly defined over the d_{mo}'s partition of X, i.e., on $X/\approx$, the set of equivalence classes of X under $\approx$. Notice that by restricting the concatenation of described consequences to the d_{mo}'s perspective (i.e., the operation of concatenation can only be performed between what is consequentially equivalent, that is, between what is contained within equivalence classes of X defined by $\approx$), such concatenations are logically independent from d_{ma}'s preferences, which are defined over X i.e., over all described consequences irrespective of their equivalence classes from the d_{mo}'s perspective. This can be seen as a prerequisite to avoid the standard model's problem of logical omniscience, i.e., that the d_{ma} knows every logical implications of his knowledge (Lipman, 1999); or more precisely, that the d_{ma} knows and agrees with every implications of his knowledge derivable from the d_{mo}'s logic when the

[10]Maybe the former level would be more fitted for linguists, logicians or philosopher of language. And the work done by psycholinguists or psychologists inspired by psycholinguistics on framing effects can be interpreted as been done at the latter level (Keren, 2011a, esp. Part. II). Though some of their results may be useful for further modeling or empirical work, there is a potential lack of tractability worth noting due to, as *they* show, the potentially infinite makings of synonymous expressions (e.g., "alive", "perish") and the complex asymmetries between negations and contraries.

latter is identified with a decision theorist.[11] That furthermore implies, in the standard model, that the d_{ma}'s indifference relation implicitly holds among all the elements of a given equivalence class, that is, among all different descriptions of the same consequence.

Within the present framework, it is possible to characterize three relevant cases of preferences under framing with different degrees of discrimination of consequences. The first case corresponds to what is usually called *description invariance* in the literature. This axiom is invisible in the standard model because the latter works in an *unframed descriptive structure* where preferences are only representable over consequences, e.g., over the d_{mo}'s equivalence classes of consequences $[x]_{\approx}$, $[y]_{\approx}$, etc. but not over described consequences, i.e., over x_1, y_1, x_2, y_2, etc. By virtue of the structure of our choice set, we are always working in a *framed descriptive structure* where preferences over described consequences are representable, hence description invariance can be made formally explicit. The standard model's case where it is impossible to discriminate different descriptions of a given consequence corresponds here to the case where the d_{ma}'s indifference relation holds among all the elements of a given equivalence class:[12]

$$\forall x, y \in X, \quad x \approx y \Rightarrow x \sim y.$$

The second case is when regardless of the d_{ma}'s preferences among all possible descriptions of the same consequence, his preferences among different consequences are "ordered". We call this case *tidy descriptive dependence*. Formally,

$$\forall x, y, z \in X, \text{ suppose } x \approx z \text{ and } x \succsim z, \text{ then } x \succsim y \text{ and } y \succsim z \Rightarrow y \approx x.$$

Tidy descriptive dependence prevents cases where the d_{ma} prefers one description of a consequence to another description of *another* consequence and yet prefers a further description of the latter consequence to a further description of the former (i.e., a framing effect under certainty, e.g., from Table 1, $y_1 \succsim x_1$ and $x_4 \succsim y_2$). This kind of case is the third one we distinguish here and call *untidy descriptive dependence*. We characterize it only informally because there is no formal relation within the set of consequences and there is no characterizable logic without further hypotheses about the d_{ma}'s decision making process.

3.1 Axioms and results

In this section, we provide an axiomatic characterization of the standard model's unframed descriptive structure. We then suggest how to weaken the structure to allow for a formal representation of framing effects compatible with empirical results and intuitions. We first present axioms that will be required in both structures.

3.1.1 Basic axioms

We first state our axioms and then emphasize their empirical interpretations. We use a structure inspired by the so-called "extensive structure" from Krantz et al. (1971) – the distinctions being that we add the idempotency axiom, our concatenation operation is not total and we do not use any Archimedean axioms.

[11] Logical omniscience is however plausible in *some* situations such as the glass example. If a glass of water is described as haf-full to the d_{ma}, it is very likely that he can unproblematically infer that the glass is also half-empty. But this is less plausible in other more complex and economically relevant situations such as the Asian Disease examples, and other ones discussed in the literature on framing effects. A non-trivial and intuitive example of such situations involve inflation, where the d_{mo}'s perspective establishes *equivalence* of purchasing powers (i.e., economic *consequences*) in terms of income and prices, but that equivalence is often not shared from d_{ma}'s perspective; i.e., the inference of equivalent purchasing powers at different levels of income and prices (the combinations of which are then just different descriptions of the same economic consequences) is not so straightforward. See Arrow (1982) for an early discussion, and Shafir et al. (1997) for empirical support and further discussion.

[12] We are not the first one to *state* description independence in this way, see, e.g., Bacharach (2003, p.65), Blume et al. (2013, p.9), Lerner (2014, p.40) and esp. Giraud (2004a, p.52). We discuss these in the next section, where the influence of Giraud's work on our framework is acknowledged. It can be argued that '*the*' standard model is in fact characterized by $x \approx y \Rightarrow [x] \sim [y]$. If we would have defined preferences over consequences, this would be the natural way of characterizing description invariance; however, with our preference defined over described consequences, we feel that the above characterization is more natural.

Axiom 1 (Descriptive structure). $\langle X, \approx, \circ \rangle$ *satisfies Definition 1.*

Axiom 2 (Weak order). $\succsim$ *is complete and transitive.*

Axiom 3 (Weak commutativity). *If* $x \circ y$ *and* $y \circ x$ *are defined, then* $x \circ y \sim y \circ x$.

Axiom 4 (Weak associativity). *If* $(x \circ y) \circ z$ *and* $x \circ (y \circ t)$ *are defined, then* $(x \circ y) \circ z \sim x \circ (y \circ z)$.

Axiom 5 (Weak idempotency). *If* $x \circ x$ *is defined, then* $x \circ x \sim x$.

Axiom 1 corresponds to the structure of objects of choice from the d_{mo}'s point of view. Its empirical interpretation has already been given above. Axiom 2 is a standard requirement about d_{ma}'s preference relation over the objects of choice, that is, his preference relation is a weak order.[13] Axioms 3 and 4 state that whenever the concatenation is defined, it does not matter for the d_{ma} in what order described consequences are concatenated. The order of the described consequences being concatenated (weak commutativity) and the order of the concatenations of described consequences (weak associativity) may be reversed without altering informational content, hence the d_{ma}'s indifference. Notice that these axioms are plausible within the scope of the framing effects with which we are concerned; but would need to be relaxed to study order effects under framing, with which we are not concerned (in any case, empirically, it has been shown that there is no impact of the order of presentation in framing effects of the Asian Disease type).

Example 1. From Table 1, $x_1 \circ x_3$, $x_3 \circ x_1$, $(x_1 \circ x_3) \circ x_2$ and $x_1 \circ (x_3 \circ x_2)$ can be defined by the d_{mo} to correspond to:

$$x_1 \circ x_3 \equiv \text{ “200 people will be saved and 400 people will not be saved”,}$$
$$x_3 \circ x_1 \equiv \text{ “400 people will not be saved and 200 people will be saved”.}$$

and

$$(x_1 \circ x_3) \circ x_2 \equiv \text{ “200 people will be saved and 400 people will not be saved and also 400 people will die”,}$$
$$x_1 \circ (x_3 \circ x_2) \equiv \text{ “200 people will be saved and also 400 people will not be saved and 400 people will die”.}$$

Hence, what remains implicit in the standard model is that because all the generators (x_1, x_2, x_3) and the concatenations are consequentially equivalent, the d_{ma} is indifferent between the order of the described consequences being concatenated ($x_1 \circ x_3 \sim x_3 \circ x_1$ by weak commutativity) and between the order of the concatenations of described consequences ($(x_1 \circ x_3) \circ x_2 \sim x_1 \circ (x_3 \circ x_2)$ by weak associativity).

Axiom 5 states that the d_{ma} is indifferent between a described consequence and the concatenation of this described consequence with itself. Formally, since the concatenation of a described consequence with itself is a logical possibility, it is necessary to include it in the basic setting. Intuitively, whether repetitions of described consequences do or do not increase informational content is an open question. For the d_{mo}, repeating the same information does not increase the informational content about the consequence. But the very fact of an information being repeated might impact the informational content about the consequence from the d_{ma}'s point of view, rationally or not. In any case, notice that such repetitions are not involved in the framing effects we are interested in, and there has been no empirical test of this to our knowledge.[14]

[13] Notice that axiom 2 can be weaken to a preorder (reflexive and transitive binary relation) quite easily. In that case, axioms 3-5 would implicitly assume a minimal degree of comparability among different descriptions of a same consequence. This would be in line with the growing body of authors who consider completeness over consequences not to be a sound requirement of rationality. With a partial order, our results on unframed descriptive structure would also show that, if the preferences of the d_{ma} are not complete, they are still complete within classes of consequentially equivalent descriptions.

[14] A generalization of our framework should indeed dispense with the idempotency axiom as it is clear from both cognitive and social psychology that d_{ma}s are anything but indifferent to informational repetitions and redundancies (Cacioppo and Petty, 1989; Judd and Brauer, 1995; Moons et al., 2009; Kuhl and Anderson, 2011; Mulligan and Peterson, 2013; English and Visser, 2014). Technically, such weakening might not be too difficult: the underlying

3.1.2 Framed descriptive structure with description invariance

The following axiom (also implicit in the standard model), together with the basic ones presented above, make our framework equivalent to the standard model, i.e., preference relations can only hold between consequences but not between different descriptions of the same consequences, among which the d_{ma} is necessarily indifferent.

Axiom 6 (Independence of common description)**.** *If $x \circ z$ and $y \circ z$ are defined, then $x \sim y$ iff $x \circ z \sim y \circ z$.*

The independence of common description (ICD) axiom states that d_{ma}'s indifference between two described consequences is not affected by concatenation with a common described consequence. And conversely, d_{ma}'s indifference between two concatenations sharing a common described consequence is not affected by cancellation of the common described consequence.

The following implication of ICD is the crucial one that prevents a formal account of the framing effects we are interested in. We already said that the standard model cannot (without *ad hoc* assumptions) capture that different descriptions of the same consequence can convey different information; it only captures that different descriptions of the same consequence can convey the same information. Thus, it cannot discriminate between the following three cases. First, when different information are conveyed by different descriptions, if the information of one description is included in the information of another description, then we can say that the latter description *absorbs* the former: the informational content of their concatenation is identical with the informational content of the latter (absorbing) description. Second, when the same information is conveyed by different descriptions, then these descriptions are *mutually absorbent*. And third, when the information conveyed by a description absorbs all possible other information conveyed by other descriptions, we say that the former description is *maximally absorbent*. Formally, $x \in X$ absorbs $y \in X$ if and only if, $x \circ y \sim x$; $x, y \in X$ are mutually absorbent if and only if, $x \circ y \sim x \sim y$; $x \in X$ is maximally absorbent if and only if, for all $y \in X$, $x \circ y \sim x$. ICD states that either there exists no absorbing description or that all descriptions are maximally absorbent. But together with Axioms 1-5, ICD implies that every description of the same consequence is maximally absorbent, so that all description of the same consequence are mutually absorbent. In other words, the standard model cannot discriminate relative absorbency among different descriptions of the same consequence.

Example 2. From Table 1, $x_1 \circ x_2$ and $x_3 \circ x_2$ can be defined by the d_{mo} to correspond to:

$$x_1 \circ x_2 \equiv \text{ “200 people will be saved and 400 people will die”,}$$
$$x_3 \circ x_2 \equiv \text{ “400 people will not be saved and 400 people will die”.}$$

ICD implies that if the d_{ma} is indifferent between these two concatenations of described consequences, then he is indifferent between the two described consequences “200 people will be saved” and “400 people will not be saved”, *and conversely*.

When Axioms 1-6 hold, we are in a *framed descriptive structure* with *description invariance*, which is equivalent to the standard model's *unframed descriptive structure*.

Proposition 1. *Let X be a nonempty set, $\approx$ and $\succsim$ nontrivial binary relations on X, and $\circ$ a partial binary operation on X. Suppose that the quadruple $\langle X, \approx, \succsim, \circ \rangle$ satisfies axioms 1-6. Then*

$$\forall x, y \in X, \quad x \approx y \Rightarrow x \sim y.$$

All proofs are in the appendix.

algebraic structure associated to a consequence will no longer be a band but a commutative semigroup. Empirically, we did conduct some informal classroom experiments on the Asian Disease to check whether some of our axioms were or not respected. For the idempotency axiom, the results are very sensitive to the empirical counterpart of the concatenation. Roughly, using just a coma without logical connectors to concatenate the repetition could change the preferences between this concatenation and just one instance of the generator (and with Tversky and Kahneman's original generators, the overall preference reversal goes in the same direction than in their); but the use of a logical connector that made it clearer that it was a repetition (e.g., “so that”) did not change the preferences (with Tversky and Kahneman's original generators, the overall preference reversal disappeared). We are currently preparing lab experiments on framing effects along these lines.

3.1.3 Framed descriptive structure with weaker descriptive dependence

We will propose two ways of characterizing a *framed descriptive structure* with *descriptive dependence* in order to suitably represent the framing effects we are interested in, i.e., two ways our model can depart from the standard model's unframed descriptive structure. This subsection presents what is common to both ways; the next one presents the two characterizations and explains their differences. We keep the basic axioms and propose to weaken the ICD axiom in the same way for both characterizations. The ICD axiom can be decomposed in two parts:

Axiom 6.1 (Weak substituability)**.** *If $x \circ z$ and $y \circ z$ are defined, then $x \sim y$ implies $x \circ z \sim y \circ z$.*

Axiom 6.2 (Weak simplifiability)**.** *If $x \circ z$ and $y \circ z$ are defined,, then $x \circ z \sim y \circ z$ implies $x \sim y$.*

Example 2 above already illustrated the empirical interpretation of weak simplifiability, weak substituability was only implicitly illustrated through the "and conversely" clause. Weak substituability is the "if" direction of ICD: if the d_{ma} is indifferent between two informational contents, then the simultaneous adjunction of another (consequentially equivalent) informational content preserves indifference. Weak simplifiability is the "only if" direction of ICD: if the d_{ma} is indifferent between two informational contents conveyed by two descriptions of the same consequence, then the retrieval of a common informational content preserves indifference between the remaining informational contents. It is this latter part that we drop to give a formal account of framing effects. To see why we drop weak simplifiability, consider the following example.

Example 3. From Table 1, $x_1 \circ (x_1 \circ x_2)$ and $x_2 \circ (x_1 \circ x_2)$ can be defined by the d_{mo} to correspond to:

$x_1 \circ (x_1 \circ x_2) \equiv$ "200 people will be saved and also 200 people will be saved and 400 people will die",
$x_2 \circ (x_1 \circ x_2) \equiv$ "400 people will die and also 200 people will be saved and 400 people will die".

Firstly, the basic axioms together with weak substituability imply that the d_{ma} is indifferent between these two described consequences.[15] Such indifference here is plausible, both intuitively because the concatenations can be seen as clarifying the informational content implied by either x_1 or x_2 and empirically because experimental results on the Asian Disease with the sure thing being completely described indeed does not reveal preference reversals (cf. previous section).

Secondly, if weak simplifiability were to hold, it would further imply that the d_{ma} is indifferent between x_1 and x_2, i.e., between "200 people will be saved" and "400 people will die". Recall that, as explained in the previous section, this is descriptively inaccurate from the classical results of Tversky and Kahneman (1981); and normatively questionable from the results of Sher and McKenzie (2008) on informational leakage. We have therefore good reasons to drop weak simplifiability.

Without the weak simplifiability axiom, the result from the previous subsection does not hold anymore: the d_{ma} is no longer necessarily indifferent between different descriptions of the same consequence. Consequently, he may exhibit preferences within a d_{mo}'s equivalence class, e.g., it is possible that, within $[x]_{\approx}$, $x_1 \succ x_2$ (or the reverse). It is because such cases of *descriptive dependence* are representable when axioms 1-5 and 6.1 hold (i.e., when 6.2 is dropped) that we say that axioms 1-5 and 6.1 characterize a framed descriptive structure with descriptive dependence. In this structure, it is possible that different descriptions of the same consequence can convey different information, i.e., all descriptions are not necessarily maximally absorbent anymore, especially generators, and the d_{ma} is not necessarily indifferent among them, he can prefer one description to another. But some indifference relations can still be implied.

[15] This is so because by weak idempotence, $x_1 \circ x_1 \sim x_1$ and $x_2 \circ x_2 \sim x_2$. Weak substituability implies that $(x_1 \circ x_1) \circ x_2 \sim x_1 \circ x_2$ and $x_1 \circ (x_2 \circ x_2) \sim x_1 \circ x_2$. Rearranging by weak associativity, weak commutativity and transitivity of $\sim$ imply that

$$x_1 \circ (x_1 \circ x_2) \sim x_2 \circ (x_1 \circ x_2).$$

Proposition 2. *Let X be a nonempty set, $\approx$ and $\succsim$ non-trivial binary relations on X, and $\circ$ a partial binary operation on X. Suppose that the quadruple $\langle X, \approx, \succsim, \circ \rangle$ satisfies axioms 1-5 and axiom 6.1. Then*

$$\forall x, y \in X, \quad x \text{ and } y \text{ are generated by the same generators } \Rightarrow x \sim y.$$

Moreover, for all $x \in X$, if $[x]_{\approx}$ is finitely generated by n generators, then $[x]_{\approx}$ has at most $2^n - 1$ equivalence classes under $\sim$.

Proposition 2 is weaker than proposition 1 in the sense that the d_{ma}'s indifference relation among described consequences is further constrained by their generators. Two different generators of the same consequence are not necessarily mutually absorbent and the d_{ma} is not necessarily indifferent between them. But two different descriptions generated by the same generators are necessarily mutually absorbent and the d_{ma} is necessarily indifferent between them. It also states that if the number of generators are limited within equivalence classes, then the structure of our objects of choice (described consequences) does not entail an infinity of indifference classes for a given consequence: the infinite number of described consequences does not allow an infinity of indifference classes (which would be absurd). The number of informational content possibilities is the number of distinct indifference classes, which is limited by the number of generators. This results can therefore be of practical use for experimenters who wish to ensure that they have tested (or at least considered) every logical possibility of different framings of the same decision problem.

Example 4. With x, y defined as in example 1, there exists, despite the infinite number of described consequences they can generate, at most 3 distinct indifference classes for the d_{ma}: $[x_1]_{\sim}$ generated by x_1, $[x_2]_{\sim}$ generated by x_2 and $[x_1 \circ x_2]_{\sim}$ generated by x_1 and x_2 (notice that they are all subclasses of one d_{mo} equivalence class, $[x]_{\approx}$ here). As already noted in the discussion of the idempotency axiom, it is an open question whether such repetitions of exactly the same descriptions of consequences (i.e., the empirical counterparts of these concatenations) does or does not increase informational content. This is not what is at stake in the framing effects we are interested in. What is at stake is the possibility of representing preferences between (but not within) elements of, respectively, $[x_1]_{\sim}$, $[x_2]_{\sim}$, and $[x_1 \circ x_2]_{\sim}$.

In the remaining of the paper we work under two hypotheses. First, generators are finite within equivalence classes; we just argued for the plausibility of this hypothesis. Second, the number of consequences is finite (as in a given experimental setup, or in a given supermarket). This greatly facilitates the exposition of our next results.[16]

3.1.4 Tidy and untidy descriptive dependence

The following two main ways of characterizing descriptive dependence are worked out with axioms 1-5 and 6.1 holding. We wish to discriminate between cases of *tidy descriptive dependence* where the d_{ma}'s preferences over consequences are necessarily well-ordered, e.g., $[x]_{\approx} \succ [y]_{\approx}$ such that $x_1 \succ x_1 \circ x_2 \succ x_2 \succ y_1 \succ y_4 \succ y_2 \circ y_3$, from cases of *untidy descriptive dependence*, where preferences over consequences are not necessarily well-ordered, e.g., $[x]_{\approx} \succ [z]_{\approx}$ and $[z]_{\approx} \succ [x]_{\approx}$ (tantamount to a framing effect or preference reversal under certainty) as with $x_1 \succ x_1 \circ x_2 \succ z_3 \succ x_2$. The following two axioms characterize the discrimination between tidy and untidy descriptive dependence in so far as when they hold, it is necessarily the case that the d_{ma}'s preferences are only tidily dependent to the descriptions of consequences, while when they are both dropped, his preferences can exhibit untidy descriptive dependence.

[16] The issue of the finite or infinite number of consequence is related to the issue of whether or not to include and Archimedean axiom in our framework. If such an axiom is included, we would have a less extensive structure in Krantz et al.'s sense, allowing us to explore utility representation which we do not explore in the section dedicated to that below, notably cases such as $u([a \circ b]_{\sim}) = u([a]_{\sim}) + u([b]_{\sim})$ or $u([a \circ b]_{\sim}) = \lambda u([a]_{\sim}) + (1 - \lambda) u([b]_{\sim})$. The empirical interpretation of such representations remains an open question. Formally, if we stay under the hypothesis that the number of consequences are finite, then we would need an infinite number of axiom (as in Fishburn 1970, p.42). That would however not be the case anymore if we worked under the hypothesis that the number of consequences are infinite. In any case, at a behavioral level, the empirical interpretations we have given of our framework under the finite consequence assumption and without an Archimedean axiom are enough to deal with all the framing effects presented in the previous section and to be presented in the next one. Finally, with an infinite number of consequences and without an Archimedean axiom, we would have to deal with lexicographic preferences.

Axiom 7 (Non-triviality)**.** *If $x \not\approx y$, then there exist $x' \in [x]_{\approx}, y' \in [y]_{\approx}$ such that $x' \not\sim y'$.*

Axiom 8 (Independence of common description across consequences)**.** *If $x \not\approx z$, $x \circ y$ and $z \circ t$ are defined, then $x \succsim z$ iff $x \circ y \succsim z \circ t$.*

Non-triviality requires that for each pair of different consequences there is at least one description of each that actively affect the d_{ma}'s strict preference relation $\succ$. In other words, it ensures that there is at least one way of describing two different consequences so that the d_{ma} is not indifferent between them. Independence of common description across consequences (ICDAC) requires that for each pair of different consequences, neither concatenations nor unyoking of both change the d_{ma}'s preference ordering between them. Note that if ICD holds, ICDAC holds trivially, but not the converse. ICDAC together with non-triviality preclude cases of indifference between pairs of consequences implying indifference among all descriptions of both consequences, which is exactly what we want to avoid regarding strict framings of consequences.

Proposition 3. *Let X be a nonempty set, $\approx$ and $\succsim$ non-trivial binary relations on X, and $\circ$ a partial binary operation on X. Suppose that the quadruple $\langle X, \approx, \succsim, \circ \rangle$ satisfies axioms 1-5, 6.1, 7, and axiom 8. Then*

$$\forall x, y, z \in X, \text{ if } x \approx z \text{ and } x \succsim z, \text{ then } (x \succsim y \succsim z \Rightarrow y \approx x).$$

As said above, when both are dropped, we are within a framed descriptive structure with untidy descriptive dependence. Evidently, we could see further shades of tidiness and untidiness by weakening ICDAC. But we do not need to go so far to capture the strict framings of consequences we are interested in.

3.2 Utility representation and choice function

3.2.1 Representation theorems

The previous results can be translated in terms of utility representations. When only axioms 1-2 or the full set of basic axioms (1-5) hold, under the finiteness assumptions, there exists a real-valued function u on X, which represents d_{ma}'s preferences, that is, for all $x, y \in X$

$$x \succsim y \Leftrightarrow u(x) \geq u(y).$$

To see this, note that the finiteness assumptions imply that X is countable. Hence, representation in this case is a basic result. With the basic axioms, the image of X by u, denoted by $u(X)$, is at most countable. The results of propositions 1 and 2 imply that $u(X)$ is finite when we add ICD or just weak substituability. In the latter case, we are within a descriptive structure with untidy descriptive dependence, and the utilities of a given consequence lie in a bounded set, hence are contained in a finite interval that may overlap with other consequences' finite intervals of utilities. Within a descriptive structure with tidy descriptive dependence, i.e., with axioms 7-8 added, there is no such overlap. The following proposition summarize the relevant properties of the representation theorems (which are practically trivial to prove, see the appendix):

Proposition 4. *Let $\langle X, \approx, \circ \rangle$ be a descriptive structure such that $X/\approx$ is finite and $[x]_{\approx}$ is finitely generated for all $x \in X$. For a binary relation $\succsim$ on X the following four statements hold:*

(i) If $\succsim$ satisfies axioms 1 and 2 (and possibly one or more axioms among 3-5), then there exists an utility function on X that has an at most countable image;

(ii) If $\succsim$ satisfies axioms 1-6, then there exists an utility function on X that has a finite image. Moreover, for all $x, y \in X$, $x \approx y$ implies $u(x) = u(y)$;

(iii) If $\succsim$ satisfies axioms 1-5 and 6.1, then there exists an utility function on X that has a finite image. Moreover, for all $x, y \in X$, if x and y are generated by the same generators, then $u(x) = u(y)$;

(iv) If in addition, to the above conditions, $\succsim$ satisfies axioms 7 and 8, then for all $x, y \in X$, such that $x \approx y$ and $y \succsim x$, for all $z \in X$, $u(z) \in [u(x), u(y)]$ implies that $z \approx x$.

3.2.2 Choice function

The goal now is to characterize the implications in terms of revealed preferences of the framework presented in the previous subsections. Salant and Rubinstein (2008) have provided a framework for what they call "choice with frames". However, none of their examples, applications and discussions consider the classical problem of description-dependence underlying Kahneman and Tversky's original framing effects. It may be thought that extending their framework through ours would allow to deal with violations of description invariance from a revealed preference perspective. This is not the case, and showing how allows to highlight the contribution of our framework.

Salant and Rubinstein extend the traditional approach in terms of choice function on the set of available alternatives A (i.e., a choice problem) from the set of alternatives X by constructing choice functions on pairs (A, f), where f is one frame from a set of frames F. Their framework can be extended to deal with framing effects due to different descriptions of the choice problem as follows. Extend the relation $\approx$ on the set of alternatives to the set of choice problems, denote this extended relation by $\approx_{\mathcal{P}}$. Define the pair $([A]_{\approx_{\mathcal{P}}}, f)$ where $[A]_{\approx_{\mathcal{P}}}$ is a standard choice problem with consequences and f indicates the description under which this choice problem is presented to the d_{ma}. Consider a descriptive structure without concatenation $\langle X, \approx\rangle$ such that X is finite (or equivalently, $X/\approx$ and $[x]_{\approx}$ are finite). Let $\mathcal{P}^*(X)$ be the set of all nonempty subsets of X and $\mathcal{P}^*(X/\approx)$ be the set of all nonempty subsets of $X/\approx$. For nonempty subsets of X, we will use notation $A, B, \ldots$, and for nonempty subsets of $X/\approx$, we will use notation $I, J, \ldots$. The relation $\approx_{\mathcal{P}}$ on $\mathcal{P}^*(X)$ is defined as: for all $A, B \in \mathcal{P}^*(X)$,

$$A \approx_{\mathcal{P}} B \text{ iff } (\forall x \in A, \exists y \in B, x \approx y) \text{ and } (\forall y \in B, \exists x \in A, y \approx x).$$

It is readily seen that the relation $\approx_{\mathcal{P}}$ on $\mathcal{P}^*(X)$ is an equivalence relation. More precisely, $\approx_{\mathcal{P}}$ extends the d_{mo}'s perspective on consequential equivalence in choice problems. A *described choice problem* is a set of descriptions of alternatives. We can illustrate the previous definitions with the following example.

Example 5. Consider three consequences. One inspired from Levin and Gaeth (1988): eating ground beef $[G]$ described either as "25% fat ground beef" g_1 or "75% lean ground beef" g_2. The two other consequences are eating chicken C and eating beans B, for each of which we consider only one description, respectively c and b, which need not to be defined for the purpose of the example. Choosing between ground beef and beans is a choice problem $\{[G], [B]\}$ that can give rise to two different described choice problems: $\{g_1, b\}$ and $\{g_2, b\}$. From the d_{mo}'s perspective, they are two descriptions of the same choice problem: $\{g_1, b\} \approx_{\mathcal{P}} \{g_2, b\}$ because $g_1 \approx g_2$ and trivially $b \approx b$.

Moreover, to a choice problem we can associate all its descriptions, in particular there is a natural correspondence between $\mathcal{P}^*(X)/\approx_{\mathcal{P}}$ and $\mathcal{P}^*(X/\approx)$.

Proposition 5. *Let X be a set and $\approx$ an equivalence relation on X. There exists a bijection r from $\mathcal{P}^*(X/\approx)$ to $\mathcal{P}^*(X)/\approx_{\mathcal{P}}$.*

The above proposition justifies the following definition: an extended choice problem is a pair $([A]_{\approx_{\mathcal{P}}}, f)$ where $[A]_{\approx_{\mathcal{P}}}$ is a standard choice problem with consequences and f is a described choice problem, that is, any subset $A \subset X$ that belongs into $[A]_{\approx_{\mathcal{P}}}$.[17]

Following Salant and Rubinstein's approach, an *extended* choice function c assigns a chosen consequence $[x]_{\approx}$ to every pair $([A]_{\approx_{\mathcal{P}}}, f)$. Given a weak order $\succsim$ on X, the induced extended choice function $c_{\succsim}$ is defined by

$$c_{\succsim}([A]_{\approx_{\mathcal{P}}}, f) = \{[x]_{\approx} \in X/\approx \mid \exists x' \in [x]_{\approx} \cap f,\ x' \succsim y,\ \forall y \in f\}.$$

If X is finite, this set is necessarily nonempty. To see how their approach fails to capture framing effects violating description invariance, we propose the following counter example. From the previous example, suppose that the d_{ma} likes meat very much and derives more utility from

[17] More formally, a standard choice problem with consequences is a subset $I \subseteq X/\approx$. By proposition 5, there exists an unique $[A]_{\approx_{\mathcal{P}}}$ such that $I = r^{-1}([A]_{\approx_{\mathcal{P}}})$, consequently with a slight abuse of notation and for the sake of clarity we denote a standard choice problem with consequences by $[A]_{\approx_{\mathcal{P}}}$ instead of I.

what looks healthy, so that he prefers the ground beef described as "75% lean" over chicken over beans over the ground beef described as "25% fat", i.e., he exhibits the following profile of preferences: $g_2 \succ c \succ b \succ g_1$. Hence his preferences are untidy, there is no possible weak order on $X/\approx$. Furthermore the order on the described consequences violates Salant and Rubinstein's property ensuring the existence of an equivalence between an extend choice function and a weak order on X. They call this property γ^+-extended, which, adapted to deal with different descriptions of choice problems, can be stated as follow: if $c([A]_{\approx_\mathcal{P}}, f) = [x]_\approx$, $c([B]_{\approx_\mathcal{P}}, g) = [y]_\approx$ and $[y]_\approx \in [A]_{\approx_\mathcal{P}}$, then there exists a described choice problem h such that $c([A]_{\approx_\mathcal{P}} \cup [B]_{\approx_\mathcal{P}}, h) = [x]_\approx$. To illustrate a violation, note that according to d_{ma} preferences: $c(([B],[G]),(b,g_1)) = [B]$, $c(([C],[G]),(c,g_2)) = [G]$ and $[G] \in ([B],[G])$. However, $c(([B],[C],[G]),(b,c,g_1)) = [C]$ and $c(([B],[C],[G]),(b,c,g_2)) = c(([B],[C],[G]),(b,c,g_1,g_2)) = [G]$, hence $[B] \notin c(([B],[C],[G]),h)$ for all description h of the choice problem $([B],[C],[G])$.

To bypass the weaknesses of γ^+-extended to deal with descriptions of choice problems we propose the following modification:

γ^+-extended for descriptions: If $[x]_\approx \in c([A]_{\approx_\mathcal{P}}, f)$, $[y]_\approx \in c([B]_{\approx_\mathcal{P}}, g)$, $[x]_\approx \cap f$ is a singleton, and $[y]_\approx \cap g \subseteq f$, then for all described choice problem $h \in [A]_{\approx_\mathcal{P}} \cup [B]_{\approx_\mathcal{P}}$ such that $[x]_\approx \cap f \subseteq h \subseteq f \cup g$, it holds that $[x]_\approx \in c([A]_{\approx_\mathcal{P}} \cup [B]_{\approx_\mathcal{P}}, h)$.

This modification of γ^+-extended states roughly that if we are under the 'same frame' (e.g., *either* under the save frame *or* under the die frame in the Asian Disease), then there are no preference reversals. With this modification, Salant and Rubinstein's framework can deal with all preference reversals violating description invariance. However, the weak order generated by an extended choice function is not necessarily unique, preventing an account of tidy descriptive dependence.

Proposition 6. *Let X be a finite choice set of described consequences such that $X/\approx$ is not trivial and c an extended choice function. Then, c is an extended choice function that satisfies γ^+-extended for descriptions if, and only if, there exists a weak order $\succsim$ on X (not necessarily unique) such that $c = c_{\succsim}$.*

The non uniqueness of the generated weak order makes it impossible to represent the modal preference of Gaeth and Levine's d_{ma}s, i.e., $g_2 \succ g_1$ if there are no other consequences between them. For instance, the following two profiles of preferences generate and are generated by the same extended choice function whereas the behavior is indeed not the same: $g_2 \succ g_1 \succ b$ and $g_1 \succ g_2 \succ b$. That is, their framework can represent $g_2 \succ g_1$ only by transitivity if $g_2 \succ c \succ g_1$.

To sum up, Salant and Rubinstein's framework, when extended to deal with descriptions of choice problems, does not allow to discriminate between the unframed descriptive structure of the standard model and the framed descriptive structure with tidy descriptive dependence of our framework. That is, if γ^+-extended for descriptions is satisfied we are necessarily in a case of untidy-descriptive dependence if framing effects are observed, but if no such framing effects are observed, then we cannot know whether we are in a case of description invariance or tidy-descriptive dependence.

In order to discriminate between tidy and untidy descriptive dependence in terms of revealed preferences, we propose to translate all our axioms in terms of choice functions. Let X be a set of described consequences, a choice function for X is a correspondence c from the set of all finite and nonempty subsets of X to X such that $\emptyset \neq c(S) \subseteq S$ for all $S \subseteq X$. The following conditions are translations of our axioms on preferences to be imposed on c:

Condition 1 (Descriptive structure)**.** *$\langle X, \circ, \approx \rangle$ satisfies Definition 1.*[18]

Condition 2 (Contraction consistency and Expansion consistency (Sen 1969))**.**

Contraction consistency: If $x \in c(S)$ and $x \in T \subseteq S$, then $x \in c(T)$;

[18] Axioms 1 does not change because it does not involve the d_{ma}'s choice function.

Expansion consistency: if $x, y \in c(S)$, $S \subseteq T$, and $y \in c(T)$, then $x \in c(T)$.

Condition 3 (Commutativity consistency)**.** *If $x \circ y \in c(S)$, then $x \circ y, y \circ x \in c(S \cup \{y \circ x\})$.*

Condition 4 (Associativity consistency)**.** *If $(x \circ y) \circ z \in c(S)$, then $(x \circ y) \circ z, x \circ (y \circ z) \in c(S \cup \{x \circ (y \circ z)\})$.*

Condition 5 (Idempotency consistency)**.** *If $x \in c(S)$, then $x, x \circ x \in c(S \cup \{x \circ x\})$.*

Condition 6.1 (Substituability consistency)**.** *If $x, y \in c(S)$ and $x \circ z, y \circ z \in T$, then $x \circ z \in c(T)$ iff $y \circ z \in c(T)$.*

Condition 6.2 (Simplifiability consistency)**.** *If $x \circ z, y \circ z \in c(S)$ and $x, y \in T$, then $x \in c(T)$ iff $y \in c(T)$.*

Condition 7 (Non-triviality)**.** *If $x \not\approx b$, then there exist $S \subset X$, finite, $x' \in S \cap [x]_{\approx}$, $y' \in S \cap [y]_{\approx}$ such that $x' \in c(S)$ and $y' \notin c(S)$.*

Condition 8 (ICDAC consistency)**.** *Suppose $x \not\approx z$, $x \approx y$ and $z \approx t$.*

If $x, z \in S$, $x \in c(S)$, then there exists $T \subset X$, finite, such that $x \circ y, z \circ t \in T$ and $x \circ y \in c(T)$;

if $x \circ y, z \circ t \in S$, $x \circ y \in c(S)$, then there exists $T \subset X$, finite, such that $x, z \in T$ and $x \in c(T)$.

These conditions characterize the behavioral dimension of description invariance and its violations discussed previously. The revealed preferences will satisfy *description invariance* if conditions 1-5 and conditions 6.1 and 6.2 are satisfied, *tidy descriptive dependence* if conditions 1-5 and conditions 6.1, 7 and 8 are satisfied (but not condition 6.2), and *untidy descriptive dependence* if conditions 1-5 and condition 6.1 are satisfied (but not conditions 6.2 and 8).

Proposition 7. *Let X be a set of described consequences and c be a choice function for X. Then, c is a choice function that satisfies condition C_i if, and only if, there exists a binary relation $\succsim$ on X such that $c = c_{\succsim}$ and $\succsim$ satisfies axiom A_i.*

4 Discussion: applications, extensions and related literature

To show how our framework can be used to deal with framing effects and decision problems more generally, we apply it to four sets of phenomena, namely to some cases of (1) framing of consequences under risk, (2) framing of acts and of contingencies (that we discuss altogether and under both risk and uncertainty), (3) framing under certainty and (4) individuation of consequences in the Allais paradox. For each of these, we contrast our framework with existing formal contributions in economics and sometimes show that the compatibility between both allow for some non-trivial extensions.

4.1 The framing of consequences under risk

Given the illustrative example used throughout the construction of our framework, its most natural application is to the original Asian Disease problem under risk. It consists in extending, through our framework, existing models of decision under risk that respect an axiom of weak coalescing, mainly by working with these models on our choice set[19]. For instance, models in expected utility theory, rank-dependent utility theory and cumulative prospect theory respect that condition. Let X be a nonempty set of described consequences (space of risk-less outcomes) and $\Delta(X)$ be the set of all simple distributions on X. If $l \in \Delta(X)$, we denote by $s(l)$ the support of l. The descriptive

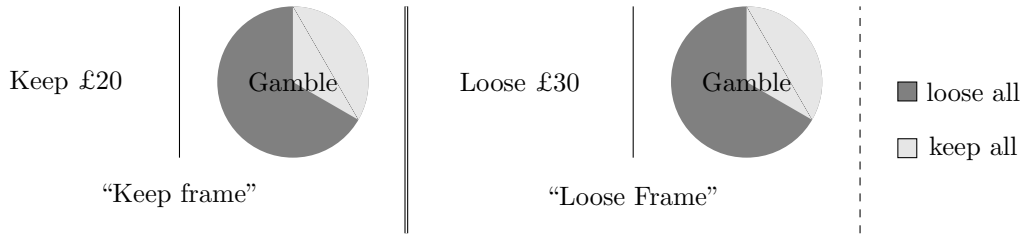
[19]Weak coalescing holds if for all $x, y \in X$ such that $x \sim y$, $(x, p; y, q; z, 1-p-q) \sim (x, p+q; z, 1-p-q)$. This property is stronger than coalescing with $x = y$.

structure is now $\langle \Delta(X), \approx, \circ \rangle$. This descriptive structure induces another descriptive structure $\langle X, \approx_X, \circ_X \rangle$ on described consequences[20]. It follows that we can use the utility of described consequences in theories that respect weak coalescing. Our results ensure that this replacement does not alter the axioms underlying these theories. Formally, using the notation from Table 1, the Asian Disease can be represented straightforwardly in expected utility theory as:

$$u(x_1) > \frac{1}{3}u(y_1) + \frac{2}{3}u(z_1)$$
$$\frac{1}{3}u(y_2) + \frac{2}{3}u(z_2) > u(x_2)$$

These inequalities can be rationalized by one weak order in the sense that $u(.)$ is its numerical representation and that it generates and can be generated by an extended choice function as defined in the previous section. Furthermore, the same weak order can also rationalize all the other empirical regularities on the Asian Disease problem.

It is worth noting that under expected utility, the implied risk attitudes are dependent on the descriptions of the consequences, and not the consequences itself, which is the whole point of Asian Disease problems in the first place. The same reasoning carries identically to other framing of consequences besides the Asian Disease. Two further illustrations are worth providing. One comes from DeMartino et al. (2006) and has nearly the same empirical structure as the Asian Disease's. d_{ma}s are told "You Receive £50", and then a graphical twist in the two frames shows how the underlying type of preference reversals is driven by the framing of the sure consequence:



In other words, the two different frames have identical descriptions of the probabilistic options, they differ only by different descriptions of the sure options. Formally, from the d_{mo}'s perspective, let $x_1 \equiv$ "Keep £20", $x_2 \equiv$ "lose £30" $\in [x]_{\approx}$, $y_1 \equiv$ "keep all" $\in [y]_{\approx}$ and $z_1 \equiv$ "lose all" $\in [z]_{\approx}$. The modal perspective from d_{ma}s can be represented straightforwardly in expected utility theory:

$$u(x_1) > \frac{1}{3}u(y_1) + \frac{2}{3}u(z_1)$$
$$\frac{1}{3}u(y_1) + \frac{2}{3}u(z_1) > u(x_2)$$

Notice the formal representation of the distinction between, on the one hand, a difference in descriptions of the same consequence (for the sure one), and, on the other hand, an identity of descriptions of the same consequence (for the probabilistic ones). The last illustration comes from McNeil et al. (1982)[21] and has a different empirical structure than Asian Disease-like problems. d_{ma}s are presented with hypothetical choices between two ways of treating lung cancer, i.e., surgery vs. radiation therapy, in one of two frames. In one frame the cumulative probabilities are presented as descriptions of survival rates:

[20] Another way of extending our framework under risk would be to follow Giraud (2004b, pp.7-11): Start from $\langle X, \approx, \circ \rangle$ and extend it to $\Delta(X)$. It is natural to extend $\approx$ on X as follows: for all $l, m \in \Delta(X)$, $l \approx m$ iff $(\forall x \in s(l), \exists y \in s(m), x \approx y, l(x) = m(y))$ and $(\forall y \in s(m), \exists x \in s(l), y \approx x, m(y) = l(x))$, and $\circ$ pointwise. This approach indeed works for the framing effects we are interested in, but leave aside logically equivalent lotteries (for instance the ones in splitting effects, i.e., violating coalescing, see Birnbaum 2005). To avoid this drawback, $\approx$ mus be extended on $\Delta(X)$ by: $l \approx m$ iff the induced lotteries on consequences are equal. In this case, the definition of $\circ$ becomes more tedious.

[21] The exact empirical wording of the problem appears only in Tversky and Kahenman (1986).

> **Surgery**: Of 100 people having surgery 90 live through the post-operative period, 68 are alive at the end of the first year and 34 are alive at the end of five years.
>
> Radiation Therapy: Of 100 people having radiation therapy all live through the treatment, 77 are alive at the end of one year and 22 are alive at the end of five years.

In another frame, they are presented as descriptions of death rates:

> Surgery: Of 100 people having surgery 10 die during surgery or the post-operative period, 32 die by the end of the first year and 66 die by the end of five years.
>
> **Radiation Therapy**: Of 100 people having radiation therapy, none die during treatment, 23 die by the end of one year and 78 die by the end of five years.

In this framing problem, the objects of choice are streams of dated binary lotteries. In each lottery, the binary consequences are mutually exclusive (i.e., being dead or alive) and from the d_{mo}'s perspective, presenting only one of the two branches (consequence plus associated probability) allows to infer the other branch and thus the complete lottery. Furthermore, we can define a logical equivalence between objects of choice over time (as was for the extension of our framework for decision under risk). Under the standard assumption of inter-temporal separability, we can focus only on lotteries independently of their place in the stream.

Formally, let

$$\left.\begin{array}{l} a_1 \equiv \text{“}\frac{90}{100} \text{ chance of living through the post-operative period”} \\ a_2 \equiv \text{“}\frac{10}{100} \text{ chance of dying during the post-operative period”} \end{array}\right\} \in [a]_{\approx}$$

$$\left.\begin{array}{l} b_1 \equiv \text{“}\frac{68}{100} \text{ chance of living in one year”} \\ b_2 \equiv \text{“}\frac{32}{100} \text{ chance of dying in one year”} \end{array}\right\} \in [b]_{\approx}$$

$$\left.\begin{array}{l} c_1 \equiv \text{“}\frac{34}{100} \text{ chance of living in five years”} \\ c_2 \equiv \text{“}\frac{66}{100} \text{ chance of dying in five years”} \end{array}\right\} \in [c]_{\approx}$$

$$\left.\begin{array}{l} d_1 \equiv \text{“}\frac{100}{100} \text{ chance of living through the post-operative period”} \\ d_2 \equiv \text{“}\frac{0}{100} \text{ chance of dying through the post-operative period”} \end{array}\right\} \in [d]_{\approx}$$

$$\left.\begin{array}{l} e_1 \equiv \text{“}\frac{77}{100} \text{ chance of living in one year”} \\ e_2 \equiv \text{“}\frac{23}{100} \text{ chance of dying in one year”} \end{array}\right\} \in [e]_{\approx}$$

$$\left.\begin{array}{l} f_1 \equiv \text{“}\frac{22}{100} \text{ chance of living in five years”} \\ f_2 \equiv \text{“}\frac{78}{100} \text{ chance of dying in five years”} \end{array}\right\} \in [f]_{\approx}$$

The modal perspective from d_{ma}s can be represented straightforwardly with the subscripts 0, $1y$ and $5y$ denoting respectively the postoperative period, one year after the operation and five years after the operation:

$$u_0(a_1) + u_{1y}(y_1) + u_{5y}(c_1) > u_0(d_1) + u_{1y}(e_1) + u_{5y}(f_1)$$
$$u_0(d_2) + u_{1y}(e_2) + u_{5y}(f_2) > u_0(a_2) + u_{1y}(b_2) + u_{5y}(c_2)$$

Again, the results of our framework are enough to ensure that there is a binary relation of preference underlying u and that it generates and can be generated by an extended choice function.

Let us now contrast our framework with other formal accounts of framing effects under risk. The main difference with all of these accounts is that we considered a richer set of behavioral data from the psychology literature to guide our axiomatic constructions. Among these accounts, the closest from our framework is Giraud's (2004b;2004a;2005). We were indeed inspired from what he calls a "normative" equivalence relation on the choice set, which is close to the perspective of the d_{mo} in our framework. However, we did not necessarily identify the d_{mo}'s perspective with a normative one that the d_{ma} would like to take and we characterized their relations through an algebraic structure on descriptions. Hence we were able to decompose our invariance axiom (ICD)

and characterize how a utility and choice function still exist when this axiom is weakened. That is, we went further than imposing the axiom and forcing a choice between taking it to be true or false[22]. This considerably simplifies applications, especially because Giraud cannot formalize a direct dependence of preferences on descriptions without interpreting it as a failure of the d_{ma} to recognize that different descriptions are about the same consequences. To bypass this shortcoming, he gives an account of the original Asian Disease which is more complex than ours, where the preference reversal is explained through a nonadditive decision weight function, making the d_{ma}'s beliefs directly dependent on descriptions and preferences indirectly so when revealed (see Giraud 2004a;b). Finally, he makes more assumptions about the underlying psychological processes of the d_{ma} on which we are agnostic, and while we narrowly focused on the role of descriptions, his framework is also built to deal with violations of context independence (i.e., preference reversals triggered by expansion and/or contraction of the choice set) and of procedure invariance (i.e., preference reversals triggered by different elicitation methods). Hence, because we believe that our frameworks are compatible, incorporating our choice set in his approach could make it more tractable while expanding the explanatory and empirical scope of ours.

The few other formal accounts of framing effects in the literature have all been conducted in frameworks that are less close to the standard one than Giraud's and ours. Nevertheless under one form or another, the differences just discussed resurface.

Take for instance Blume, Easley, and Halpern (2013). They account for framing effects within their "contructive decision theory" framework where objects of choice are syntactic descriptions of events in a simple programming language. They show how this language naturally formalizes the two frames of McNeil et al. (Surgery vs. Radiation) as two logically non-equivalent syntactic descriptions. Description invariance is not derived and then weakened but imposed by a subset of the syntactic descriptions that describes the same events according to the d_{ma}'s *beliefs*. This allows to represent both d_{ma} revealing framing effects and those who don't (simply by putting the two syntactic descriptions of the two frames in different subsets or by regrouping them within the same subset, respectively).

Or take Gold and List (2004). They use the resources of predicate logic to construct a choice set in which the elements are either "target" propositions about the d_{ma}'s preferences or various "background" propositions about the decision situation. Framing effects are formalized as arising from a sequential process whereby *beliefs* about two different sets of background propositions can influence contradictory beliefs about the same target proposition, e.g., the d_{ma} believes that it is true that he prefers the sure program over the probabilistic one and the dm_a believes that it is false that he prefers the sure program over the probabilistic one (see pp.267-268). They make description invariance formally explicit but not as quite straightforwardly as in our and others' frameworks.

Take finally Bourgeois-Gironde and Giraud (2009). They analyze framing effects through the framework of Bolker-Jeffrey expected utility where preferences are over propositions. They make explicit the implicit axiom of invariance in this framework by showing how it is hidden both in the mathematical structure of the Boolean algebra with which the choice set is constructed and in its interpretation as a set of propositions. Once made explicit, they however do not weaken it, but propose to "bypass" it (p.390) by introducing a new set of information ("good news" and "bad news") to be associated with each events, hence refining the d_{ma}'s *beliefs* about the latter. Following McKenzie and co-authors (see section 2 above), framing effects are thus rationalized by refining invariance to the informational (and not only logical) structure of decision problems.

Among these three contributions, the closest to ours in spirit is Bourgeois-Gironde and Giraud (2009) because of their inspiration from McKenzie and co-authors' results to motivate an account of framing effects as not necessarily irrational. They however do not consider the further results from psychology that motivated our formalization of a direct dependence of preferences on descriptions. Bacharach (2003) provides an analysis of framing effects under risk in this spirit, but it is quite informal and less motivated by behavioral data (concerning the direct dependence of preferences on descriptions)[23].

[22] We thank Giraud for explaining to us these differences in these terms

[23] Both Ryan (2005) and Lanzi (2011) claim to provide a framework to formalize framing effects. Their accounts do not seem very tractable; in any case they do not provide concrete applications to framing effects showing how the formalization should work (they do so for other phenomena). Lerner (2014) provides an account of framing effects

Summing up, the main difference between ours and others' frameworks is the axiomatic approach through which we make description invariance explicit and then weaken it to allow for a direct dependence of preferences on descriptions. The main advantages of this approach are its simplicity (both of the framework itself and in its application) and its fit with existing behavioral data on framing effects in psychology. Notice that these data do not include only the ones presented in section 2, but also Kahneman and Tversky's original ones. One of the points of their early experiments was to demonstrate framing effects by framing consequences, acts and events *independently* of each others. This threefold empirical and conceptual distinction seems lost in others' contributions, all of which introduce events and beliefs in the formalization of problems such as the Asian Disease and other framing effects under risk where there are no events and where only the consequences are framed[24]. What does our framework have to say on beliefs and the framing of events and acts?

4.2 The framing of acts and events, under risk and uncertainty

To deal with this question, we propose to contrast our framework with Ahn and Ergin (2010). The latter is primarily designed to account for framing effects due to the framing of events in cases such as the following (ibid, p.655). Consider these two health insurance contracts stipulating the consequences (i.e., the amounts of money in their left sides) occurring contingently upon the realization of events (i.e., described in their right sides):

$$\begin{pmatrix} \$500 & \text{surgery} \\ \$100 & \text{prenatal care} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} \$500 & \text{laminotomy} \\ \$500 & \text{other surgeries} \\ \$100 & \text{prenatal care} \\ \vdots & \vdots \end{pmatrix}$$

Notice that "laminotomy" and "other surgeries" are both included in "surgery", and the union of the two former is equal to the latter. Hence we have two different descriptions of the same contract. Ahn and Ergin's axiomatic approach consists in making explicit the axiom of invariance to such redescriptions that is implicit in the standard model, and then weaken it to provide what they call a "partition-dependent expected utility" theory.

Our framework cannot deal directly with such framing effects. It could if properly extended for acts as objects of choice instead of consequences. Such extension would be in line with Ahn and Ergin's framework because, due to our finiteness assumptions, we would need to weaken description invariance with a finite state-space à la Anscombe and Aumann (1963) as they do, i.e., not within an infinite one à la Savage)[25]. In a way, Ahn and Ergin's elegant results would make it easy for us to carry such an extension, which may not be trivial in the sense of accounting for at least two further types of framing phenomena for which they cannot in their framework, namely those due to different descriptions (1) of the same consequence or (2) of the same events when the redescription is "pure", i.e., not made through the packing or unpacking of an inclusion relation. The two following examples respectively illustrate each case and are inspired from the field studies and lab experiments of Sinaceur et al. (2005) about the impacts on various cognitive and behavioral measures of different descriptions of the same disease either with the ordinary label "Mad Cow" or with the scientific label "bovine spongiform encephalopathy" (BSE). Imagine that you ate beef and the consequences of that act are that you get the disease in the event that the U.K. trades beef with France (abbreviated by "trades with U.K.") and that you don't get the disease otherwise. Here are two different descriptions of that act:

$$\begin{pmatrix} \text{get Mad Cow} & \text{trades with U.K.} \\ \text{don't get Mad Cow} & \text{no trades with U.K.} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} \text{get BSE} & \text{trades with U.K.} \\ \text{don't get BSE} & \text{no trades with U.K.} \\ \vdots & \vdots \end{pmatrix}$$

through intensional logic. Though she illustrates how her framework formalizes the Asian Disease, it is again not very tractable.

[24] With the exception of Bacharach (2003, p.66), who insists that "*Framing is logically prior to believing.*"

[25] Or to do so in Savage's framework, we could work our extension trhough Gul's (1992) modification of the former for finite state-space.

Now imagine that you ate beef at Mike's place, and the consequences of that act is that you get sick in the event that Mike bought infected beef and don't get sick otherwise. Here are two different descriptions of that act:

$$\begin{pmatrix} \text{get sick} & \text{Mike bought Mad Cow beef} \\ \text{don't get sick} & \text{Mike bought healthy beef} \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} \text{get sick} & \text{Mike bought BSE beef} \\ \text{don't get sick} & \text{Mike bought healthy beef} \\ \vdots & \vdots \end{pmatrix}$$

It can be argued that Ahn and Ergin's impossibility to deal with this latter example is problematic in the sense that their model is devised to account for the effects of redescriptions of the same event, while it is less problematic that they cannot deal with the former because doing so is not their contention. On the other hand, we could deal with the former case quite straightforwardly because it is the kind of framing effects we designed our framework for, while the latter case would require more work. As we see it, a state-dependent account may be the sensible way to go here. We believe so because state-dependent utility finds a natural interpretation that is well in line with the work of McKenzie on information leakage which motivated our framework. The interpretation is that, it is not the descriptions of the event *per se* on which the d_{ma}'s preferences are dependent, but on *the fact that an entity chose a given description* over other possible ones. Hence preference reversals due to 'who said what' could be formally represented in a neat way.

Finally, let us consider what Tversky and Kahneman (1981, p.454) call a framing of "act" under risk, though "act" is not to be taken in a Savagean sense because there are no events in their example:

> [First frame]
>
> Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.
>
> Decision (i). Choose between:
>
> A. a sure gain of \$240
>
> B. 25% chance to gain \$1000, and 75% chance to gain nothing
>
> Decision (ii). Choose between:
>
> C. a sure loss of \$750
>
> D. 75% chance to lose \$1000, and 25% chance to lose nothing
>
> [Second frame]
>
> Choose between:
>
> $A\&D$. 25% chance to win \$240, and 75% chance to lose \$760.
>
> $B\&C$. 25% chance to win \$250, and 75% chance to lose \$750.

In this framing problem, the object of choice are lotteries that are either composed in the first frame or reduced in the second one. From the d_{mo}'s perspective, both forms of lotteries are equivalent, i.e., $(A, D), A\&D \in [AD]_{\approx}$ and $(B, C), B\&C \in [BC]_{\approx}$. The modal preference from d_{ma}s, i.e., $(A, D) \succ (B, C)$ and $B\&C \succ A\&D$, exhibit untidy descriptive dependence and hence violate ICDAC. However, to fully represent these choices in terms of utility functions on described consequences with their associated probabilities, we would need to incorporate our framework in models allowing for violations of first-order stochastic dominance (see the ones presented by Birnbaum 2005).

4.3 Framing under certainty

In this subsection we briefly apply our framework to two famous cases of framing under certainty to illustrate possible interpretations of the d_{ma}-d_{mo} relation. The first case is from Quattrone and Tversky (1988). It can be thought of as an analogue of the Asian Disease under certainty where the choice is between two public policies with different trade-offs between (un)employment and inflation:

If **program A** is adopted, 5% of the work force would be unemployed, while the rate of inflation would be 17%.

If program B is adopted, 10% of the work force would be unemployed, while the rate of inflation would be 12%.

If program C is adopted, 95% of the work force would be employed, while the rate of inflation would be 17%.

If **program D** is adopted, 90% of the work force would be employed, while the rate of inflation would be 12%.

In our framework, there are two straightforward ways to represent these modal preferences. Firstly, the d_{mo} is an economist who classes objects of choice in terms of their economic implications:

$$\left.\begin{array}{l} a_1 \equiv \text{“5\% of the work force would be unemployed, while the rate of inflation would be 17\%”} \\ a_2 \equiv \text{“95\% of the work force would be employed, while the rate of inflation would be 17\%”} \end{array}\right\} \in [a]_{\approx}$$

$$\left.\begin{array}{l} b_1 \equiv \text{“10\% of the work force would be unemployed, while the rate of inflation would be 12\%”} \\ b_2 \equiv \text{“90\% of the work force would be employed, while the rate of inflation would be 12\%”} \end{array}\right\} \in [b]_{\approx}$$

The modal perspective from d_{ma}s can be represented straightforwardly as $u(a_1) > u(b_1)$ and $u(b_2) > u(a_2)$. If the d_{mo} wish to take a normative perspective on these preferences, he can either argue that they are rational because representable by one utility function and one choice function, or irrational because they exhibit untidy descriptive dependence and hence violate ICDAC.

Secondly, the d_{mo} is a politician (e.g., Donald Trump seems like a candidate fit for the following interpretation) who classes objects of choice in terms of whether they look like good news or like bad news, respectively:

$$\left.\begin{array}{l} a_1 \equiv \text{“5\% of the work force would be unemployed, while the rate of inflation would be 17\%”} \\ a_2 \equiv \text{“90\% of the work force would be employed, while the rate of inflation would be 12\%”} \end{array}\right\} \in [a]_{\approx}$$

$$\left.\begin{array}{l} b_1 \equiv \text{“10\% of the work force would be unemployed, while the rate of inflation would be 12\%”} \\ b_2 \equiv \text{“95\% of the work force would be employed, while the rate of inflation would be 17\%”} \end{array}\right\} \in [b]_{\approx}$$

In this case, the modal perspective from the d_{ma}s can be represented straightforwardly as $u(a_1) > u(b_1)$ and $u(a_2) > u(b_2)$. If the d_{mo} wishes to take a normative perspective on these preferences he can argue that they are rational not only because they are representable as utility maximization and choice behavior, but also because they exhibit tidy descriptive dependence and hence do not violate ICDAC. Alternatively, the d_{mo} could argue for their irrationality because they exhibit tidy descriptive dependence and hence violate ICD.

The second case of framing under certainty is Tversky and Kahneman's (1981) lost bill/lost ticket example, which has a rather different empirical structure than the previous one:

[Lost bill frame]

Imagine that you have decided to see a play where admission is \$10 per ticket. As you enter the theater you discover that you have lost a \$10 bill.

Would you still pay \$10 for a ticket for the play? **Yes** No

[Lost ticket frame]

Imagine that you have decided to see a play and paid the admission price of \$10 per ticket. As you enter the theater you discover that you have lost the ticket.

Would you pay \$10 for another ticket? Yes **No**

The main distinction with the other framing effects discussed so far is twofold. On the one hand, the two frames are about two different situations, not two descriptions of the same situation. On the other hand, beyond the lab, this problem cannot really be interpreted as an external d_{mo} posing a decision problem to the d_{ma}. In the real world you would be both the d_{mo} and the d_{ma} and construct your own counterfactuals to make up your mind. Nevertheless, it is possible to capture both these differences in our framework, and again in two different but straightforward manner. Firstly, the d_{mo} is either an observing economist or a self of a multiple-selves d_{ma}, both of whom class objects of choice by their consequences in terms of opportunity costs:

$$\left.\begin{array}{l} a_1 \equiv \text{“paying for a \$10 ticket after having lost a \$10 bill”} \\ a_2 \equiv \text{“paying for a \$10 ticket after having lost a \$10 ticket”} \end{array}\right\} \in [a]_{\approx}$$

In this case, the modal preference of d_{ma}s can be represented straightforwardly as $u(a_1) > u(a_2)$. If the d_{mo} wishes to take a normative perspective on these preferences, he can either argue that they are rational because representable by a utility and choice functions, or irrational because they exhibit tidy descriptive dependence and hence violate ICD.

Secondly, the d_{mo} is a self of a multiple-selves d_{ma} who classes objects of choice by their consequences in terms of their experienced utility, considering that the experience of being upset from having lost a ticket is different from the experience of of being upset from having lost a bill, respectively:

$$\begin{array}{r} a_1 \equiv \text{“paying for a \$10 ticket after having lost a \$10 ticket”} \in [a]_{\approx} \\ b_1 \equiv \text{“paying for a \$10 ticket after having lost a \$10 bill”} \in [b]_{\approx} \end{array}$$

$$\begin{array}{r} a_1 \equiv \text{“paying for a \$10 ticket after having lost a \$10 ticket”} \in [a]_{\approx} \\ b_1 \equiv \text{“paying for a \$10 ticket after having lost a \$10 bill”} \in [b]_{\approx} \end{array}$$

In this case, the modal preference of d_{ma}s can be represented straightforwardly as $u(a_1) > u(b_1)$. If the d_{mo} wishes to take a normative perspective on these preferences, he can either argue that they are rational because representable by a utility function and a choice function, or that they are irrational because they exhibit untidy descriptive dependence and hence violate ICDAC.

By presenting the possibility of multiple interpretations from our framework we do not aim to take a relativistic perspective on the normative issues underlying framing effects. Instead, we believe that these issues are important and that the flexibility of our framework allows to make precise discussions of the conditions under which a given interpretation of observed preferences can be seen as rational or irrational. In that sense, though formally quite different, we take a position similar to Gilboa, Maccheroni, Marinacci, and Schmeidler (2010), i.e., that a formal framework can help organize rational discussions and even debates over normative issues underlying individual rationality in economics.

4.4 Individuation and redescription of consequences

In this subsection, we suggest that something very close to our framework was presupposed in a set of arguments constitutive of the debates around Allais' paradoxes. Recall that the classical Allais paradox (a version of the so-called common consequence effect) goes as follows (Allais, 1953, p.527; consequences are monetary, originally in French francs):

> **A**: The certainty of winning 100 millions
>
> B: 10% chance of winning 500 millions, 89% chance of winning 100 millions, and 1% chance of winning nothing

Notice that both A and B share a common consequence, namely “winning 100 millions”, with certainty in A and with 89% chance in B. Thus both A and B share at least 89% of winning 100 millions (and A has an extra 11% chance for it). So, if we retrieve this common consequence with the common probability of occurrence, the preference between A and B should not change to respect the independence axiom. But it does:

C: 11% chance of winning 100 millions

D: 10% chance of winning 500 millions and 90% chance of winning nothing

As reviewed by Broome (1991, chap.5), most defenses of expected utility theory consisted in arguing that Allais' paradoxes are not paradoxes anymore when the consequences are redescribed to add information about, for instance, some feelings of disappointment if the decision maker does not win anything. Such redescriptions, e.g., "winning nothing" and "winning nothing and being disappointed" are then taken as different descriptions of *different* consequences, hence breaking the equivalence between the two initial pairs of problem, and making it impossible to observe an inconsistency from expected utility theory[26]. Within our framework, this reasoning could be formalized as follows. The d_{mo} is a decision theorist who classes consequences in terms of their experienced utility, considering that the experience of being disappointed when winning nothing is different from the experience of winning nothing:

$$a_1 \equiv \text{“winning nothing”} \in [a]_{\approx}$$
$$b_1 \equiv \text{“winning nothing and being disappointed”} \in [b]_{\approx}$$

By contrast, those who argued against this interpretation held a position that can be formalized as :

$$\left.\begin{array}{l} a_1 \equiv \text{“winning nothing”} \\ a_2 \equiv \text{“winning nothing and being disappointed”} \end{array}\right\} \in [a]_{\approx}$$

It can be argued that the condition of possibility for this debate is the standard notion of a consequence, which, following Savage (1972 [1954], pp.13), is taken to mean "anything that may happen to the person", i.e., to the d_{ma}. This includes any "things, or experiences, regarded as consequences" (ibid, p.14). It may be asked, regarded as a consequence by whom? In standard decision theory, consequences are, in principle, regarded as such by d_{ma}s. But by virtue of revealed preference methodology, only the d_{ma}'s obervable choice behavior should be a primitive in the analysis which excludes his subjective perception of what counts as one consequences. Hence, in practice, consequences are regarded as such by a decision theorist guessing or postulating what a d_{ma} would count as a consequence. The first position in the debate on redescriptions in the Allais paradox shows that one drawback of this permissive notion of a consequence is that it prevents to account for the simple fact that one d_{ma} (or even one decision theorist for that matter) can entertain different perspectives on the *same* consequence. That is, it prevents to account for the recognition by the d_{ma} that different presentations of the same consequence (or "things") trigger different preferences (or "experiences"). Instead, it can be argued that we shall try to do justice to the subjectivity of a d_{ma} who recognizes that one and the same consequence can trigger different mental states in different situation, without conveniently postulating that the differences were "in fact" about different consequences. This is the second position in the debate, which was notably held by Sen (1986a, 1986b) and Machina (1989, sect. 6.6). Following these two authors, we argue that in order to do justice to the decision maker's values and hence make the related normative analysis easier, we shall not blend all the factually relevant observations into one single theoretical construct (here of "consequence"). The empirical stakes are just as high as the normative ones here, since too permissive a notion of a consequence makes any utility theory not amenable to empirical falsification *in principle*, a methodological position which is not quite the same as not being falsifiable *in practice*. Therefore, what underlies our whole framework is not the standard notion of a consequence, but rather what Machina (1989, p.1660) calls "the operational definition of a consequences". We do not see this as a drawback, as it can be argued that the whole of experimental economics (or at least its subpart that focuses on decision theory) does indeed not rely on the standard notion of a consequence. We do not see how framing effects could even be intelligible with that notion.

In short, we argue that different descriptions of one same consequence that can be described as such by a d_{mo} and recognized as such by a d_{ma} should be represented as such by the decision theorist

[26]It should be noted that Savage's (1972 [1954], p.103) famous response to Allais does not involve redescriptions of the consequences, but additions of events to 'materialize' Allais' probabilities (from abstract numbers to numbered lottery tickets). Savage then argued that his version and Allais' original one were two different descriptions of the same decision problem, and that his preferences in his version made him change his original preferences in Allais' original version.

(if he is not already the d_{mo}). By virtue of the formal discriminations of different descriptions of a same consequence, our framework allows to do this in a straightforward manner. With it, we have tried to shed some lights on a debate around the Allais paradox which is not a framing effect per se. Yet the underlying issues are not unconnected with framing effects (for a sustained argument in that direction, see Giraud 2004a, pp.146-7). Whether out framework can help to resolve deeper conceptual issues related to the permissive notion of consequence in decision theory (see Aumann 1987 [1971]; Savage 1987 [1971]) is an open question that we left for further work.

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5 Proofs

Proof of proposition 1. Let $x \in X$. By axiom 1, $\approx$ is an equivalence relation, define $[x]_{\approx} := \{a \in X \mid a \approx x\}$ the equivalence class of x by $\approx$. $[x]_{\approx}$ is a subset of X and $\sim$ is an equivalence by axiom 2. Let $[x]_{\approx}/\sim$ be the set of equivalence classes of $[x]_{\approx}$ under $\sim$. Define $\star$ on $[x]_{\approx}/\sim$ as follows: $[a] \star [b] = [a \circ b]$. By axioms 1, if $a, b \in [x]_{\approx}$, then $a \circ b$ is defined and belongs to $[x]_{\approx}$. $\star$ is a closed binary operation on $[x]_{\approx}/\sim$. Observe that $\star$ is well defined since if $a, a', b, b' \in [x]_{\approx}$, $a \sim a'$ and $b \sim b'$, then, by axiom 1, $a \circ b$ and $a' \circ b'$ are defined and by two applications of axiom 6, $a \circ b \sim a' \circ b \sim a' \circ b'$, so $[a] \star [b] = [a'] \star [b']$. We show that $\langle [x]_{\approx}/\sim, \star \rangle$ is a cancellative idempotent commutative semigroup. If $a, b, c \in [x]_{\approx}$, then by axioms 1, $a \circ b$, $b \circ a$, $(a \circ b) \circ c$ and $a \circ (b \circ c)$ are defined. Associativity follows from axiom 4; commutativity follows from axiom 3; idempotence follows from axiom 5; and cancellation follows from axiom 6. It is a classical result that a commutative semigroup can be embedded in a group if and only if it is cancellative. Note that $\langle [x]_{\approx}/\sim, \star \rangle$ is idempotent, it follows that the group extension of $\langle [x]_{\approx}/\sim, \star \rangle$ is a group in which every element is idempotent. Every group has exactly one idempotent element: the identity. It follows that for all $a, b \in [x]_{\approx}$, $a \sim b$. Hence,

$$\forall a, b \in X, \quad a \approx b \Rightarrow a \sim b.$$

■

Proof of proposition 2. As in proof of Proposition 1, define $[x]_{\approx} := \{a \in X \mid a \approx x\}$ the equivalence class of x by $\approx$; and $\star$ on $[x]_{\approx}/\sim$ as follows: $[a] \star [b] = [a \circ b]$. As axioms 1-5 and 6.1 hold, it is clear that $\star$ is well defined and that $\langle [x]_{\approx}/\sim, \star \rangle$ is an idempotent commutative semigroup. We show that if $a \in X$ and $b \in X$ are generated by the same generators, then $a \sim b$. Let B be an idempotent commutative semigroup generated by $b_1, \ldots, b_n$. For each $x \in B$, the length of x, denoted by $|x|$, is the minimum k such that $x = x_1 x_2 \ldots x_k$, where $x_k \in \{b_1, \ldots, b_n\}$, $k \geq i \geq 1$. Note that $|x| \geq 1$ for all $x \in B$. The largest possible length of an element of any idempotent commutative semigroup with n generators is n. Now, let $a, b \in X$, generated by the same generators. The previous reasoning applies and by axiom 6.1, it is clear that $a \sim b$. Suppose that $[x]_{\approx}$ is finitely generated by n pure descriptions, then $\langle [x]_{\approx}/\sim, \star \rangle$ is finitely generated by n elements. By analogy with the cardinal of the power set of a finite set $\langle [x]_{\approx}/\sim, \star \rangle$ has at most $2^n - 1$ elements. ■

Proof of proposition 3. In view of proposition 2, we show that axioms 7 and 8 imply that

$$\forall x, y, z \in X, \text{ if } x \approx z \text{ and } x \succsim z, \text{ then } (x \succsim y \succsim z \Rightarrow y \approx x).$$

Assume that $\succsim$ satisfies axioms 7 and 8, take $x, y, z \in X$ such that $x \approx z$, $x \succsim z$, and $x \succsim y \succsim z$), and suppose that not $y \approx x$, hence not $y \approx z$. By axiom 7 there exist $z_1 \approx z$, $x_2 \approx x$, and $y_1, y_2 \approx y$ such that not $z_1 \sim y_1$ and not $x_2 \sim y_2$. Assume $z_1 \succ y_1$ or $y_2 \succ x_2$, applying axiom 8 twice, we obtain $y \succ z$ or $z \succ x$. This contradicts $x \succsim y \succsim z$. Hence, we have $y_1 \succ z_1$ and $x_2 \succ y_2$. By similar reasoning, $x \sim y$, $y \sim z$ and $x \sim z$ are impossible. We can suppose that $x \succ y \succ z$, but

again, applying axiom 8 twice, by transitivity we obtain $x \circ z \succ x \circ y$, a contradiction. Therefore, $y \approx x$ (and $y \approx z$). ■

Proof of proposition 4. Let $\langle X, \approx, \circ \rangle$ be a descriptive structure such that $X/\approx$ is finite and $[x]_\approx$ is finitely generated for all $x \in X$. Let $x \in X$, we can explicitly enumerate, in a countable fashion, all possible concatenations on $[x]_\approx$. Since this provides a listing of all elements of $[x]_\approx$, $[x]_\approx$ is countable. $X/\approx$ is finite, whence it is the union of a finite number of countable sets, hence $X/\approx$ is countable. It is well-known that a countable weakly ordered set admits an utility representation, hence statement (i) is proven. Statement (ii) (respectively (iii), (iv)) is a consequence of (i) and Proposition 1 (respectively Proposition 2, 3). ■

Proof of proposition 5. Define $r : \mathcal{P}^*(X/\approx) \to \mathcal{P}^*(X)/\approx_\mathcal{P}$ by

$$r(I) := \left[\bigcup_{[x]_\approx \in I} [x]_\approx \right]_{\approx_\mathcal{P}}.$$

We will show that r is onto. Let $A \in \mathcal{P}^*(X)$. Set $I = \{[x]_\approx \mid x \in A\}$, we will show that $\bigcup_{[x]_\approx \in I}[x]_\approx \approx_\mathcal{P} A$, that is $r(I) = [A]_{\approx_\mathcal{P}}$. If $x \in A$, by definition, $x \in \bigcup_{[x]_\approx \in I}[x]_\approx$. Conversely, if $y \in \bigcup_{[x]_\approx \in I}[x]_\approx$, there is an $x \in A$ such that $y \approx x$. Therefore $\bigcup_{[x]_\approx \in I}[x]_\approx \approx_\mathcal{P} A$. Next we show that r is one-to-one. Suppose that $r(I) = r(J)$ for some $I, J \in \mathcal{P}^*(X/\approx)$. Let $[x]_\approx \in I$, then as $\bigcup_{[x]_\approx \in I}[x]_\approx = \bigcup_{[x]_\approx \in J}[x]_\approx$, and $[x]_\approx \subseteq \bigcup_{[x]_\approx \in I}[x]_\approx$, we have $[x]_\approx \subseteq \bigcup_{[x]_\approx \in J}[x]_\approx$, whence $[x]_\approx \in J$. Hence $I \subseteq J$, by symmetry $I = J$. ■

Proof of proposition 6. Let X be a finite choice set of described consequences such that $X/\approx$ is not trivial and c an extended choice function. Let $c = c_{\succsim}$ where $\succsim$ is complete and transitive relation on X. To see that c satisfies property γ^+-extended for descriptions, note that if $[x]_\approx \in c([A]_{\approx_\mathcal{P}}, f)$, $[y]_\approx \in c([B]_{\approx_\mathcal{P}}, g)$, $[x]_\approx \cap f$ is a singleton, and $[y]_\approx \cap g \subseteq f$, then there exists a unique $x' \in f$ such that $x' \approx x$ and $x' \succsim z$ for all $z \in f$ and there exists $y' \in [y]_\approx$ such that $y' \in f$ and $y' \succsim z$ for all $z \in g$. It follows that $x' \succsim z$ for all $z \in f \cup g$. Let $h \in [A]_{\approx_\mathcal{P}} \cup [B]_{\approx_\mathcal{P}}$ such that $[x]_\approx \cap f \subseteq h \subseteq f \cup g$, whence $x' \in h$, hence $[x]_\approx \in c([A]_{\approx_\mathcal{P}} \cup [B]_{\approx_\mathcal{P}}, h)$.

In the other direction, let c be an extended choice function satisfying γ^+-extended for descriptions. Define $\succsim_c$ on X as follows: $x \succsim_c y$ iff $x \not\approx y$ and $\exists A \subseteq X$ such that $x, y \in A$, $[x]_\approx \cap A = \{x\}$, $[y]_\approx \cap A = \{y\}$, and $[x]_\approx \in c([A]_{\approx_\mathcal{P}}, A)$. If $x \not\approx y$, since c is nonempty by definition, it is straightforward to show that either $x \succsim_c y$ or $y \succsim_c x$. Suppose that $x \succsim_c y$, $y \succsim_c z$ and $x \not\approx z$. By definition, $x \succsim_c y$ means $x \not\approx y$ and $\exists A_{xy} \subseteq X$ such that $x, y \in A_{xy}$, $[x]_\approx \cap A_{xy} = \{x\}$, $[y]_\approx \cap A_{xy} = \{y\}$, and $[x]_\approx \in c([A_{xy}]_{\approx_\mathcal{P}}, A_{xy})$. And $y \succsim_c z$ means $y \not\approx z$ and $\exists A_{yz} \subseteq X$ such that $y, z \in A_{yz}$, $[y]_\approx \cap A_{yz} = \{y\}$, $[z]_\approx \cap A_{yz} = \{z\}$, and $[y]_\approx \in c([A_{yz}]_{\approx_\mathcal{P}}, A_{yz})$. Let A_{xz} be the maximal subset (w.r.t inclusion) of $A_{xy} \cup A_{yz}$ which belongs to $[A_{xy}]_{\approx_\mathcal{P}} \cup [A_{yz}]_{\approx_\mathcal{P}}$ and includes x, z as only representative of $[x]_\approx, [z]_\approx$ respectively. Whence, $[x]_\approx \in c([A_{xy}]_{\approx_\mathcal{P}}, A_{xy})$, $[y]_\approx \in c([A_{yz}]_{\approx_\mathcal{P}}, A_{yz})$, $[x]_\approx \cap A_{xy}$ is a singleton, and $[y]_\approx \cap A_{xy} = \{y\} \subseteq A_{xy}$. Hence, γ^+-extended for descriptions applies, as A_{xz} belongs to $[A_{xy}]_{\approx_\mathcal{P}} \cup [A_{yz}]_{\approx_\mathcal{P}}$ and $[x]_\approx \cap A_{xy} = \{x\} \subseteq A_{xz} \subseteq A_{xy} \cup A_{yz}$, it is true that $[x]_\approx \in c([A_{xz}]_{\approx_\mathcal{P}}, A_{xz})$. Hence $x \succsim_c z$ because $x \not\approx z$. Now, if $S \subseteq X$ is such that for all $x, y \in S$, $x \not\approx y$ whenever $x \neq y$, then the restriction of $\succsim_c$ is a weak order on S. It is easy to see that $\succsim_c$ is Suzumura consistent, let $\succsim_c^{tr}$ be its transitive closure. If there exits x, y incomparable for $\succsim_c^{tr}$, then necessarily $x \approx y$ and there exist z_1, z_2 such that $z_1, z_2 \not\approx x, y$ and $z_1 \succsim_c^{tr} x, y \succsim_c^{tr} z_2$ (or just z_1 if x, y are bottom elements or just z_2 if x, y are top elements). We can easily complete $\succsim_c^{tr}$ consistently. Let $\succsim_c^{o}$ be such completion. It is straightforward to show that $c = c_{\succsim_c^{o}}$. ■

Proof of proposition 7. Let X be a set of described consequences and c be a choice function for X. Define $\succsim_c$ on X as follows: $x \succsim_c y$ iff $\exists S \subseteq X$, finite, such that $x, y \in S$, and $x \in c(S)$. Given a binary relation $\succsim$ on X, the induced choice function $c_{\succsim}$ is defined by

$$c_{\succsim}(S) = \{x \in S \mid x \succsim y,\ \forall y \in S\}.$$

It is a classical result that c satisfies $C2$ if, and only if, there exists a binary relation $\succsim$ on X such that $c = c_{\succsim}$ and $\succsim$ satisfies axiom A_2, that is, $\succsim$ is a weak order. We prove the proposition for the conditions $C5$ (Axiom 5) and $C6.1$ (Axiom 6.1). The other cases are similar and/or follow from

basic definitions. Let $c = c_{\succsim}$ where $\succsim$ is a binary relation on X that satisfies axiom 5. Suppose that $x \in c(S)$ for some S finite. By definition, $x \succsim y$, $\forall y \in S$, but $x \sim x \circ x$ by hypothesis, whence $x \sim x \circ x \succsim y$, $\forall y \in S$, it follows that $x, x \circ x \in c(S \cup \{x \circ x\})$. Conversely, let c be a choice function satisfying $C5$ and $\succsim_c$ the binary relation induced by c. By definition, $\emptyset \neq c(\{x\}) \subseteq \{x\}$, then clearly $c(\{x\}) = \{x\}$. By $C5$, it is also clear that $x \succsim_c x \circ x$ and $x \circ x \succsim_c x$. Thus $\succsim_c$ satisfies axiom 5. Finally, we prove the proposition for $C6.1$. Let $c = c_{\succsim}$ where $\succsim$ is a binary relation on X that satisfies axiom 6.1. Suppose that $x, y \in c(S)$ and $x \circ z, y \circ z \in T$ for some S, T finite. It is clear that $x \sim y$, whence $x \circ z \sim y \circ z$. It follows that $C6.1$ holds. Conversely, let c be a choice function satisfying $C6.1$ and $\succsim_c$ the binary relation induced by c. Suppose that $x \circ z$ and $y \circ z$ are defined and that $x \sim_c y$. Set $S = \{x, y\}$ and $T = \{x \circ z, y \circ z\}$. Then $C6.1$ applies, $c(T)$ is nonempty by definition, it follows that $x \circ z \sim_c y \circ z$. ■

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