Groupe de REcherche en Droit, Economie, Gestion
UMR CNRS 7321

AMBIGUITY, AGENCY RELATIONSHIPS AND ADVERSE SELECTION

Documents de travail GREDEG
GREDEG Working Papers Series

Gérard Mondello

GREDEG WP No. 2012-06

http://www.gredeg.cnrs.fr/working-papers.html

Les opinions exprimées dans la série des **Documents de travail GREDEG** sont celles des auteurs et ne reflèlent pas nécessairement celles de l'institution. Les documents n'ont pas été soumis à un rapport formel et sont donc inclus dans cette série pour obtenir des commentaires et encourager la discussion. Les droits sur les documents appartiennent aux auteurs.

*The views expressed in the **GREDEG Working Paper Series** are those of the author(s) and do not necessarily reflect those of the institution. The Working Papers have not undergone formal review and approval. Such papers are included in this series to elicit feedback and to encourage debate. Copyright belongs to the author(s).*

Ambiguity, Agency Relationships and Adverse Selection

Gérard Mondello[♣]

September 2012

Summary

This paper applies recent results and advances in the field of ambiguity theory to adverse selection in a delegation process. Our results are the following ones: i) a relevant second-best contract induces no production distortion considering the efficient agent. This alike to the standard case. But the principal will pay him an higher information rent compared the standard case; ii) This is due to the level of transfer paid to the inefficient agent which is higher than under the complete information system (i.e. the first best…); The above results are reached when the agent has neither fully optimistic nor optimistic beliefs. When, he feels an extreme feeling then, the information rent and second best transfers are inside bounds similar to the SEU case; iv) as a consequence, the principal has to adopt a flexible behavior consisting in acquiring new information for becoming either entirely optimistic or pessimistic to minimize transfers and information rent in the proposed delegation contract.

Keywords: Asymmetric information, agency theory, adverse selection, uncertainty, ambiguity theory, irreversibility, information arrival.

JEL:D80**,** D82, D86

[♣] University of Nice Sophia Antipolis, GREDEG,UMR 6727,CNRS.
250, rue Albert Einstein, 06560 Valbonne Sophia Antipolis- France.
Tel.: + 33-4-93954327-fax:+ 33-4-93653798,
gerard.mondello@gredeg.cnrs.fr

0. Introduction

.

This paper applies recent results and advances in the field of ambiguity theory to adverse selection in a delegation process[1]. The question is to know to what extent ambiguity theory can change the nature of incentives relationships in an asymmetric information framework. Adverse selection arises when a principal wants to delegate a charge to an agent. As specialized literature showed it, this choice can be motivated either by the will to benefit from increasing returns (division of work) or to perform efficiently several tasks. Concrete examples do not miss. Are concerned all situations that involve contractual relationships between two entities, (firm, agents or group or agents, etc.). The delegation process generates information asymmetry because the agents can know private information (true ability, cost structure and technologies or hidden knowledge). Using efficiently the economic resources involves making symmetric private information between all parties and the price for this consists in an information rent allowed to the privately informed agent. Hence as a second best, the principal has to disburse an informational rent for receiving the desired amount of merchandise.

We focus on the simplest formulation of the one-shot adverse selection model of Laffont and Martimort (2002) to study ambiguity in the context of asymmetric information. One shot means no-repetition, no-adjustment process and the necessity to formulate a contractual framework. This standard model will help to implement the argument of ambiguity theory and compare the results. Hence, the main goal of this paper is to characterize the rent extraction-trade-off process faced by the principal when designing his contractual offer to the agent under "true" uncertainty. For standard view, both the principal and the agent are rational and optimize their own individual expected utility. The simplest representation considers two types of agent the efficient and the inefficient ones. The principal does not know the agent's private information and he cannot distinguish between them and their proportion (probability distribution) of each is common knowledge. Considering "real" uncertainty, and as a consequence, ambiguity about it, it appears that beliefs can no longer be represented by a unique probability distribution. This has been shown by the early works of Ellsberg and a huge literature, both highly theoretical and empirical that

[1] The references about these advances are presented in the following of the introduction and in section 1 and appendix 1.

followed the publication of the so called “Ellsberg” paradox in 1961 (See Teitelbaum (2007) for a huge view about it). In our model, the principal forms beliefs about the probability distribution about the states of nature. We define the principal’s objective function is a specific Choquet integral i.e. a“neo-additive capacity” as described by Chateauneuf Eichberger and Grant (2007), or still, Eichberger, Kelsey and Schipper (2005). This objective function is substituted to the standard expected utility function and this one is optimized within the set of incentive and participation constraints. More precisely, this function is understood as the weighted sum of the maximum value, the minimum one and the average he can expect from his transaction with the agent.

The adverse selection literature literature considers that uncertainty is described through Savage expected utility theory in which the distribution of probability of uncertain events is additive (Savage (1954)). It shows that, as the first best contracts cannot be enforced under Savage uncertainty, then, second best contracts involve a distortion in the volume of goods or services due to the payment of an informational rent to the efficient agent. This deters him to mimic the inefficient agent. Substituting neo-capacity to standard Savage utility expectation (SEU) involves that the the principal faces ambiguous beliefs and may feel either optimism or pessimism about the efficiency of the agent.

Then, our main results consist in showing:

- First, that a relevant second-best contract produces no production distortion of production considering the efficient agent. This is conform to the standard case and an information rent will be paid to him by the principal. However, the level of information rent, which depends partly on the remuneration of the inefficient agent is higher than under the standard case.
- Second, concerning the inefficient agent, the level of transfers is higher than under the complete information system (i.e. the first best situation…).
- Third, the above results are reached when the agent feels neither fully optimistic nor optimistic. When, he feels an extreme feeling then, the information rent and second best transfers are inside bounds similar to the SEU case.
- Fourth, from the above results we deduce that the principal has to adopt a flexible behavior consisting in acquiring new information for becoming either entirely optimistic or pessimistic to minimize transfers and information rent. Hence, introducing a neo-additive capacity as utility function instead of SEU involves that the principal takes into account opportunity costs linked to potentially irreversible

choices. The tools of information theory under true uncertainty have to be used to form the contractual relationship between the principal and the agent.

Obviously, applying ambiguity to adverse selection relationships is not that new. For instance, Wakker, Timmermans and Machielse (2007) analyse empirically this question in the field of insurance theory: they test how agents are wary about ambiguity. More theoretical is the dissertation of Martinez-Correa (2012) who is interested, among others, in testing the impact of ambiguity on Borch-Arrow insurance markets. He shows that the level of coinsurance under ambiguity, compared to risk only, will depend on the relative importance of risk and ambiguity for each party and the differences in beliefs which are translated by the ratios of the risk aversion and ambiguity aversion parameters. These works are rather far from our own purpose which is to study the behavior of a principal endowed with non-additive utility function when involved in an agency relationship.

In a first part, we remind the theoretical background involved by the using of neoadditive capacities compared to standard capacities. In a second part, we describe the set of allocations that the principal can attain in spite of the lack of information he undergoes. He has to induce the agents to reveal their true nature by defining a set of incentive compatibility constraints and participation constraints. Then, in a third part we introduce neo-additive capacities in the Principal's program. In a fourth part, we sketch the features of the informational belief system corresponding to the acquisition of new information. A fifth part concludes.

1. Background

For the last sixty years, the leader uncertainty theory of choice has been the subjective Savage expected utility theory (SEU) (Savage (1954)). This theory combines de Finetti's (1937) calculus of subjective probability and the expected utility theory of von Neumann and Morgenstern (1947). Since its early formulation, this leadership has always been put into question from theoretical and empirical basis (Allais (1952)). The critics study violations of its main rationality axioms and this raised legitimatedoubts about the foundations of SEU. We recall that in SEU, the decision makers are endowed with personal probabilities defined on several uncertain states (states of nature). They express preferences on these states and the SEU axioms show the conditions under which they can be represented by a numerical expected utility. Subjective probabilities are inferred from their choices.

The well-known "Ellsberg's paradox in 1961 (Ellsberg (1961)) revitalized the distinction between risk (known probabilities as for a dice) and uncertainty (probabilities unknown) made by Keynes and Knight at the beginning of the XXth century. The Ellsberg's paper is based on an assessment of the behavior of people placed in a situation similar to the one analyzed by SEU. Ellsberg analyses the behavior of individuals who have to make a similar choice under two situations. Considering the first one (A), the probabilities of the choice are known and, for the second one, these probabilities are unknown (B). He shows that, that rational individuals will prefer to choose for the known proportion case, rather than for the unknown one. This matter of fact violates the sure-thing principle of the SEU. Implicitly, even ignoring the true probability of (B), by their choice in A, they do "as" if the chance of success in B were lesser. He deduces that such agents feel aversion for ambiguity. This can be shown in the following example that will introduce our further argument.

We give now a simple illustration of the consequences of true uncertainty on the choices of a manager (principal). Let us assume that he has the opportunity to hire the services of an agent in a location A. A priori he does not know whether the worker is skilled enough or not. If this last one is qualified the principal will gain $\underline{S}$, if not, his gain will be limited to $\overline{S}$, with $\underline{S} > \overline{S}$, ($\underline{S}$, $\overline{S} \in \mathbb{R}_+$). The principal knows only the proportion of skilled and unskilled workers in the A area, i.e. respectively $\underline{p}$ and $\overline{p} = 1 - \underline{p}$, $(1 \geq \underline{p} \geq 0)$ this proportion. Let us assume now that he has the opportunity to earn the same amounts $\{\overline{S}, \underline{S},\}$ in a location B, but, the proportion ($\underline{p_f}$) between skilled and inexperienced workers$(1 - \underline{\mathrm{p}}_{\mathrm{f}})$ is unknown to him. We can express the expectation of gain for location A as $E[S/A] = \underline{p}\ \underline{S} + (1 - \underline{p})\overline{S}$ and for the location B, $[S/B] = \underline{p_f}\ \underline{S} + (1 - \underline{p_f})\overline{S}$.

If the principal chooses A rather than B, that means that $\mathrm{E[S/A]} > E[\mathrm{S/B}]$ or still:

$\underline{p}\ \underline{S} + \left(1 - \underline{p}\right)\overline{S} > \underline{p_f}\ \underline{S} + (1 - \underline{p_f})\overline{S}$ *or,* $\overline{S} + \underline{p}(\underline{S} - \overline{S}) > \overline{S} + \underline{p_f}(\underline{S} - \overline{S})$

And, naturally: $\underline{p} > \underline{p_f}$.

Hence, by choosing A, the principal has implicitly allocated a lower probability to meet a skilled work in B. He has expressed his aversion for ambiguous situation. This does not prevent him to meet a non-experimented worker in A. We can go further and consider the following relationships:

i) $\underline{p}\ \underline{S} + (1 - \underline{p})\overline{S} > \underline{p_f}\ \underline{S} + \left(1 - \underline{p_f}\right)\overline{S} \Rightarrow \left(\underline{p} - \underline{p_f}\right)\underline{S} > \left(1 - \underline{p_f} - 1 + \underline{p}\right)\overline{S}$ and naturally,

$$\underline{S} > \overline{S}$$

ii) However, if we consider the same relationships under the following side i.e.:

$$\underline{p}_f\,\underline{S} + \left(1-\underline{p}_f\right)\overline{S} < \underline{p}\,\underline{S} + (1-\underline{p})\overline{S} \Rightarrow \left(\underline{p}_f - \underline{p}\right)\underline{S} < \left(1-\underline{p}\right)\overline{S} - (1-\underline{p}_f)\overline{S}$$

$$\left(\underline{p}_f - \underline{p}\right)\underline{S} < \left(1-\underline{p}\right)\overline{S} - (1-\underline{p}_f) <$$

vanishes and, $\underline{S} < \overline{S}$.

We are facing a contradiction because $\underline{S} > \overline{S}$, this involves that ii) should be writes as $\underline{p}_f\,\underline{S} + \left(1-\underline{p}_f\right)\overline{S} > \underline{p}\,\underline{S} + \left(1-\underline{p}\right)\overline{S}$ which means that $\underline{p}_f > \underline{p}$. With this result, implicitly, we describe the situation in which the principal would have chosen B rather than A.

Hence, Ellsberg (1961) shows that beliefs cannot be represented by usual distribution probabilities: the agents do not use probability in a linear manner as does the expected utility model. Ellsberg's paradox highlights the non-additivity of "subjective probability" (or "beliefs") attributed to complementary events. Ellsberg shows that agents' preferences do not respect the principles of the Savage theory of uncertainty (pre-order principle and the sure-thing principle). This is considered by Schmeidler (Gilboa and Schmeidler (1989), Schmeidler (1989)) as the "first principle of bayesianism" i.e. the representation of uncertainty by a single measure of additive probability. This author used Choquet expected utility (henceforth CEU), which involves representing individuals' beliefs with non-additive probabilities (or capacities). Following this approach, individuals maximize the expected value of a utility function with respect to a non-additive belief and this corresponds to a Choquet integral.

Besides Shmeidler's approach which generated a proper theoretical trend, Ellsberg's paradox gave rise to empirical experiments. Laboratory tests showed distortions between losses and gains in the mind of people and the importance of the nature of uncertainty. Nowadays, the literature about uncertainty and ambiguity is huge and several fields are being studied. One among the more remarkable contribution are the works of Gonzalez and Wu (1999) who attempt to bring together theoretical and empirical studies in the aim at defining the effective behavior of individuals facing uncertainty. Their work is largely inspired by the studies of Kahneman, and Tversky (1979), Tversky and Kahneman (1986) and 1992) or still Tversky and Fox (1995) or Tversky and Wakker (1995). They try to understand how individuals distort outcomes and probabilities. They show that agents systematically tend to overweight low probabilities and underweight high ones. Furthermore they misweight near the extremes of the probability measure. Hence, this involves that they conform their beliefs to the well-known inverse-S-shaped probability weighting function. This empirical approach has been systematized by a trend of theoretical works that simplified also the quite difficult applicable Schmeidler approach.

Considering the critical approach of Schmeidler and his followers Eichberger, Kelsey and Schipper (2005) noted that the general CEU model is hard to apply, first because, of its mathematical complexity and, second, because there are too many free parameters. They show that a capacity on a set with n elements involves2^n parameters, while n-1 parameters describe a probability distribution on the same set. Hence, Chateauneuf, Eichberger and Grant (2007) and Eichberger, Kelsey and Schipper (2005) develop the concept of neoadditive capacity. This rather new concept is a probability weighting function and "Neo" is an abbreviation for Non-extremal outcome additive. This approach helps to assimilate the contribution associated to the inverse-S-shaped probability weighting function (Gonzalez and Wu 1999). Indeed, under Choquet expected utility with a neo-additive capacity, an agent's preferences are represented by a weighted sum of the minimum utility, the maximum utility, and the expected utility. This links together experimental analysis and theoretical approaches that were quite disconnected into the literature about decision under uncertainty. Concretly, preferences are represented by the choice of a concrete act:

$$V_p(a) = \lambda M(a) + \gamma m(a) + (1 - \gamma - \lambda)E_p(a)$$

Where $M(a)$ represents the maximum value of act a, and $m(a)$ the minimum one. Parameters λ and γ express respectively the level of optimism and the degree of belief of the agent in the expected value of a. For instance consider that if $\gamma = 0$, the agent will assess his preferences by the expected value of a and the maximum value

$$V_p(a) = \lambda M(a) + (1 - \lambda)E_p(a)$$

Hence, λ measures the level of optimism of the decision-maker, while γ expresses his level of pessimism[2]. As a consequence, the agent will over-weight a number of good and bad outcomes compared to expected utility theory. Considering neo-additive-capacities only the best and worst outcomes are over-weighted. In appendix one, we provide a simplified version of these developments.

2. The basic framework

In what follows we consider that a principal (firm, consumer, regulator) wants to delegate to an agent the production of q positive units of a good. For the principal the value of these q units of good is $S(q)$ where the derivatives are the following:

$S'(q) > 0$, (increasing in q), and negative or null in the second derivative $S''(q) \leq 0$. Hence $S(q)$ is assumed to be concave with $S(0) = 0$. We assume that there are no fixed costs

[2] See Teitelbaum (2007)

(because here they add nothing to the model and are eliminated in the following of the argument). As remuneration, the agent receives a transfer equal to t for having produced q and supplied a value of $S(q)$ to the principal. As usual under the scope of asymmetric information, cost productions of agent's are assumed unobservable to the principal, but, it is common knowledge that the marginal cost belongs to the set: $\Psi = \{\underline{\theta}, \overline{\theta}\}$ with the fact that $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$. The index $\underline{\theta}$ points out efficient agent while a $\overline{\theta}$ index corresponds to an inefficient one. Past experience shows that the Principal will meet efficient $\underline{\theta}$-agent with a probability of ϑ and inefficient $\overline{\theta}$-agent with a probability $1 - \vartheta$. We deduce then the cost function that is assumed linear in q:

- $C(q, \underline{\theta}) = \underline{\theta} q$ with a probability ϑ or, (1)
- $C(q, \overline{\theta}) = \overline{\theta} q$ with a probability $1 - \vartheta$. (2)

The linearity of cost will be our working assumption.

2.1. The contracting sets under complete and incomplete information

A contract is basically defined by a quantity of goods to supply from the agent to the principal and the transfer as the due recompense (payment) from the principal to the agent. Let $\mathcal{B}$ be the set of feasible contractual allocations where:

$$\mathcal{B} = \{(q, t): q \in \mathbb{R}_+, t \in \mathbb{R}\} \quad (3)$$

A third party can observe and verify these variables (generally authors refer to a court of law or a specialized agency).

In the usual asymmetric information framework, it assumed that the agent discover his type, then the Principal offers a contract that will either be accepted or rejected by the agent, and if accepted, the contract will be accepted. This process is called the timing of the contractual relationship.

2.1.1. The complete Information Optimal Contract

This case will be used as the benchmark of this agency relationship. As a consequence, we make the assumption that there is no information asymmetry between the principal and the agent. The efficient productions levels are get by seeking for the first order conditions of the social welfare function (W^{CI}) under complete information:

$W^{CI}(q, \theta) = S(q) - C(q, \theta), \theta \in \Psi$.

We deduce then:

$$S'\left(\underline{q}^*\right) = C'\left(\underline{q}^*, \underline{\theta}\right) = \underline{\theta} \quad (4)$$

And,

$$S'(\overline{q}^*) = C'(\overline{q}^*, \overline{\theta}) = \overline{\theta} \quad (5)$$

According the nature of the agent, efficient or not, under complete information, the production level should be $\underline{q}^*$ or $\overline{q}^*$, if their social value $W^{CI}\left(\underline{q}^*, \underline{\theta}\right) = S\left(\underline{q}^*\right) - C\left(\underline{q}^*, \underline{\theta}\right)$ or $W^{CI}\left(\underline{q}^*, \underline{\theta}\right) = S(\overline{q}^*) - C(\overline{q}^*, \overline{\theta})$ are obviously non-negative.

We have to note that the condition that the social value of production achieved with an efficient agent is always higher than an inefficient one, i.e. :

$$S\left(\underline{q}^*\right) - \underline{\theta}\,\underline{q}^* \geqq S(\overline{q}^*) - \underline{\theta}\overline{q}^* \geqq S(\overline{q}^*) - \overline{\theta}\overline{q}^* \quad (6)$$

This is a direct result of $\Delta\theta > 0$.

An important result for the following is that $\underline{q}^* > \overline{q}^*$, i.e. the optimal production of an efficient agent is greater than an inefficient one because the principal's marginal value of output is decreasing. Implementing the first best production levels entails for the principal to make a take-it-or leave-it offer to the agent which involves a zero profit opportunity. Then, the efficient contracts under asymmetric information, $\mathcal{B}^{\mathrm{AI}*} = \left\{(\underline{t}^*, \underline{q}^*) \text{ and } (\overline{t}^*, \overline{q}^*)\right\} \sqsubseteq \mathcal{B}$, will have to meet the participation constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq 0 \quad (7)$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq 0 \quad (8)$$

(with equality respected for an optimal contract).

Constraints (1.7) and (1.8) are called "participation constraints of the agent" and mean that the principal has to supply that level of transfer which insures to the agent a utility level at least as high as the agent could obtain without doing anything. Hence an optimal contract under complete information has to verify that $\underline{t}^* = \underline{\theta}\,\underline{q}^*$ and $\overline{t}^* = \overline{\theta}\overline{q}^*$.

2.1.2. The asymmetric information case

We assume now that the efficiency type of the agent $\Psi = \{\underline{\theta}, \overline{\theta}\}$ is private information. It is well known that the set of efficient contracts $\mathcal{B}^{\mathrm{CI}*}$ defined under complete information cannot be implemented because the efficient type could simulate the behavior of the inefficient one and, if profitable to get an informational rent $(\overline{t}^* - \underline{\theta}\overline{q}^*)$. Consequently, the efficient allocation set under asymmetric information $\mathcal{B}^{\mathrm{AI}*}$ will have to satisfy the following incentive compatibility constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq \overline{t} - \underline{\theta}\overline{q} \quad (9)$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq \underline{t} - \overline{\theta}\,\underline{q} \quad (10)$$

The condition for accepting a menu is to fulfill the following participation constraints:

$$\underline{t} - \underline{\theta}\,\underline{q} \geqq 0 \tag{11}$$

$$\overline{t} - \overline{\theta}\,\overline{q} \geqq 0 \tag{12}$$

Asymmetric information adds more constraints on the allocation of resources and a menu of contract is incentive feasible if it satisfy the incentive compatibility constraints (1.9) and (1.10) and the participation ones (1.11) and (1.12). If we add (1.9) and (1.10) we get

$\underline{t} - \underline{\theta}\,\underline{q} + \overline{t} - \overline{\theta}\,\overline{q} \geqq \overline{t} - \underline{\theta}\overline{q} + \underline{t} - \overline{\theta}\,\underline{q}$ and $-\underline{\theta}\,\underline{q} - \overline{\theta}\,\overline{q} \geqq -\underline{\theta}\overline{q} + -\overline{\theta}\underline{q}$ and

$(\overline{\theta} - \underline{\theta}\,)\,\underline{q} \geqq (\overline{\theta} - \underline{\theta}\,)\overline{q}$ and, naturally:

$$\underline{q} \geqq \overline{q} \tag{13}$$

We have developed this well-known result, because it will be useful in the following of our argument, (1.13) is an enforcement condition.

It is also well known that under a complete information system, the level of information rent is null, that means that if $\underline{U}$ and $\overline{U}$ are the utility level respectively the efficient and the inefficient agent, then, $\underline{U}^* = \underline{t}^* - \underline{\theta}\,\underline{q}^* = 0$ and $\overline{U}^* = \overline{t}^* - \overline{\theta}\overline{q}^* = 0$. That means that agents cannot benefit from extra-rents extracted under asymmetric information. This is no longer the case under asymmetric information. The efficient agent will get

$$\underline{U} = \overline{t} - \underline{\theta}\overline{q} = \overline{t} - \overline{\theta}\,\overline{q} + \Delta\theta\overline{q} = \overline{U} + \Delta\theta\overline{q}, \tag{14}$$

if he decides to mimic the inefficient agent. This relationship means that the information rent is carried out by the informational gain of the agent over the principal.

The information rent of each category of agent will be denoted as:

$$\underline{U} = \underline{t} - \underline{\theta}\,\underline{q} \tag{15}$$

$$\overline{U} = \overline{t} - \overline{\theta}\,\overline{q} \tag{16}$$

These writings (1.15) and (1.16) will be used in the following of the argument.

3. The Principal's choice under ambiguity

We consider now that the principal has beliefs represented by neo-capacities as defined above in the background of the study. Neo-capacities play a similar role to a subjective probability in the expected theory of utility.

a) the Choquet utility expectation function of the principal

Coming back to considerations about uncertainty, we define the utility function of the principal as a Choquet expected utility the structure of which writes as ever mentioned as the weighted sum of the maximum utility, the minimum utility, and the expected utility:

$$V_p(a) = \lambda M(a) + \gamma m(a) + (1-\gamma-\lambda)E_p(a)$$

In appendix 1 we recall the conditions that lead to build neo-additive capacities and adaptated Choquet integrals. To apply this to our model, we have to consider the space of states $\Psi = \{\underline{\theta}, \overline{\theta}\}$ to which is associated the skillness of the agents Consequently, under asymmetric information the outcome space will be:

$X = \{M(a), m(a)\}$. Then, in what follows (by abuse), $X = \{M(q, \underline{\theta}), m(q, \overline{\theta})\}$

$$M(q, \underline{\theta}) = S(\underline{q}) - \underline{t} \tag{17}$$

$$m(\overline{\theta}) = S(\overline{q}) - \overline{t} \tag{18}$$

The regulator's beliefs about the nature of the agent are given by a neoadditive capacity μ based on the prior probability distribution $p = \{\vartheta(\underline{\theta}), \vartheta(\overline{\theta})\}$, where $\vartheta(\overline{\theta}) = 1 - \vartheta(\underline{\theta})$ and in the following $\vartheta(\overline{\theta}) = 1 - \vartheta$ and $\vartheta(\underline{\theta}) = \vartheta$.

$$\mu(X) = \lambda M(\underline{q}) + \gamma m(\overline{q}) + (1-\gamma-\lambda)p(X) \tag{19}$$

Where λ, γ are real numbers such that $0 \leq \gamma \leq 1$ and $0 \leq \lambda \leq 1-\gamma$. Hence, we say that the regulator is optimistic with $\gamma = 0$ (and with $\lambda = 1$ extremely optimistic) . That means that he gives lesser weight to the probability of revealing the inefficient agent and he is pessimistic whenever $\lambda = 0$ (extremely pessimistic with $\gamma = 1$). When $\lambda = \gamma = 0$ the capacity is additive and this correspond to the expected utility function. We define then the Choquet expected utility of the regulator as

$$V_p(X) = \lambda M(\underline{q}) + \gamma m(\overline{q}) + (1-\gamma-\lambda)E_p(X) \tag{20}$$

Or still,

$$V_p(X) = \lambda\left(S(\underline{q}) - \underline{t}\right) + \gamma\left(S(\overline{q}) - \overline{t}\right) + (1-\gamma-\lambda)\Big(\vartheta\left(S(\underline{q}) - \underline{t}\right) + (1-\vartheta)\left(S(\overline{q}) - \overline{t}\right)\Big) \tag{20'}$$

We can check that with neo-additive capacity, the Choquet utility is the weighted average of the minimum, the maximum and the mean pay-offs. We quote Eichberger, Kelsey and Schipper (2005, p. 6) "*Intuitively a neo-additive capacity describes a situation in which the individual believes the likelihood of events is described by the additive probability*

measure : However (s)he lacks confidence in this belief. In part (s)he reacts to this in an optimistic way measured by λ and in part the reaction is pessimistic, measured by γ ."

b) The program of the principal

We can define then the program of the principal that for heuristic reason we write in extenso:

$$Max_{\{(\underline{q},\underline{t}),(\overline{q},\overline{t})\}}\{\lambda\left(S\left(\underline{q}\right)-\underline{t}\right)+\gamma(S(\overline{q})-\overline{t})+(1-\gamma-\lambda)\left(\vartheta\left(S\left(\underline{q}\right)-\underline{t}\right)+(1-\vartheta)(S(\overline{q})-\overline{t})\right)\} \quad (21)$$

This under the incentive and participation constraints (9) to (12).

Using the same change of variable as Laffont and Martimort(2002) in (15) and (16) (i.e. $\underline{U}=\underline{t}-\underline{\theta}\,\underline{q}$ and $\overline{U}=\overline{t}-\overline{\theta}\,\overline{q}$), the program writes now:

$$\underset{\{(\underline{q},\underline{U}),(\overline{q},\overline{U})\}}{\text{Max}}\{V_p(X)\}=$$

$$\underset{\{(\underline{q},\underline{U}),(\overline{q},\overline{U})\}}{\text{Max}}\{\lambda\left(S\left(\underline{q}\right)-\underline{\theta}\,\underline{q}\right)+\gamma(S(\overline{q})-\overline{\theta}\,\overline{q})++(1-\gamma-\lambda)\left(\vartheta\left(S\left(\underline{q}\right)-\underline{\theta}\,\underline{q}\right)+(1-\vartheta)(S(\overline{q})-\overline{\theta}\,\overline{q})\right)-\overline{U}\left(\gamma+(1-\gamma-\lambda)((1-\vartheta))\right)-\underline{U}(\lambda+(1-\gamma-\lambda)\vartheta)\} \quad (22)$$

Under the modified set of constraints:

$$\underline{U}\geqq\overline{U}+\Delta\theta\overline{q} \quad (23)$$

$$\overline{U}\geqq\underline{U}-\Delta\theta\underline{q} \quad (24)$$

Under the participation constraints:

$$\underline{U}\geqq 0, \quad (25)$$

$$\overline{U}\geqq 0 \quad (26)$$

In order to solve this program, as practiced by Laffont and Martimort (2002), we focus on the reduction of constraints due to the fact that the ability of the $\underline{\theta}$-agent to mimic the inefficient involves that the participation constraint (25) is always strictly satisfied. Then, as (30) is fulfilled and combining it with (23) involves that (25) is always strictly verified. Indeed, when a set of contracts allows an inefficient agent to reach his minimum utility level ($\overline{U}=0$), consequently the efficient agent reaches a higher level strictly. The constraint(24) is irrelevant: the inefficient agent cannot claim that he is efficient when obviously he is not. Hence, only two constraints are relevant (23) and (26) and both are binding. Hence, we get:

$$\underline{U}=\Delta\theta\overline{q} \quad (27)$$

And,

$$\overline{U}=0 \quad (28)$$

Replacing these values in the program, we get a reduced form:

$$\text{Max}_{\{\underline{q},\overline{q}\}}\Big\{\lambda\left(S\left(\underline{q}\right)-\underline{\theta}\,\underline{q}\right)+\gamma\big(S(\overline{q})-\overline{\theta}\,\overline{q}\big)+(1-\gamma-\lambda)\Big(\vartheta\left(S\left(\underline{q}\right)-\underline{\theta}\,\underline{q}\right)+(1-\vartheta)\big(S(\overline{q})-\overline{\theta}\,\overline{q}\big)\Big)-(\lambda+(1-\gamma-\lambda)\vartheta)\,\Delta\theta\overline{q}\Big\} \quad (29)$$

Hence, looking for the first order conditions considering $\underline{q}$ and $\overline{q}$ gives:

$$\frac{\partial V_p(X)}{\partial \underline{q}}=0\Rightarrow S'\left(\underline{q}^{SB}\right)=\underline{\theta} \quad (30)$$

And,

$$\frac{\partial V_p(X)}{\partial \overline{q}}=0\Rightarrow S'\big(\overline{q}^{SB}\big)=\overline{\theta}+\Delta\theta\left(-1+\frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) \quad (31)$$

We can see that if $\lambda=0$ and $\gamma=0$, then,

$$S'\big(\overline{q}^{SB}\big)=\overline{\theta}+\Delta\theta\,\frac{\vartheta}{1-\vartheta} \quad (31')$$

This last expression corresponds to the solution where the principal is governed by a Savage expected utility function (SEU-Principal in the following). Rewriting the above equation as : $(1-\vartheta)(S'\big(\overline{q}^{SB}\big)-\overline{\theta})=\Delta\theta\,\vartheta$ we can see that as Laffont and Mortimort (2002) showed it, in this state of nature, the increasing of the inefficient agent's output by an infinitesimal amount *dq* increases allocative efficiency.

The SEU-principal's expected payoff improves by a term equal to the left-hand side of the equation times *dq*. Simultaneously, the extremely small change in output also increases the efficient agent's information rent and this diminishes the principal's expected payoff by a term equal to the right-hand side above times *dq*. At this level are equated the expected marginal efficiency cost and the expected marginal cost of the rent brought about by an infinitesimal change of the inefficient type's output. Under this result, it is clear that there is a tradeoff between rent extraction and efficiency. However, with a principal that feels ambiguity, if this result goes on to be correct, it involves an increment of the efficient rent at a higher level than expected in the standard case.

This is the object of proposition 1 :

Proposition 1: *Under asymmetric information between the principal and the agent, with a principal governed by a Choquet Expected Utility function, when costs are linear, the optimal menu of contracts entails:*

i) As for a SUE-Principal, no output distortion for the efficient agent with respect to the first-best, $\underline{q}^{SB}=\underline{q}^{*}$. *Conversely to the SEU case, concerning the inefficient type, there is an upward distortion,* $\overline{q}^{SB}\geq\overline{q}^{*}$ *with* $S'\big(\overline{q}^{SB}\big)=\overline{\theta}+\Delta\theta\left(-1+\frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)$ *where* $\left(-1+\frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)<0$.

However, $\underline{q}^* \geq \overline{q}^{SB} \geq \overline{q}^*$.

ii) *The efficient agent gets a positive information rent corresponding to:*

$$\overline{U}^{SB} = \Delta\theta\overline{q}^{SB} \quad (32)$$

iii) *The second-best transfers corresponds respectively to(for the efficient agent):*

$$\underline{t}^{SB} = \underline{\theta}\,\underline{q}^* + \Delta\theta\overline{q}^{SB} \quad (33)$$

And, for the inefficient one:

$$\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB} \quad (34)$$

(Proof in appendix 2).

The similarity of this proposition with the equivalent one in the SEU case is only an appearance. Without doubt, it is a noticeable result that, under a Choquet Expected Utility, as for the SEU case, one can verify the lack of production distorsion regarding the efficient agent. Indeed, i) of proposition 1 shows that $\underline{q}^{SB} = \underline{q}^*$. This result was not a priori intuitive. Things change considerably with respect to the inefficient agent. Here, the level of production supplied by this agent is not only higher than the agent of a SEU-principal, but, furthermore, the second best production level is such that, $\overline{q}^{SB} > \overline{q}^*$, that is to say, it is higher than under the perfect information case.

This state of matter has consequences on the level of transfers that the principal offers in each state of nature. Indeed, considering these transfers (in (33) and (34)), it appears that they are higher than for the complete information case, respectively: ($\underline{U}^* = \underline{t}^* - \underline{\theta}\,\underline{q}^* = 0$ and $\overline{U}^* = \overline{t}^* - \overline{\theta}\overline{q}^* = 0$). This establishes our proposition 2:

Proposition 2: *Under asymmetric information between the principal and the agent, with a principal gifted with a Choquet Expected Utility function, when costs are linear,then the level of transfers that the principal has to supply, taking into account the information rent of the efficient agent is higher than under the complete information case.*

Proof: The proposition is drawn from the above argument.

The question here, even if important, is not to compare this result with the situation of a SEU-principal. Indeed, once accepted the methodological change in utility functions, the question to relate the performances of both scheme is of few interest once showed that results are different. Much more interesting is to see here whether the principal is ready to accept the situation of having to pay more in uncertainty than under complete information. Putting the terms as such is somewhat absurd because one may answer that the agents cannot choose the kind of universe they are living in… In fact, the effective question is to know if the principal could act in such a way that he has not to pay such an amount. To give an answer to this question we will examine the cases of pure optimism and pessimism.

Proposition 3: *Under asymmetric information between the principal and the agent, with a principal gifted with a Choquet Expected Utility function, when costs are linear, and when the principal is either fully optimistic* ($\gamma = 0$) *or fully pessimistic* ($\lambda = 0$), *then:*

i) *the level of quantity supplied by the less efficient agent is lesser than the one offered under complete information.*

ii) *In the fully optimistic case, the second-best transfers corresponds respectively to(for the efficient agent):*

$$\underline{t}^{opt} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{opt} \quad (35)$$

And, for the inefficient one:

$$\overline{t}^{opt} = \overline{\theta}\overline{q}^{opt} \quad (36)$$

(*Where* $\underline{q}^{op}$ *and* $\overline{q}^{op}$ *correspond to the second best solutions when the principal feels fully optimistic*),

iii) *In the fully pessimistic case, the second-best transfers corresponds respectively to(for the efficient agent):*

$$\underline{t}^{psm} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{psm} \quad (37)$$

And, for the inefficient one:

$$\overline{t}^{psm} = \overline{\theta}\overline{q}^{psm} \quad (38)$$

(*Where* $\underline{q}^{psm}$ *and* $\overline{q}^{psm}$ *correspond to the second best solutions when the principal feels fully pessimistic*).

Proof of proposition 3 in appendix 2.

Proposition 3 establishes that if the principal forms extreme beliefs such as an absolute optimism or pessimism ($\gamma = 0$) or ($\lambda = 0$), then, under a second best situation, the production distorsion will be of the same order than for the situation where Principal is a SEU-Principal. Hence, this production level is lower than his first best level (complete information case):

$$\overline{q}^{psm} < \overline{q}^{*} \text{ or } \overline{q}^{op} < \overline{q}^{*}.$$

This situation is for the principal highly preferable compared to the case in which ($\gamma \neq 0$) and ($\lambda \neq 0$). Indeed, that means that when beliefs are extreme, the level of transfers is less than when they are not. The difference expresses the gain that would feel the principal if he could switch from the situation of moderated beliefs to the extreme one. Consequently from (33) and (34) and from proposition 3 ii) and iii), we deduce (for the fully optimistic case) :

$$\underline{t}^{SB} - \underline{t}^{opt} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{SB} - \left(\underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{opt}\right) = \Delta\theta\left(\overline{q}^{SB} - \overline{q}^{opt}\right) > 0 \quad (38)$$

And, for the inefficient one:

$$\overline{t}^{SB} - \overline{t}^{SB} = \overline{\theta}\,(\,\overline{q}^{SB} - \overline{q}^{\,opt}) \quad (39)$$

(the argument is the same for the pessimistic case).

That means that, when he has to define a contract for the efficient agent, if the principal cannot change his beliefs when he feels not fully optimistic or pessimistic, he will incur more costly transfers compared to the fully optimistic or pessimistic situations. If, however, the principal could change its state of mind and move towards these "extreme cases", then he could incur less transfer costs. This could be done, if possible, by waiting for more information. Before going further we deduce from proposition 3, the proposition 4 that states that:

Proposition 4 : *When* $\gamma > \lambda$, *then* $S'(\overline{q}^{op}) > S'(\overline{q}^{psm})$ and $\overline{q}^{op} < \overline{q}^{psm}$ *and the reverse for* $\gamma < \lambda$.

This proposition means that the ratio of pessimism to optimism is fundamental to determine what situation will bring more advantage. This result is important for the following.

4. The principal's choice under irreversibility possibility

The situation described above is very similar to the one where an economic agent has to make choices under uncertainty with irreversible consequences. This agent has to take into account the opportunity costs linked to the impulsive decision to choose immediately rather than to wait better chance in the future. Here irreversibility lies in the fact that the principal can incur higher transfers cost if he proposes an immediate contract rather than to wait to reach a better information structure. Hence, implicitly, we admit that the principal has the opportunity to wait further information and to choose the right moment to propose a contract. This moment can be the situation under which he becomes either fully optimistic or fully pessimistic knowing that in these situations the level of transfers will minimize the level of information rent of the efficient agent. Compared to the "usual" irreversible choice literature, the difference lies in the fact that reducing uncertainty means becoming either fully optimistic or fully pessimistic. The arrival of new information changes the probability of states of nature on a Bayesian basis (Epstein (1980)) and, thus, this process modifies the neoadditive capacities and the question is then the convergence process towards either one situation (full optimism) or the other one (full pessimism). To study the full implication of such a process deserves to write a full article about it and this is not the goal followed here. We limit ourselves to underline the specificity and the condition for convergence.

Reaching this point, we can only sketch the information arrival process, which is not very far from the theory of choices facing irreversibility. To give an illustration rather than a full discussion, which would require too much space, we come back to our earlier example

about the location of an activity. We recall that the principal knows only the proportion of skilled and unskilled workers in the A area, i.e. respectively $\underline{p}$ and $\overline{p} = 1 - \underline{p}, (1 \geq \underline{p} \geq 0)$ this proportion. This ratio will be the prior probability on which he forms beliefs.

Let be λ, γ respectively the level of optimism and pessimism with as before $1 \geq \gamma \geq 0$ and $1 - \gamma \geq \lambda \geq 0$. Then the Choquet expected utility of the principal is then:

$$ChE(S) = \lambda \underline{S} + \gamma \overline{S} + (1 - \lambda - \gamma)\left(\underline{p}\underline{S} - \left(1 - \underline{p}\right)\overline{S}\right) \quad (40)$$

To deal with this question we operate a variable change where:

$$\lambda = \alpha\beta, \gamma = \alpha(1 - \beta) \text{ and, naturally, } (1 - \lambda - \gamma) = 1 - \alpha \quad (41)$$

Then,

$$ChE(S) = \alpha\beta \underline{S} + \alpha(1 - \beta)\overline{S} + (1 - \alpha)\left(\underline{p}\underline{S} - \left(1 - \underline{p}\right)\overline{S}\right) \quad (42)$$

This writing is conforming to the publication Chateauneuf and alii (2005) that in their early writing used (40) rather than (42). The meaning of these writings improves the understanding of pessimism and optimism which are represented respectively by $(1 - \beta)$ and β. Then, α represents the preference for ambiguity. The higher α, the lesser will be the weight of the linear expectation in the principal opinion. Then, $(1 - \alpha)$ represents the aversion for ambiguity (see Teitelbaum (2007) for a more complete analysis).

We can consider that $ChE(S)$ is the expected gain for a given initial level of information in present time $k = 0$, let be φ_0 this level we rewrite it as $ChE(S|\varphi_0)$.

Considering the second best optimum level we get the following level of transfers from (33) and (34), $\underline{t}^{SB} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{SB}$ and $\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB}$.

Let us assume that the principal has the opportunity waiting a second period (or stage) for getting new information. By assumption, with this information arrival, he can become either fully optimistic $\beta = 1$ or fully pessimistic $\beta = 0$. Its level of aversion/preference for ambiguity can stay identical (fixed α). At stage 2, the information will be either positive φ^+, that means that he becomes fully optimistic, or negative φ^- (fully pessimistic), (let us note that the terms "positive" or "negative" are only conventional). Then the set of information in stage 1, is then:

$$\alpha\beta \underline{S} + \alpha(1 - \beta)\overline{S} + (1 - \alpha)\left(\underline{p}\underline{S} - \left(1 - \underline{p}\right)\overline{S}\right) \quad (43)$$

$\varphi = \{\varphi^-, \varphi^+\}$ and let be $\beta(\varphi)$ the level of the optimism ratio after the coming of new information with:

$$\beta(\varphi) = \begin{cases} 1 \text{ if } \varphi = \varphi^+ \\ 0 \text{ if } \varphi = \varphi^- \end{cases} \quad (44)$$

Then the expected Choquet utility for stage 1, after the arrival of new information is then:

$$ChE(S|\varphi) = \alpha\beta(\varphi)\,\underline{S} + \alpha(1-\beta(\varphi))\,\overline{S} + (1-\alpha)\left(\underline{pS} - \left(1-\underline{p}\right)\overline{S}\right) \quad (45)$$

For $\varphi = \{\varphi^-, \varphi^+\}$

Obviously from proposition 3 ii), for the efficient agent $\underline{t}^{SB} > \underline{t}^{opt}$ and $\underline{t}^{SB} > \underline{t}^{psm}$ and, the inefficient one: $\overline{t}^{SB} > \overline{t}^{opt}$ and $\overline{t}^{SB} > \overline{t}^{psm}$. As a consequence, without any further assumption, the problem would be very simple, because, as we know from proposition 2 and 3, here, it is more profitable to wait because in this case, by proposition 3, $\mathrm{ChE}(S|\varphi) > ChE(S|\varphi_0)$ for $\varphi = \{\varphi^-, \varphi^+\}$. Hence, whatever the relationship between α and β taking into account the change of variable (41), it is better for the principal to wait further information. Things change if the information becomes costly. Let $c, c > 0$, the cost of new information, this increases the transfer costs and makes the information waiting process more risky. For instance, by assumption we could have a situation in which:

$$ChE(S|\varphi^+) > ChE(S|\varphi_0) > ChE(S|\varphi^-) \quad (46)$$

Where :

$$ChE(S|\varphi) = \alpha\beta(\varphi)\,\underline{S}(c) + \alpha(1-\beta(\varphi))\,\overline{S}(c) + (1-\alpha)\left(\underline{pS}(c) - \left(1-\underline{p}\right)\overline{S}(c)\right) \quad (47)$$

For $\varphi = \{\varphi^-, \varphi^+\}$

The problem is transforming in a classical problem of option value and as such it will not be dealt here. Indeed, we give an over-simplified model drawn from Fisher and Arrow (1974), Fisher and Peterson (1976) or still Henry (1974) and Freeman (1985). The object is to know whether a positive quasi-option may be defined from the waiting of new information. Here, this information is supposed costly because we gave a sure alternative about the convergence of beliefs about a full optimistic situation or full pessimistic situation. In fact, the number of stages could be longer knowing that convergence towards these polar situations is not certain. The gathering of information could be organized through a Cremer and Khalil (1992) (also Crémer, Khalil and Rochet (1998)) process where the principal finds it profitable to organize competition between several agents, even though he has monopoly power and can push a single agent down to his reservation utility.

5. Conclusion

In the specific context of basic agency relationships as described in the paper, taking into account “true uncertainty” and the concomitant aversion/preference for ambiguity leads

to two kinds of results. Concerning the first one, as expected, compared to situations that consider probability additive utility functions, ambiguity theory changes somehow the standard results. Hence, in view of the second best solutions, if one can check no production distortion about the quantity supplied by the efficient agent (equal to the first best level), nevertheless, the level of transfers and the information rent is highly increased. This is due to the fact that the transfers due to the inefficient agent are higher than the one supplied under the complete information case. This increases the level of information rent. This change in results induces a change in the behavior of the principal. This matter of fact induces the second kind of results.

Indeed, it has been put into evidence that when the principal feels "extremely" pessimistic or optimistic, that means that there is no balance between optimism and pessimism, then the classical result are found again notwithstanding their size-order compared to a SEU-principal. As a consequence, if possible, the principal can minimize the level of transfer if by a way or another one, he could feel "extreme" beliefs. This could be made by getting new information. Hence, under our scheme, a principal endowed with a Choquet expected utility has the possibility to propose an immediate contract to the agent as in the standard scheme, or to wait news in the aim at excluding one feeling (optimism for instance) to keep exclusively the other one (pessimism here). If he decides to offer a delegation contract then, he knows that the level of transfer is kept in an acceptable range compared to the complete information situation. The postponement of the contract formation will depend upon the cost of information, its rate of arrival, the subjective probability of staying at the same level of uncertainty, etc.

As a conclusion, the introduction of ambiguity does not invalidate the relevancy of asymmetric information theory in the context that has been described here. It leads to consider the possibility of irreversible choices and writes it in the field of the literature dedicated to the theory of options and quasi-options because it becomes obvious that the principal has to take into account the opportunity costs of waiting. The above results have been reached for the simplest model with linear cost, further research will extend to more complicated situations.

Bibliography

Allais M., 1953. " *Le comportement de l'Homme rationnel devant le risque : critique des postulats de l'École américaine* ", *Econometrica*, 21: 503-546.

Arrow, Kenneth J., and Fisher, Anthony C., 1974. "Environmental Preservation, Uncertainty, and Irreversibility," *Quarterly Journal of Economics*, 88, May: 312-319.

Camerer, C., 1995. "Individual Decision Making. Pp. 587 – 703 in The Hand-book of Experimental Economics , edited by John. H. Hagel and Alvin. E.Roth. Princeton, N.J.: Princeton University Press.

Camerer, C., and Weber M., 1992. Recent Developments in Modeling Preferences: Uncertainty and Ambiguity. Journal of Risk and Uncertainty 5: pp.325 – 70.

Chateauneuf, A. Eichberger J.and Grant S. 2007, "Choice under uncertainty with the best and worst in mind: Neo-additive capacities", *Journal of Economic Theory*, 2007, vol. 137, issue 1, pages 538-567.

Cicchetti, Charles J. and Freeman, A. Myrick, III. 1971. "Option Demand and Consumer Surplus: Further Comment.1," *Quarterly Journal of Economics*, 85 (Aug): 528-539.

Crémer, J. and F. Khalil, 1992. "Gathering Information before Signing a Contract," *European Economic Review*, 38: 675-682.

Crémer, J., F. Khalil and J.C. Rochet, (1998), \Strategic Information Gathering before a Contracts is Offered," *Journal of Economic Theory*, 81,163-200.

Ellsberg, D., 1961. "Risk, Ambiguity, and the Savage Axioms". *Quarterly Journal of Economics* 75: 643 – 69.

Eichberger, J., Kelsey, D. and Schipper B. C. 2005, "Ambiguity and social interaction", Discussion paper n°59, *Department of Economics, University of California.*

Eichberger, J. Kelsey, D. Schipper B. C. . 2009, "Ambiguity and social interaction", *Oxford Economic Papers*, Volume 61, Issue 2, pp. 355-379.

Freeman, A.M., 1985. "Supply Uncertainty, Option Price and Option Value," Land Economics, 6, 176-181.

Fisher, Anthony C. and Peterson, F.M. 1976. "The Environment and Economics: A Survey." Journal of Economic Literature, 14(March): 1-33.

Fisher, Anthony C. and Krutilla, John V. 1974. "Valuing Long Run Ecological Consequences and Irreversibilities", *Journal of Environmental Economics and Management*, 1: 96-108.

Gilboa, I., 1987. Expected Utility with Purely Subjective Non-additive Prob-abilities. Journal of Mathematical Economics 16:65 – 88.

Gilboa, I., and Schmeidler D., 1989. "Maxmin Expected Utility with a Non-unique Prior", Journal of Mathematical Economics, 18:141 – 53

Gonzalez, R., and Wu. George 1999. On the Shape of the Probability Weighting Function. Cognitive Psychology 38:129 – 66.

Johansson, Per-O., 1987. The Economic Theory and Measurement of Environmental Benefits, Cambridge Press

Henry, Claude. 1974. "Option-Values in the Economics of Irreplaceable Assets," *Review of Economic Studies:* Symposium on The Economics of Exhaustible Resources, pp.89-104.

Kahneman, Daniel, and Amos Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. Econometrica 47:263-91

Knight, Frank H., 1921. *Risk, Uncertainty and Profit*. Boston: Hougthon Mifflin.

Laffont, J. J. and D. Martimort, 2002. The Theory of Incentives: The Principal-Agent Model Princeton University Press.

Prelec, D. (1998) "The Probability Weighting Function", Econometrica, Vol. 66, p. 497-527.

Savage, Leonard J. 1954. The Foundations of Statistics . New York: John Wiley.

Schmalensee, R. 1972. "Option Demand and Consumer's Surplus: Valuing Price Changes Under Uncertainty." American Economic Review, 62(Dec): 813-824

Teitelbaum J., 2007. "A unilateral accident model under ambiguity", *Journal of Legal Studies*, 36: 431-477.

Tversky, A., and Fox, C. R., 1995. Weighing risk and uncertainty", Psychological Review,102: 269–283.

Tversky, A. and Kahneman, D., 1986. "Rational choice and the framing of decisions". Journal of Business,59,S251–S278.

Tversky, A. and Kahneman, D., 1992. "Advances in prospect theory: Cumulative representations of uncertainty", *Journal of Risk and Uncertainty*,5,297–323.

Tversky, A., and Koehler, D. J., 1994. "Support theory: A nonextensional representation of subjective probability", Psychological Review,101,547–567.

Tversky, A., and Wakker, P. 1995. "Risk attitudes and decision weights", Econometrica,63, 1255–1280.

von Neumann, J. and Morgenstern, O., 1947. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

Weisbrod, B. 1964, "Collective Consumption Services of Individual Consumption Goods," *Quarterly Journal of Economics*, 78 (3):471-477.

Appendix 1 Neo-capacities and ambiguity

The principal forms beliefs that are represented by neo-additive capacities. Before giving an understanding of this concept, it is necessary to define the notion of capacity that plays a similar role to subjective probabilities under Savage uncertainty.

A capacity is an extension of a probability, and as such is a real valued function $\tau(.)$ that assigns a real value to the set of events $\mathcal{E}$, $\tau: \mathcal{E} \rightarrow \mathbb{R}$, where $\mathcal{E}$ is the σ-algebra of events from the finite set of states of nature A (which could be assumed to be infinite too in the general case), (with $a \in A, \{a\} \in \mathcal{E}$). $\tau(.)$ to be considered as a capacity has to fulfill two conditions

i) For all $E, F \in \mathcal{E}$, and $E \subseteq F$, then $\tau(E) \subseteq \tau(F)$ as monotonicity condition and,

ii) As normalization conditions, $\tau(\emptyset) = 0$ and $\tau(A) = 1$.

Convexity of a capacity is verified by the following relationships:

$\tau(E \cup F) \geq \tau(E) + \tau(F) - \tau(E \cap F)$ (and concave in the opposite situation).

Choquet integrals are used to integrate capacities. To do that is considered a simple function f (where simple means of finite range such that $f: A \rightarrow \mathbb{R}$, f is $\mathcal{E}$-mesurable) that takes values $\mu_1 \geq \mu_2 \ldots \geq \mu_n$.

Hence, Choquet integral of a simple function f with respect to a capacity $\tau(.)$ is defined as:

$$V(f/\tau) = \int f \, d\tau = \sum_{\mu \in f(A)} \mu[\tau(\{s/f \geq \mu\}) - \tau(\{s/f > \mu\})] \quad (16)$$

The neo-additive capacity is a special kind of capacity and it is called as such because it is additive on "non-extreme" outcomes (Neo) as defined by Chateauneuf Grant …(2007). These authors consider that a neo-capacity and its Choquet integral are a particular parametrization of the Choquet Expected Utility. "Neo-additive capacities may be viewed as a convex combination of an additive capacity and a special capacity that only distinguishes between whether an event is impossible, possible or certain" (CEG p.540.

To build the neo-additive capacity they consider that the set of events $\mathcal{E}$ is partitioned into three subsets :

i) The set of null events $\mathcal{N}$, where $\emptyset \in \mathcal{N}$ and for $G \subset H$, and $G \in \mathcal{N}$ if $H \in \mathcal{N}$.

ii) The set of "universal events" $\mathcal{W}$, in which an event is certain to occur, (complement of each member of the set $\mathcal{N}$), $\mathcal{W} = \{E \in \mathcal{E}: A - E \in \mathcal{N}\}$

iii) The set of essential events, $\mathcal{E}^*$, in which events are neither impossible nor certain. This set is composed of the following: , $\mathcal{E}^* = \mathcal{E} - (\mathcal{N} \cup \mathcal{W})$.

We can define now the neo-additive capacity:

Definition (Chateauneuf and alii (2007)

A neo additive capacity $\rho(.)$ is defined as a linear combination of

i) An additive belief $p(.)$, that corresponds to a finite additive probability where $p(E) = 0$ if $E \in \mathcal{N}$ and $p(B) = 1$ if $B \in \mathcal{W}$,

ii) A non-additive belief τ^1 where $\tau^1(E) = 0$ if $E \in \mathcal{N}$ and 1 otherwise,

iii) A non-additive belief τ^0 where $\tau^0(E) = 1$ if $E \in \mathcal{W}$ and 0 otherwise,

Then, for $\gamma \geq 0$ and $\lambda \geq 0$ such that $\gamma + \lambda \leq 1$, a neo-capacity writes as:

$$\rho(E) = \gamma \, \tau^0(E) + \lambda \, \tau^1(E) + (1 - \gamma - \lambda) p(E)$$

This for all $E \in \mathcal{E}$.

Then, we can define the Choquet integral of a neo-capacity as a weighted sum of the minimum, the maximum and the expectation of a simple function $f: \mathcal{E} \rightarrow \mathbb{R}$ such that

$z = inf\,(f)$ if $z = f^{-1}(x: x \geq z) \in \mathcal{W}$ and for $y > z, z = f^{-1}(x: x > y) \notin \mathcal{W}$ and a similar argument is developed for $z = \sup(f)$ if $z = f^{-1}(x: x \geq z) \in \mathcal{N}$ and for $y > z, z = f^{-1}(x: x > y) \notin \mathcal{N}$. Hence, we draw the following relationship:

$$V(f\,/\,p, \lambda, \gamma) = \lambda . \sup(f) + \gamma \inf(f) + (1 - \gamma - \lambda) E_p(f) \quad (.)$$

Where $E_p(f)$ is the expected value of the welfare function taking into account the type of agents, and from the linearity of the Choquet integral with respect to the capacity, $V(f / \nu_0(.)) = \inf(f)$ and $V(f / \nu_1(.)) = \sup(f)$, (proof see CFG(2002, 3).

From the definition of the Choquet integral, we can derive the following results:

i) If $\gamma, \lambda = 0$, then $\rho_{\gamma,\lambda=0}(E) = E_p(f)$ (the expected utility),

ii) $\lambda = 0$, then $\rho_{\lambda=0}(E) = \gamma \inf(f) + (1-\gamma)E_p(f)$ (pure pessimism),

iii) $\gamma = 0$, then $\rho_{\gamma=0}(E) = \lambda \sup(f) + (1-\lambda)E_p(f)$ (pure optimism),

iv) $\gamma + \lambda = 1$, $\rho_{\gamma+\lambda=1}(E) = \lambda.\sup(f) + \gamma \inf(f)$ (Hurwitz criteria).

All that means that the Choquet integral of a neo-additive utility may be represented as a weighted average of the expected utility, the maximum utility and the minimum one. That means that neo-additive capacities allow to take into consideration situatios in which the probability of events is described by the additive probability distribution $p(E)$, however the agent lacks confidence in this belief and he may feel either optimistic, (measured by λ), or pessimistic (measured by γ).

Appendix 2: Proof of the propositions

Proposition 1: *Under asymmetric information between the principal and the agent, with a principal gifted with a Choquet Expected Utility function, when costs are linear, the optimal menu of contracts entails:*

i) *As for a SUE principal, no output distortion for the efficient agent with respect to the first-best,* $\underline{q}^{SB} = \underline{q}^{*}$. *Conversely to the SEU case, concerning the inefficient type, there is an upward distortion,* $\overline{q}^{SB} > \overline{q}^{*}$ *with* $S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\underline{\vartheta}}\right)$ *where* $\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\underline{\vartheta}}\right) < 0$. *However,* $\underline{q}^{*} \geq \overline{q}^{SB} \geq \overline{q}^{*}$.

ii) *The efficient agent gets a positive information rent corresponding to:* $\overline{U}^{SB} = \Delta\theta\overline{q}^{SB}$

iii) *The second-best transfers corresponds respectively to:* $\underline{t}^{SB} = \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{SB}$ *for the efficient agent and* $\overline{t}^{SB} = \overline{\theta}\overline{q}^{SB}$

Proof:

This proof requires several steps:

Proof of i)

- *Concerning the efficient agent*, considering $S'\left(\underline{q}^{SB}\right) = \underline{\theta}$, we verify immediately that $S'\left(\underline{q}^{SB}\right) = S'\left(\underline{q}^{*}\right) = \underline{\theta}$ then $\underline{q}^{SB} = \underline{q}^{*}$.
- *Concerning the inefficient type*, from, $S(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right)$ we can verify that that:

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) \leq \overline{\theta}.$$

To show this point, let us assume that: $\overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) > \overline{\theta}$. Developing this expression, recalling that: $\overline{\theta} - \underline{\theta} = \Delta\theta$, we get: $\frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta} > 1$ or still, $\vartheta < \frac{\lambda}{(-1+\gamma+\lambda)}$.

Let us notice that $\frac{\lambda}{(-1+\gamma+\lambda)} < 0$ because $(\gamma + \lambda) \leq 1$ then $(-1 + \gamma + \lambda) < 0$. However, ϑ is a probability and $\vartheta \in [0,1]$ and the above relationship is impossible. Then,

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1 - \lambda + (-1 + \gamma + \lambda)\vartheta}\right) \leq \overline{\theta}$$

That means, because by assumption that $S'(q)$ is a decreasing function ($S''(q) \leq 0$), that $S'(\overline{q}^{SB}) \leq S'(\overline{q}^{*})$, then $\overline{q}^{SB} \geq \overline{q}^{*}$.

- It remains to show that $\underline{q}^{*} \geq \overline{q}^{SB}$

  We apply the same argument than previously and we examine the conditions for which here

$$S'(\overline{q}^{SB}) = \overline{\theta} + \Delta\theta\left(-1 + \frac{1}{1 - \lambda + (-1 + \gamma + \lambda)\vartheta}\right) < \underline{\theta}$$

  Developing this expression, recalling that: $\overline{\theta} - \underline{\theta} = \Delta\theta$, leads to study:

$\Delta\theta + \Delta\theta\left(-1 + \frac{1}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) < 0$, or,

$$\left(\frac{\Delta\theta}{1-\lambda+(-1+\gamma+\lambda)\vartheta}\right) < 0$$

As the numerator is always positive, we have to study the conditions for a negative denominator: $1-\lambda+(-1+\gamma+\lambda)\vartheta < 0$. Putting it otherwise, the condition becomes:

$$\frac{(1-\lambda)}{(1-(\gamma+\lambda)} < \vartheta$$

Obviously this cannot hold, because $(1-(\gamma+\lambda) < (1-\lambda)$ that involves that

$$\frac{(1-\lambda)}{(1-(\gamma+\lambda)} > 1$$

This is contradictory with the fact that ϑ is a probability then,

$$\frac{(1-\lambda)}{(1-(\gamma+\lambda)} \geq \vartheta$$

That means that $S'(\overline{q}^{SB}) \geq \underline{\theta}$, and consequently, $\underline{q}^* \geq \overline{q}^{SB}$, QED.

Proofs of ii) and iii):

Calculus are made on the definition of information rents and the level of constraints in a similar mode achieved by Laffont Martimort (2002) chap.2 section 2.

Proposition 3: *Under asymmetric information between the principal and the agent, with a principal gifted with a Choquet Expected Utility function, when costs are linear, and when the principal is either fully optimistic* ($\gamma = 0$) *or fully pessimistic* ($\lambda = 0$), *then:*

i) *the level of quantity supplied by the less efficient agent is lesser than the one offered under complete information.*

ii) *In the fully optimistic case, the second-best transfers corresponds respectively to(for the efficient agent):*

$$\underline{t}^{opt} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{opt} \quad (35)$$

And, for the inefficient one:

$$\overline{t}^{opt} = \overline{\theta}\overline{q}^{opt} \quad (36)$$

(*Where* $\underline{q}^{op}$ *and* $\overline{q}^{op}$ *correspond to the second best solutions when the principal feels fully optimistic*),

iii) *In the fully pessimistic case, the second-best transfers corresponds respectively to(for the efficient agent):*

$$\underline{t}^{psm} = \underline{\theta}\underline{q}^* + \Delta\theta\overline{q}^{psm} \quad (37)$$

And, for the inefficient one:

$$\overline{t}^{psm} = \overline{\theta}\overline{q}^{psm} \quad (38)$$

(*Where* $\underline{q}^{psm}$ *and* $\overline{q}^{psm}$ *correspond to the second best solutions when the principal feels fully pessimistic*).

Proof of proposition 3

Concerning point i) we proceed in two steps. The first one analyzes the situation in which the principal feels fully or purely optimistic and the second one in which he is pessimistic.

a) Pure optimism

A Principal purely optimistic means that $\gamma = 0$ (then, because $\lambda \leq 1 - \gamma$, λ can take values between 0 and 1). We keep the same program, constraints and simplification over constraints as before and we get:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\Big\{\lambda\ \Big(S\left(\underline{q}\right) - \underline{\theta}\ \underline{q}\Big) + (1-\lambda)\Big(\vartheta\Big(S\left(\underline{q}\right) - \ \underline{\theta}\ \underline{q}\ \Big) + (1-\vartheta)\big(S(\overline{q}) - \overline{\theta}\ \overline{q}\big)\Big) - (\lambda + (1-\lambda)\vartheta)\ \Delta\theta\overline{q}\Big\}$$

Looking for second best solutions under the assumption of a pure optimistic principal leads to the following results. Hence, the first order conditions considering $\underline{q}$ and $\overline{q}$ give:

$$\frac{\partial V_p(X)}{\partial \underline{q}} = 0 \Rightarrow S'\left(\underline{q}^{op}\right) = \underline{\theta}$$

And,

$$\frac{\partial V_p(X)}{\partial \overline{q}} = 0 \Rightarrow S'\left(\overline{q}^{op}\right) = \overline{\theta} + \frac{v}{(1-v)(1-\lambda)}\Delta\theta + \frac{\lambda}{1-\lambda}\Delta\theta.$$

Where $\underline{q}^{op}$ and $\overline{q}^{op}$ correspond to the second best solutions when the principal feels fully optimistic. Naturally concerning the efficient agent we can see that, as in the general case there is no distortion in production and$S'\left(\underline{q}^{op}\right) = \underline{\theta}$, and as a consequence, $\underline{q}^{op} = \underline{q}^{*}$.

We can check immediately that, $\frac{v\Delta\theta}{(1-v)(1-\lambda)}\Delta\theta > 0$ and $\frac{\lambda}{1-\lambda}\Delta\theta$ and, as a consequence $\overline{q}^{op} < \overline{q}^{*}$ because $S'\left(\overline{q}^{op}\right) > S'\left(\overline{q}^{*}\right) = \overline{\theta} + \frac{v\Delta\theta}{1-v}\Delta\theta$. The first term of the proposition 3 is proved. This result means that under ambiguity and pure optimism the level of informational distortion considering the inefficient agent is higher than under the lack of ambiguity of the standard result. The above relationship means that when the principal feels fully optimistic, he expects that the inefficient agent is more inefficient than under SEU.

b) Pure pessimism

We proceed alike as above and we consider that the principal is now absolutely pessimistic $(\lambda = 0)$.The objective function becomes then:

$$\underset{\{\underline{q},\overline{q}\}}{Max}\Big\{\gamma\big(\ S(\overline{q}) - \overline{\theta}\ \overline{q}\big) + (1-\gamma)\Big(\underline{\vartheta}\Big(S\left(\underline{q}\right) - \ \underline{\theta}\ \underline{q}\ \Big) + \big(1-\underline{\vartheta}\big)\big(S(\overline{q}) - \overline{\theta}\ \overline{q}\big)\Big) - \Big((1-\gamma)\underline{\vartheta}\Big)\ \Delta\theta\overline{q}\Big\}$$

Looking for second best solutions under the assumption of a pure optimistic principal leads to the following results. The first order conditions considering $\underline{q}$ and $\overline{q}$ give:

$$\frac{\partial V_p(X)}{\partial \underline{q}} = 0 \Rightarrow S'\left(\underline{q}^{psm}\right) = \underline{\theta}$$

And ,

$$\frac{\partial V_p(X)}{\partial \overline{q}} = 0 \Rightarrow S'\left(\overline{q}^{psm}\right) = \overline{\theta} + \frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)}$$

Where $\underline{q}^{psm}$ and $\overline{q}^{psm}$ correspond to the second best solutions when the principal feels fully pessimistic. As previously and the general case, concerning the efficient agent there is no

distorsion in production and $S'\left(\underline{q}^{psm}\right) = \underline{\theta}$, and as a consequence, $\underline{q}^{psm} = \underline{q}^{*}$. However, concerning the inefficient agent, distorsion exists and we have to check if $\frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)} > 0$. It appears immediately that naturally, $\vartheta(1-\gamma)\Delta\theta > 0$, then then to verify the relationships, we must have $1 + \underline{\vartheta}(-1+\gamma) > 0$. Let us assume that $1 + \underline{\vartheta}(-1+\gamma) \leq 0$. That means that $1 \leq (1-\gamma)\,\vartheta$ which is a contradiction because $0 < \gamma < 1$ and $0 < \vartheta < 1$. Hence, $\frac{\underline{\vartheta}(1-\gamma)\Delta\theta}{1+\underline{\vartheta}(-1+\gamma)} > 0$ and, consequently

$S'(\overline{q}^{psm}) > S'(\overline{q}^{*})$ and $\overline{q}^{psm} < \overline{q}^{*}$. The second term of the proposition is then proved.

ii) Concerning the proof of ii) and iii) we proceed in a similar way as for point iii) of proposition 1.

Proposition 4 : When $\gamma > \lambda$, then $S'(\overline{q}^{op}) > S'(\overline{q}^{psm})$ and $\overline{q}^{op} < \overline{q}^{psm}$ and the reverse for $\gamma < \lambda$.

Proof in appendix 2:

Let us assume that $S'(\overline{q}^{op}) > S'(\overline{q}^{psm})$ or

$$\overline{\theta} + \frac{v}{(1-v)(1-\lambda)}\Delta\theta + \frac{\lambda}{1-\lambda}\Delta\theta > \overline{\theta} + \frac{\vartheta(1-\gamma)\Delta\theta}{1+\vartheta(-1+\gamma)}$$

After simplification we get:

$$v > -\frac{\lambda}{\gamma-\lambda}$$

This is true for $\gamma > \lambda$. When, $\gamma < \lambda$, then $S'(\overline{q}^{op}) < S'(\overline{q}^{psm})$ QED.

DOCUMENTS DE TRAVAIL GREDEG PARUS EN 2012
GREDEG Working Papers Released in 2012

2012-01 Frédéric Marty
Les clauses environnementales dans les marchés publics : perspectives économiques

2012-02 Christophe Charlier
Distrust and Barriers to International Trade in Food Products: An Analysis of the US-Poultry Dispute

2012-03 Gérard Mondello
The Equivalence of Strict Liability and Negligence Rule: A Trompe-l'Œil Perspective

2012-04 Agnès Festré & Pierre Garrouste
Somebody May Scold You! A Dictator Experiment

2012-05 Dorian Jullien & Nicolas Vallois
A Probabilistic Ghost in the Experimental Machine

2012-06 Gérard Mondello
Ambiguity, Agency Relationships, and Adverse Selection

2012-07 Flora Bellone, Kozo Kiyota, Toshiyuki Matsuura, Patrick Musso & Lionel Nesta
Comparative Advantage, Trade Costs, and International Productivity Gap: Evidence Built from French and Japanese Firm-level Data

2012-08 Tiziana Assenza & Domenico Delli Gatti
E. Pluribus Unum: Macroeconomic Modelling for Multi-Agent Economies

2012-09 Catherine Guillemineau
Financial Reforms, International Financial Flows, and Growth in Advanced Economies

2012-10 Dino Borie
Social Decision Theory and Non-Strategic Behaviour

2012-11 Edward Lorenz
Social Capital and Enterprise Innovative Performance: A Multi-Level Analysis of Developing Nations

2012-12 Agnès Festré & Pierre Garrouste
The 'Economics of Attention': A New Avenue of Research in Cognitive Economics

2012-13 Harald Hagemann
Capitalist Development, Innovations, Business Cycles and Unemployment: Joseph Alois Schumpeter and Emil Hans Lederer

2012-14 Sophie Pommet
The Survival of Venture Capital Backed Companies: An Analysis of the French Case

2012-15 Thomas Jobert, Fatih Karanfil & Anna Tykhonenko
The Environmental Kuznets Curve Reconsidered from the Perspective of Heterogeneity: Insights for Climate Change and Energy Policy

2012-16 Francesco Quatraro
The Co-Evolution of Knowledge and Economic Structure: Evidence from European Regions