

IN SEARCH OF THE RIGHT TOOL: FROM FORMALISM TO CONSTRUCTIVIST MODELLING

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In search of the right tool

From formalism to constructivist modelling

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Criticisms of mainstream economics are not new but took on a new dimension in the wake of the biggest economic crisis of the last 80 years. “Where it went wrong – and how the crisis is changing it” titled *The Economist* in July 2009, while showing a modern economic theory textbook disintegrating into a wallow. Lack of realisticness, hyper formalisation, ultra specialisation are now criticisms that are dealt with more seriously. This paper concentrates on one dimension of this present crisis of economics, the criticism of hyper formalisation of economic theories and focuses more generally on the question of the use of mathematics in economics. One danger would be to throw the baby out with the bath water. The thesis of this article is rather that it is not mathematics as such that should be reconsidered but rather the *kind* of mathematical tools used. In other words, mathematics is not neutral and it is the specificity of the mathematical apparatus used in economics that is at the origins of much of mainstream’s limits.

The aim of this paper is to assert the non-neutrality of the mathematical tools used by economists and to show that, after half a century of domination of a peculiar vision of mathematics, economics is ready to switch to a diverse conception thanks to the availability of new mathematical tools. Such a methodological turn may dramatically change the nature of the economists’ research work.

Mathematics allows the translation into formal language of an organized thought; but through this operation, there is an interaction between syntax and semantics, between the – mathematical – form and the – economic – content. The kind of tools used constrains theorists in their reasoning; in other words, economists have tended to prioritise mathematical tractability at the expense of realism.

Modern economics is elaborated upon the standards of the mathematical formalist school and the current dissatisfaction opens the way to consideration of alternative standards, for instance those of the constructivist mathematical tradition. Formalism and constructivism are two competing philosophies of mathematics that emerge and confront each other at the occasion of the debate over the foundations of mathematics of the beginning of the 20th century (section 1). Economics witnesses in the 70s the so-called formalist revolution as Blaug (1999) called it, that shapes modern mainstream economics and its associated limits (section 2). Lots

of aspects of economic reality are not dealt with in mainstream economics simply because they are not formally tractable. The joke of the drunken man looking for his key only in the light of the street lamp is a depressingly good caricature. Mainstream economics could simply not develop alternative visions of economic reality because the suitable tools were lacking. A suggestive historical illustration concerns the Austrian economists' attitude toward the use of mathematics in economics (section 3). It is an error to consider Menger's position as a dogmatic rejection of mathematics per se; rather, he considers that the then available tools – of the kind used by Walras – were not adapted to his vision of economic reality. Now, constructivist tools are available and there are attempts to model neo-mengerian theories, in particular through multi-agent techniques. Multi-agent modelling is an instance of constructivist tool whose diffusion within the community of economists may develop into a proper methodological revolution.

Section 1 Two competing philosophies of mathematics

The term 'formalism' is ambiguous because it bears a double significance. In its commonly accepted sense, formalism indicates nothing else than the mere use of symbols and unspecified mathematical techniques to express an idea. It is not this acceptance that the term implies when it is associated with the name of mathematician David Hilbert. By formalism, one then understands a particular approach which reduces mathematics to a pure symbolic language with no a priori meaning, and is opposed to intuitionism and logicism on the question of the foundations of mathematics.

The debate on the foundations emerged among mathematicians by the end of the nineteenth century, while attempts to extend the traditional Euclidean axiomatic method to branches of mathematics other than geometry were multiplying. This method consists in accepting without demonstration a reduced set of postulates, the axioms, and deducing from it a set of theorems. For a long time, the empirical obviousness of axioms seemed to guarantee the veracity of the theorems which it was possible to deduce from them. But the growing abstraction of the mathematical practice (axioms are no more evident) and the discovery by Cantor and Russell of logical antinomies (even if axioms were obvious, contradictions could emerge) bring to the foreground the question of the consistency of formal systems. 'Consistency' refers to a precise property: a formal system is consistent when it is impossible to deduce from its axioms two contradictory theorems. Three types of answer were advanced to give back to mathematicians their confidence in the rigour of mathematical practices.

- Logicians tried to found the consistency of mathematics by defining it as a branch of logic. The *Principia Mathematica* of Whitehead and Russell, published in 1910, falls under this heading. There, the authors propose a specific representation of arithmetic, whose goal is to clarify and make explicit all the logical inferences used in the reasoning and to show that all the concepts of arithmetic can be brought back to concepts of pure logic. However, this step did not gain much support from mathematicians as it did nothing but displace the problem: the

consistency of arithmetic depended on that of logic, and the consistency of logic was then itself under discussion.

- Intuitionists, headed by Poincaré and Brouwer, placed the authority of the perception and of the intuition of the mathematician above that of the logical principles and inference rules whose historical and cultural relativity were underlined. To be consistent, a system of calculation must thus be built from obvious and unimpeachable axioms and from rules of inference subjectively considered as reliable by the mathematician.¹ Brouwer (1912, p. 125) sums up the fundamental dissension which exists between intuitionism and formalism as follows: “to the question, where shall mathematical rigor be found, the two parties give different answer. The intuitionist says, in the human intellect; the formalist says, on paper”. Thus, the consistency of a mathematical theory does not require a demonstration for intuitionists insofar as it results from the construction itself of the theory, following the principles and the procedures acceptable to the majority of mathematicians.

Intuitionism is one form of constructivist mathematics; the various constructivist streams distinguish themselves relatively to the type of deductive rules and mathematical objects that are accepted as valid. Indeed, constructivists consider that there is no such a thing as a universally valid set of deductive arguments, but rather aesthetically and epistemologically valid deductive arguments, historically and culturally determined. For instance, intuitionists reject the relevance of the principle of excluded middle in elaborating their proofs whilst finitists, the extremist constructivist stream led by Kronecker, considered that a mathematical object exists only if it is possible to construct it by elaborating a finite process that starts only from natural numbers.² More generally, constructivists all agree on the idea that understanding a mathematical object means being able to construct it, that is, to provide an accepted method (a finite method, or a method based on intuitionist logic i.e. without the excluded middle principle or the axiom choice and so on...) for creating it, using a set of approved deductive arguments. This is the proof of its consistency.

- The response of formalists to the uncertainty on foundations consisted in trying to establish rigorous evidence of consistency of the various branches of mathematics. Demonstrations of consistency initially took the form of relative proofs. Thus, Hilbert showed that the consistency of Euclidean geometry depends on that of algebra. Thereafter, he tried, with the assistance of his disciples (the first of whom was von Neumann, but also Ackermann and Bernays) to provide an absolute demonstration of consistency of arithmetic. It is at this point that the famous impossibility theorem of Gödel intervenes. In 1931, Gödel arrived at a devastating result on the question of the foundations of mathematics. He, in fact, showed that it was impossible to provide a demonstration of absolute consistency of arithmetic.³ Gödel did not

¹ As a matter of fact, intuitionists reject the logical principle of exclusion of the middle for infinite systems. On this subject, David Hilbert wrote in 1928 in his *Die Grundlagen der Mathematik* that to remove this principle from the toolbox of mathematicians would be the same thing as prohibiting use of the telescope for astronomers, or boxers the use of their fists.

² According to Kronecker famous motto, « God created the natural numbers, all else is the work of men ».

³ Gödel actually arrived at two results in his 1931 article: (1) for any formal system including arithmetic, it is possible to construct a proposition which is true in this system, but not provable in it; and (2) one cannot prove the consistency of a formal system containing a finite theory of numbers.

prove the inconsistency of arithmetic, rather, the impossibility of showing that it was consistent, leaving the door open to the potential occurrence of new logical antinomies. In his book of reference on the question, Kline (1980) presents in a provocative way the debate on the foundations of mathematics as a major intellectual rout, liquidating the hitherto-dominant design of mathematics as height of rigour and scientific exactitude. The title of his work, *The Loss of Certainty*, returns us precisely to this radical reconsideration: mathematics cannot be unanimously regarded any more as a set of firmly established eternal truths.

This result certainly cooled the enthusiasm of formalists but did not put an end to the programme of Hilbert whereof the work on foundations constitutes only one part. Formalists gave up the hope of showing that mathematics is consistent, but they did not give up their confidence in the power of modern axiomatics as an engine for discovering new scientific knowledge. As Israel and Gasca (1995) note indeed, the formalism of Hilbert is founded on the belief in a pre-established harmony between mathematics and physical reality, a harmony which makes it possible to conceive mathematics as the base of all exact scientific knowledge of nature. The normative aspect of Hilbert's programme can consequently be interpreted as follows: the mathematical analogy, understood as the systematic adoption of the modern axiomatic approach represents good scientific practice and this is so, whatever the scientific field considered.

I believe: anything at all that can be the object of scientific thought becomes dependent on the axiomatic method, and thereby indirectly on mathematics, as soon as it is ripe for the formation of a theory. By pushing ahead to ever deeper layers of axioms . . . we also win ever-deeper insights into the essence of scientific thought itself, and we become ever more conscious of the unity of our knowledge. In the sign of the axiomatic method, mathematics is summoned to a leading role in science.

(Speech by Hilbert 1918, in Ewald, 1996 and Weintraub, 1998)

The association between axiomatic method and scientific rigour thus justifies the second side of the formalist programme of Hilbert consisting concretely in trying to extend this approach to other scientific disciplines, physics initially, but also economics. Thus, Hilbert's formalism has a double finality: to solve the problem of the foundations of mathematics (and, at this level, the results of Gödel are overwhelming); and to extend modern axiomatics to other scientific disciplines. This second aspect of the programme, the aspect that can be described as the imperialist or normative side, survived to Gödel's discoveries. Weintraub (2002, p. 90) identifies these two aspects of the formalist programme.⁴ He distinguishes between the *Finitist Programme for the Foundations of Arithmetic* (FPFA) whose objective was to found the consistency of arithmetic and the *Axiomatic Approach* (AA); it is this second aspect of the formalist programme which has actually influenced the process of mathematisation of economics through the contributions of von Neumann for the strictly Hilbertian version of the AA programme, and Debreu for the Bourbakist version.⁵ As argued in the next section, modern

⁴ Leonard (1995, p. 732) also puts forward these two aspects of the formalist program of Hilbert: to base all the branches of mathematics on a sure axiomatic base; and to extend axiomatics to other fields.

⁵ For an account of the differences between the Hilbertian and Bourbakist versions of formalism applied to economics, see Philippe Mongin (2003)

economics is not simply mathematical economics, it is formalist economics.

Section 2 The formalist revolution and its limits

The objective of this section is less to give a historiographic description of the formalist revolution than to stress the consequences of the adoption of mathematical formalism upon the nature of economics and upon practices of economists.⁶

For sure, the place and role of mathematics in economics have considerably evolved since last century. Consider for instance the striking contrast Lunghini (2004) makes between Edgeworth's and Debreu's entries in the *Palgrave* dictionary, respectively *Mathematical Method in Political Economy* and *Mathematical Economics*, one century apart. To Edgeworth, the mathematical method is useful insofar as it helps get rid of erroneous theories that flood the discipline. The author however warns against possible abuses resulting from an over-evaluation of its explanatory power. In particular, he explains, mathematical economics does not worth anything if the economic phenomenon under analysis is not empirically specified. At light-years from this view, Debreu promotes the specific bourbakist version of formalism as the relevant mathematical practice in economics. Standards of proof, of rigour, of truth in modern economics flow from the adoption of this conception of mathematics by economists since the 50s onwards, what Blaug (1999, 2003) describes as the "formalist revolution" in economics. This dramatic evolution in the practices of economists is centred around the works of Arrow (1951), Arrow and Debreu (1954), Patinkin (1956), Solow (1957), Koopmans (1957), Dorfman, Solow and Samuleson (1958), Debreu (1959) and around three main institutions, Karl Menger's mathematical Colloquium in Vienna, where the first demonstrations of the existence of an economic equilibrium were provided during the interwar period (Wald, 1935 and Von Neumann, 1945/46), the Cowles Commission where members of the Vienna Circle migrated in the 40s and Princeton University, where Bourbakist economists could develop their works.

The formalist turn of economics results from a double impulse. On the one hand, economists themselves were still looking for a method that meets the requirements of rigour of natural sciences, a method that allows to escape from the subjectivity of ideology (of the kind that marks for instance the planning debate of the interwar period) or rather a method that could be used to demonstrate, objectively, the superiority of market over plan, that is, a method that could be used to insist on the qualities of the concept of competitive equilibrium.⁷ Mathematicians on the other hand, were most eager to find new domains of application to their formal systems and economics was one of these. If with Gödel, formalists gave up the hope of

⁶ For a detailed history of the introduction of formalism in economics, see Weintraub (2002), Giocoli (2003) and Blaug (1999, 2003).

⁷ As reported by Velupillai (2004, endnote 15), the Rand Corporation of which ideological orientations are well-known, would have played an active role in "planning" to some extent, the import of bourbakism in economics.

showing that mathematics is consistent, they nevertheless did not give up their confidence in the power of modern axiomatics in discovering new scientific knowledge. Indeed, the formalism of Hilbert was founded on the belief in a pre-established harmony between mathematics and physical reality, a harmony which makes it possible to conceive mathematics as the basis of all exact scientific knowledge of nature.

The so-called formalist revolution in economics marked the strict separation between (mathematical) form and (theoretical) content, and is mostly characterised by the crucial importance economists give to a specific (non-constructive) kind of demonstration of existence of equilibrium. Let's examine these two aspects in greater details.

The separation between syntax and semantic may be understood as follows: a 'formal system' is composed of (1) a set of symbols, (2) a set of rules for transforming these symbols into formulae, (3) a set of rules for transforming the formulae, and (4) a reduced number of formulae representing the axioms of the system to be observed. By construction, a formal system has no semantic content and may take on different interpretations. A 'model' is an interpretation that is given to a formal system. The clear-cut separation between syntax and semantics – between the formal aspects of the system and its various interpretations – is one of the most salient characteristics of modern axiomatics. To formalise a theory in the sense of Hilbert means indeed emptying it of all of its semantic content and giving an abstract representation of it – the formal system – in the form of symbols, formulae (among them axioms) and sequences of formulae having no more obvious bond with the theory of departure. The formal system thus formed is like an abstract box, deprived of any significance, on which the mathematician works in order to draw theorems. At this stage, the question of the realism of the axioms is completely irrelevant. But it would be erroneous to say that in axiomatics reality does not matter at all, for in the next stage of the axiomatisation process, the objective is precisely to assign models to each formal system, that is, to find an interpretation in terms of real phenomena for the formal system.¹² A model consists of an interpretation of the formal system, each symbol receiving a meaning, and the same abstract box being able to receive various interpretations. The initial theory which inspired the formal system constitutes one model, among others. Formalism as a philosophy of mathematics is attached at this level with Plato's realism consisting of supporting the thesis that mathematics does not create anything, does not invent objects, rather, discovers pre-existent objects in the intellect. The power of axiomatisation is due precisely to the fact that the 'discovery' of an abstract box, of a formal system, makes it possible to explain several real phenomena, and rests on the belief of a preset adequacy between the structure of mathematics and reality.

From the axiomatic point of view, mathematics appears thus as a storehouse of abstract forms – the mathematical structures; and so it happens without our knowing how that certain aspects of empirical reality fit themselves into these forms, as if through a kind of preadaptation.

(Bourbaki, 1950, p.231)

As noted by Debreu (2008, p.???, our italics), "...in a global historical view, the *perfect fit* between the mathematical concept of a fixed point and the social science concept of an

equilibrium stands out”, in other words, economics offers a new semantic field of development to a pre-existing conceptual structure. More precisely,

“[a]n axiomatized theory first selects its primitive concepts and represents each one of them by a mathematical object...Next, assumptions on the objects representing the primitive concepts are specified, and consequences are mathematically deduced from them. The economic interpretation of the theorems so obtained is the last step of the analysis. According to this schema, an axiomatized theory has a mathematical form that is completely separated from its economic content. If one removes the economic interpretation of the primitive concepts, of the assumptions and of the conclusions of the model, its bare mathematical structure must still stand”

(Debreu, 2008, p...) ⁸

The importance of the demonstrations of existence of equilibrium that has been at the centre of interests of mathematical economists for decades is another salient characteristic of the adoption of formalism. The obsession of formalists is to provide consistent formal systems. Proving the consistency of a formal system means proving that it is free from contradictions. The very notion of proof is one criterion of classification between the various mathematical traditions. To constructivists proving a theorem, the existence of a mathematical concept, means finding an algorithm that ends up with the elaboration of the concept itself, in other words, with providing a method for creating the object, growing it; to formalists, proving a theorem means asserting its logical necessity. Equilibrium exists because otherwise a contradiction would emerge from the formal system. Existence proofs based on the fix-point theorem are negative demonstrations in the sense that they insist on the logical necessity of the existence of equilibrium. Equilibrium exists not because the theory gives an explanation of how it is possibly reached but because it is a logical necessity. *Understanding* a phenomenon holds a specific meaning to formalists, it means logically deriving it from a limited number of assumptions. This kind of non-constructive proof (or negative proof) allows a direct jump from the axioms of the model to its final outcome and accounts for the neglect of mainstream economists in the analysis of the economic process that possibly leads to equilibrium. As far as stability is concerned, the relatively poor results neoclassical economists reach and the nature of the difficulties they face – totally irrelevant conditions of gross substitution for instance – are the symptoms of the inappropriate nature of their investigation framework to study out of equilibrium dynamics.

As soon as the community of economists accepted the standards of rigour of the formalist tradition, the so-called formalist revolution was completed and this has overwhelming consequences on the nature and practices of economics.

The adoption and generalisation of the formalist approach in economics was supposed to bring ever more generality, rigour and simplicity. The separation between syntax and semantic would lead to the discovery of new theories simply from the mere reinterpretation of a pre-established formal system⁹; economics could in this sense becomes ideologically free since a

⁸ Accordingly, Hahn (1991, p.???) describes pure theory as “the activity of deducing implications from a small numbers of fundamental axioms”.

⁹ I argue elsewhere (2010) that von Neumann’s growth model is a reinterpretation, in the realm of economics, of a formal system he first elaborated in 1928 in game theory; another direct example is given by Debreu

formal system is supposed to convey no specific vision of the world before the interpretative step (we shall return to this claim at the end of this section); the axiomatic approach aims at eliminating the possibility of a logical contradiction by rigorously making explicit the primitive concepts and the chains of deduction leading to theorems so that if a contradiction was to emerge, it would be easier to identify with precision its origin in order to eliminate it. The position of the Bourbakist programme is for this reason evocative: the objective of this radical version of formalism is not to found mathematics any more, rather, to clarify, through the linking of formal systems with one another, the architecture and unity of mathematics. The mathematician must face contradictions, if they emerge, on a case-by-case basis.

Absence of contradiction, in mathematics as a whole or in any given branch of it, thus appears as an empirical fact, rather than as a metaphysical principle. The more a given branch has been developed, the less likely it becomes that contradictions may be met with in its further development. [...] What will be the working mathematician's attitude when confronted with such dilemmas? It need not, I believe, be other than strictly empirical. We cannot hope to prove that every definition, every symbol, every abbreviation that we introduce is free from potential ambiguities, that it does not bring about the possibility of a contradiction that might not otherwise have been present. Let the rules be so formulated, the definitions so laid out, that every contradiction may most easily be traced back to its cause, and the latter either removed or so surrounded by warning signs as to prevent serious trouble. This, to the mathematician, ought to be sufficient;

(Bourbaki, 1949, p.3)

As a consequence of this divorce between form and content, the work of the theoretical economist evolved, became more and more abstract, directed by “[...] the quest for the most direct link between the assumptions and the conclusions of a theorem. Strongly motivated by aesthetic appeal, this quest is responsible for more transparent proofs in which logical flaws cannot remain hidden, and which are more easily communicated” (Debreu, 2008, P.??). Progress in the formalist sense means elaborating consistent formal systems based upon less and less primitive concepts and assumptions. Starting from the walrasian formalist reformulation taken as a benchmark, mathematical economists search for ever weaker assumptions, refinements and extensions that still permit ~~for~~ the existence proof, modifying one part of the original formal system in order to find new models, ending up with a pile of models that stand more and more distant from economic reality. Walras’ applied problem of the possibility of a multi-market equilibrium in a real economy transformed into a mathematical problem to be solved with the standard of rigour of formalism. In his famous 1972 article, Kaldor (p. 1238-9), referring to the axiomatic approach, already warns against the consequences of this practice: “In terms of gradually converting an “intellectual experiment” (to use Professor Kornai’s phrase) into a scientific theory – in other words, into a set of theorems directly related to observable phenomena – the development of theoretical economics was one of continual *degress*, not *progress*”. Indeed, the “relevant” assumptions are relevant in relation to the capacity of the related formal system to exhibit a rigorous existence proof and not in relation to the realism of the related model flowing from this formal system. The risk is,

(2008,p???) and concerns economic theory of uncertainty: Arrow in 1953 without modifying the formal structure of the model, just added the state of the world in the traditional characteristics of a commodity.

as Debreu (2008, p.??) himself self-consciously conceded, that mathematical economists may be tempted to concentrate on the form to the detriment of the content, excluding from their analysis those economic phenomena not readily amenable to this form of mathematisation. Mathematical tractability overtakes the degree of realism of the modelling activity. Von Neumann was already well-aware of the danger of a radical formalist approach.

As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from "reality" it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely *l'art pour l'art*. This need not be bad, if the field is surrounded by correlated subjects, which still have closer empirical connections, or if the discipline is under the influence of men with an exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical source, or after much "abstract" inbreeding, a mathematical subject is in danger of degeneration. At the inception the style is usually classical; when it shows signs of becoming baroque, then the danger signal is up. It would be easy to give examples, to trace specific evolutions into the baroque and the very high baroque, but this, again, would be too technical.

(von Neumann, 1947, p.195)

Of course, these critics do not concern economics specifically but the most abstract practices of mathematicians, and more precisely the radical extension of formalism conveyed by the Bourbakist programme. By substituting the term 'mathematical' by 'economical' in the preceding quotation, the criticism remains valid to some extent, testifying to the success of the imperialist incursion of formalism in economics. This evolution into the "baroque", toward a practice where modelling activity becomes "*l'art pour l'art*" is the thread economics haven't been able to avoid.

Beyond the supremacy of form over content, connexion with reality deteriorates also as a consequence of the adoption of the formalist conception of explanation. "Explaining" a phenomenon simply means proving its consistency. So the competitive economic equilibrium is "understood" through existence proof directly linking assumptions to the final state of rest, no matter if the dynamics of the process that brings to equilibrium is totally left outside the realm of the analysis. As Blaug explains (2003, p.146), "[t]he Formalist Revolution made the existence and determinacy of equilibrium the be all and end all of economic analysis", with no need to investigate any more on the coordination mechanisms that leads prices to their equilibrium level, with no need thus to take into account non-price coordinating devices such as institutions in the broad sense of the term. The issues of uniqueness and stability do receive much less attention from economists than existence proofs and with relatively poor results.

The relative neglect by mainstream economics of economic processes, traditionally the object of study of the heterodoxy – Austrians, Evolutionists, Institutionalists – results also from an ontological circumstance. Economic processes are the result of the interaction of a multiplicity of individual agents interacting in a specific environment. Clearly, economic processes are investigation objects of a totally diverse nature than economic states and the

fiction of the representative agent is nothing else but an expedient allowing sidestepping the mathematically non tractable dimension of individual interaction. If equilibrium states can be expressed through structural simplification involving linear dynamics as a solution of a set of equations, processes are complex phenomena that can be expressed in terms of replication process. At this point it becomes obvious that the choice of the tools conveys an ideological dimension: the adoption of the formalist approach implies a focus exclusively on linear phenomena which may be fully captured through specific formal system whereas complex phenomena should be analysed through a totally different kind of approach which outlines are nowadays, in our field, still to be defined.

Section 3 An illustration: Austrians and mathematics

Menger is often presented in textbooks as one of the three leaders of the marginalist revolution whose originality simply lies in his firm rejection of the use of mathematics in economic analysis. The whole Austrian tradition is from that moment on characterised by this position regarding formalisation, as an inheritance from the scarce technical competences of its founding father.¹⁰ This characteristic refusal of Austrians to use mathematics is often quoted as the essential element of distinction with mainstream economics.

Two objections can be advanced to this assertion: first, Menger develops economic theories that by many aspects, and not only the form, are distinct from the marginalist stance; modern Austrian economics develops precisely around this originality and can definitely not be reduced to a literary version of neoclassical economics; second, the rejection of mathematisation is only the consequence of this originality and does not reveal any systematic opposition to the use of mathematics per se: the mathematical tools available at Menger's times and used by his contemporaries were not adapted to his vision of economic phenomena, in the same way as today, formalism does not allow modern Austrians to provide an accurate representation of their ideas. On the contrary, it seems that the Austrian tradition is much more in line with constructivist tools and in particular it may offer an interesting field of application to multi-agent modelling.

The scientific approach defended by Menger is purely analytical and consists in breaking down complex economic phenomena into their most simple elements, a logical decomposition in terms of relations of causality. On a methodological level, his objective is:

[...] to reduce the complex phenomena of human economic activity to the simplest elements that can still be subjected to accurate observation, to apply to these elements the measure corresponding to their nature, and constantly adhering to this measure, to investigate the manner in which more complex phenomena evolve from their elements according to definite principles.

¹⁰ Menger began to throw himself into self-taught mathematics in the 1890s. However, his son Karl, himself a great mathematician, ruthlessly declared : 'I am afraid that he did not acquire an operative knowledge, let alone a critical insight into calculus' (K. Menger, 1973, p.45).

Human behaviour seeking to satisfy needs is the simplest premise upon which everything may be built, thereby defining economics upon a strict subjectivist basis. Menger defines it as the principle of 'economizing'.¹¹

In his 1883 methodological work, Menger continues and goes deeper into the methodological foundations which, in his opinion, should underlie any theoretical science and economics in particular. Essentialism and universalism, the two principles at the core of Menger's methodology which were already introduced in the *Grundsätze* of 1871, are here confirmed and justified. The scientific approach, whose ultimate aim is to acquire general knowledge of phenomena, consists in systematically researching ultimate causes which are the very essence of these phenomena, by establishing general laws having a universal character, i.e. knowing no exceptions:

The goal of scholarly research is not only the *cognition*, but also the *understanding* of phenomena. We have *gained cognition* of a phenomenon when we have attained a mental image of it. We understand it when we have recognized the reason for its existence and for its characteristic quality (the reason for its *being* and for its *being like it is*).

(Menger, [1883] 1963, p. 43)

Understanding an economic phenomenon means identifying the causal process which brings it into being, starting from its most elementary cause – economizing principle – to the most complex manifestation of the phenomenon under analysis.

Clearly, Menger's conception of economics clashes with marginalism. The opposition has been first made explicit by Hans Mayer. Mayer ([1932] 1995, p. 57) distinguishes between two types of theoretical approach to the question of how economic prices are formed: *causal-genetic theories* which, 'by explaining the formation of prices, aim to provide an understanding of price correlations via knowledge of the laws of their genesis', and *functional theories* which, 'by precisely determining the conditions of equilibrium, aim to describe the relation of correspondence between already existing prices in the equilibrium situation'.

The adoption of a causal-genetic way of thinking in economics has a direct consequence as regards the use of mathematical tools. Indeed, according to Mayer (1932), Menger does not dogmatically reject any recourse to mathematics in economics, but he rejects mathematics in the only form that was available at the end of the 19th century, that is functional mathematics which are not adapted to Menger's conception of economic explanation. The Austrian position regarding the use of mathematics is the result of an ontological investigation, of an investigation on the nature of economic phenomena and economic understanding. Menger's position in this respect is straightforward and explicitly clashes with Walras' position. To the marginalist leader, mathematics is much more than just a mere demonstrative tool enabling him

¹¹ The term used by Menger ([1871] 1950, p. 116) is *Bedürfnissbefriedigung*, literally the satisfaction of needs and desires.

to give a simpler and more rigorous presentation than a literary equivalent. Mathematics is indeed a real investigative tool, a 'research method' in itself.

For the non-mathematician it is natural to believe that mathematical form, where it can be used, conveys nothing more than ordinary language, and is only used to explain things to those who cannot understand them in any other form; however this assertion will bring a smile to the lips of anyone aware of services rendered by mathematics to all sciences to which they can be applied: mechanics, astronomy, physics, chemistry.

(Letter by Walras, 16 January 1882)¹²

It is precisely here that the methodological rift between Walras and Menger lies.

The object of my research is to reduce complex economic phenomena to their true causes, and to seek out laws according to which these complex phenomena of political economy are repeated. The results of my research may be represented by mathematical formulae. Mathematical representations may help with the demonstrations: however, the mathematical method of representation is in no way the essential part of the task I have undertaken.

(Letter by Menger, 1 June 1883)¹³

The mathematical method used by Walras seems far from appropriate to Menger's objective, i.e. knowing how to determine the *essence* of complex economic phenomena.

"My opinion is actually that the method that should be adopted within pure economics cannot be simply called *mathematical* or *rational*. We should not only investigate relations between magnitudes but also the *essence* of economic phenomena. But how could we know this essence, for instance, the essence of value, of entrepreneurial profit, of labour distribution, of bimetallism, etc. in a mathematical way? Even if the mathematical method was purely and simply justified, in any case, it would not fit with the solution of the mentioned part of the economic problem.

However, I cannot accept the mathematical method at all, even for the determination of the laws of economic phenomena [...].

The problem which you consider to be the most important is the formation of the *laws* according to which goods exchange with goods. Using the name of goods, we, German people, include means of production as well as products, more precisely all the things that contribute directly or indirectly to the satisfaction of human needs. Are the quantities of goods which we exchange in trading (quantities that change according to time and places!) arbitrary or are they ruled by fixed *laws*? This is the question. Did I understand you well in considering that the investigation of these laws is the main concern of your research?

Now, if this is the case, it is at the same time clear that the purpose of your investigations will never be reached through the mathematical method. It is rather necessary that we come back to the simplest elements of phenomena which are generally very complex -therefore that we determine *analytically* the last constitutive factors of phenomena [...].

Let us consider the theory of prices. If we want to have access to the knowledge of the laws which rule goods exchange, it is first necessary to come back to the motives which lead men to act within exchanges, to the facts which do not depend on the will of traders, which have a causal relation with goods exchange.

¹² Our translation from French, from Antonelli (1953, p.269, footnote n°3).

¹³ Our translation from French, from Antonelli (1953, p.272).

We should come back to the needs of men, to the importance they give to the satisfaction of needs, to the quantities of different goods which different economic agents own, to the subjective importance (subjective value) that different economic agents confer to given quantities of goods and so on"

(Letter from Menger, translated from Antonelli, 1953, pp. 279-281)

As soon as the aim of the theorist is to understand the process of emergence of a phenomenon through causal decomposition into its primary elements, formalisation in the form of a system of simultaneous equations is inappropriate since it turns a blind eye to the sequence leading to the formation of the phenomenon, focusing exclusively upon the ultimate outcome of the process. Rejection of mathematical formalism by Austrians is thus justified because there exists a direct correspondence between formalist tools (mainly functional relationships, derivative and systems of simultaneous equations, topology and fix point theorem) and the functional approach defined by Mayer. Menger's refusal of functional mathematics should thus be analysed as an ontological awareness of the specificity of the economic *explanation* rather than the evidence of his weakness in mathematic techniques.

The modern Austrian tradition although not monolithic, picks up again and deepens the Mengerian causal-genetic orientation associated with Hayekian and Misesian insights: the object of investigation is the economic process; this process is a complex phenomenon resulting from the interaction of purposeful agents, confronting one another in a specific environment of diffuse and incomplete knowledge, each agent taking its decision on the basis of its individual subjective plans. Austrians do not make any concession to formalism in the sense that mathematical tractability does not constrain the assumptions of their analysis. For instance, they never rely on the fiction of the representative agent to build models and are very critical towards an assumption that has no meaning in a context where strategic interaction of distinct cognitive agents is precisely the object of investigation. In the same vein, Austrians do not introduce market clearing assumptions ensuring the existence of economic equilibrium and focus on the coordination properties of markets. In other words, not being committed with formalism allows Austrians to deal with a series of non tractable assumptions such as the heterogeneity of agents, multiple equilibria, learning processes, competitive processes and so on.

The alternative however, is not between formal unrealistic models and literary exposition. If this was the choice Menger was facing at the end of the 19th century, this is not anymore the case today with the development of constructivist tools, associated with diverse standards of rigour and proof, that allow for at least an heuristic insight into complex economic phenomena.

A possible, promising, constructive approach to economic theory is provided by agent-based modelling which allows for a quantitative study of the aspects put forward in a qualitative manner by the Austrian tradition. It consists in object oriented computer programming aiming at simulating market processes understood as dynamic complex systems of interactive agents in order to identify the macro regularities that emerge from the local interactions. As Tesfatsion defines it (2006, p 7), a complex system is characterised by two elements: it is composed of interactive units and exhibits emergent properties that is, properties distinct from those of the interactive units, in the sense that what goes out is more than the sum

of what goes in. The degree of abstraction is considerably reduced in the sense that there could be no one model of the market process but plenty of models of market processes according to the taxonomy of agents selected (arbitragists, speculators, intermediaries...) and the characteristics of the environment (nature of the institutions) in which they interact. Lachmann (1977, p.34), one of the three leaders of the modern Austrian revival of the 70s along with Kirzner and Rothbard, calls for a reduction of the degree of abstraction associated with “late classical formalism” as he names the set of functional theories in Mayer’s sense, and defends an ideal-typical approach to market processes, which, to this extent, seems in perfect fit with the agent-based approach.

Agent-based modelling is part of constructivist techniques in the sense that it offers a generativist understanding of phenomena. As Epstein (2006, p.1587) sums up, “if you don’t grow it, you didn’t explain it”: understanding a phenomenon means being able to generate it starting from local interactions of heterogeneous agents with bounded rationality.

“To explain a macro-x, please show how it could arise in a plausible society. Demonstrate how a set of recognizable – heterogeneous, autonomous, boundedly rational, locally interacting- agents could actually get there in reasonable time. The agent-based computational model is a new, and especially powerful, instrument for constructing such demonstrations of generative sufficiency.” (Epstein, P.1587)

This is exactly the kind of tools adapted to Menger’s approach, a tool in accordance with causal-genetic explanation, that allows to focus on the process of emergence of social regularities – organic institutions in Menger’s terms – and that takes the individual and his specific rationality – economizing principle – as the essence, the primary cause, of complex economic phenomena.

Despite some isolated attempts to engage Austrian economics in the way of agent-based modelling (Holian and Graham 2011, Chad and Seagren 2011, Nell 2010, Vriend 2002) or more generally towards the use of constructivist tools (Oprea and Wagner 2003, Vaughn 1999, Lavoie and Al. 1990), one cannot but notice that modern Austrians have been committing themselves in that direction rather gingerly, most papers being methodological discussions rather than modelling proper, justifying to some extent the criticism of dogmatism they are addressed. Given the apparent perfect fit that exists between agent-based modelling and the Austrian conception of economic phenomena, this may appear as a real conundrum. There is a rapid and simple answer to that paradox. Agent-based modelling seems to be the answer to Hayek’s analysis of spontaneous order resulting from the interaction of individual agents in a context of dispersed and tacit knowledge. The Hayekian concept of spontaneous order conveys however, a narrower meaning than that of emergence, which may hamper the use of these techniques by Austrian economists. Contrary to the more general idea of emergent properties, a spontaneous order conveys implicit welfare properties. The whole Austrian edifice is oriented towards the normative objective of assessing the qualities of decentralised market processes whereas multi-agent modelling of market processes has no a priori reason to end up with efficient regularities. Simulations may display back and forth between coordination patterns and uncoordinated situations, with the possible occurrence of numerous catastrophes. In other

words, agent-based modelling does not guarantee the free market point of view.¹⁴ No doubt that the adoption of a specific mathematical tool is decided on the basis of the adequacy of this tool with the ontological perception of the scientist (his conception of explanation, of rigour, of truth), with his investigation field (the kind of phenomena he wants to capture, complex or linear phenomena, equilibrium state or processes...) but also with his ideological stance (the normative value of his approach); as far as the Austrian tradition is concerned, the primacy of this latter dimension above the rest is definitely an obstacle to the widespread use of agent-based modelling.

Conclusive remarks

The object of this paper has been to assert the implications, in terms of practices of the community of economists and constraints for their analysis, of the adoption of specific mathematical tools. Our thesis is opposite to the position of Samuelson (1952) who claims the neutrality of mathematics in economic analysis, in the sense that there is, according to him, a strict equivalence between the mathematical language, of any kind, and prose, the former simply allowing for simplest handling of long chain of reasoning. Mathematics is reduced to a mere medium of expression strictly identical to literary exposition “There is no place you can go by railroad that you cannot go afoot and I might add, vice-versa” (Samuelson, 1952, p.56). The author examines the odd position of Menger with respect to Jevons and Walras, but he simply concludes that, of the three marginalists, Menger was probably the less interesting given his focus on qualitative features of economic phenomena and his search for the identification of the essence of these phenomena. The thesis here defended is that Menger could not engage in his research program given the nature of the mathematical tools then available. Nor could modern Austrians investigate the nature of the market process with the help of formalist tools. Constructivist tools, with their associated distinct conception of explanation, may provide an interesting alternative.

Ultimately, the view defended here is that mathematics matters, it matters to the extent that one could, like Giocoli (2003), use it as demarcation line in order to classify theories instead of or in addition to the traditional considerations concerning the nature of assumptions. On one side formalists theories, characterised by “[...] non-descriptive content, the adoption of topological tools, the faith in non-constructive existence proofs” (Giocoli, 2003, p.33), on the

¹⁴ At least two other reasons may be put to the fore to explain the Austrian reluctance: a first obstacle is the anti-constructivist stance of Hayek who in the 40s criticises the scientist point of view consisting in adopting the same investigation methods in both natural and social sciences (Hayek 1942). The position of the author has however evolved later on when, in *The Sensory Order* he adopts a distinct classification between sciences, those which concentrate on complex phenomena and those which concentrate on non complex phenomena (see Caldwell 2004). A second obstacle may emerge from the strict methodological individualism of Austrians that is only apparently in phase with agent-based modelling where by “agent” is meant not only cognitive agents but also organisations, institutions and so on. Agent-based modelling allows to understand the emergence of institutions – a very Austrian issue indeed since Menger’s analysis of the emergence of money – but also the influence of institutions on economic individual decisions – much more difficult to integrate in an Austrian framework (see Gloria-Palermo 1999 for a discussion of these difficulties).

other side, constructivist theories, characterised by realist starting points, simulation tools, generative explanations.

Weintraub (2008 Pal) provides a history of mathematical economics where each rupture in the *body* of economics – set of theorems proved, of techniques used...– corresponds to the adoption of a changing *image* of the mathematical knowledge – concept of mathematical rigor, of a good proof...¹⁵. Weintraub identifies three steps: (1) by the end of the nineteenth century mathematics is tantamount to Euclidian axiomatic; axioms being evident, theorems logically flowing from them were without doubt true and economics engages in the verificationist hypothetico-deductive approach of the Ricardian school. (2) The rise of non Euclidian geometry questions this correspondence between theorems and truth; successful mathematical models are then those susceptible of a physical interpretation and economics adopts the mechanical analogy. It is the age of Walras, Pareto, Marshall, Edgeworth and Fisher. (3) the third step refers to the formalist revolution under scrutiny in this paper, with the problem on the foundations of mathematics opening the way to the adoption of the bourbakist standards of rigour.

The image of mathematics is changing again, Colander (2000) suggests that once again economists follow mathematicians in the adoption, this time, of the paradigm of complexity and its associated constructivist philosophy.¹⁶

Criticisms of mainstream economics are not new, but they may have a major impact today simply because, as soon as one realises that many of its limits are the direct consequence of the use of specific mathematical tools, the very existence of alternative techniques may help to overcome the lock-in in which our discipline has been engaged for decades.

Being aware of the non neutrality of mathematics is thus the first step. « ...[A]s economic analysts we are directed by, if not *prisoners* of, the mathematical tools that we possess » (Sargent 1987, p.xix, italics added). The next step consists in analysing the consequences of the adoption of such or such tool in order to help to free us from unnecessary constraints and discard the tools associated with a set of value judgements (on the ontological or ideological levels) not adapted to one's conception of economic phenomena.

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¹⁵ The distinction between body and image is borrowed from Corry (1996).

¹⁶ This intuition is to a certain extent shared by Weintraub (2008) who mentions the constructivist lead noticing that mathematicians that build models are now enjoying a respect as high as that traditionally enjoyed by mathematicians that prove theorems; The potentialities of the constructivist trajectory is also examined in details by Velupillai (1996, 2004).

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